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Criteria for Shell-and-Tube Heat Exchangers According to Part UHX of ASME Section VIII-Division 1

ASME

SETTING THE STANDARD

PTB-7-2014

CRITERIA FOR SHELL-AND-TUBE HEAT EXCHANGERS ACCORDING TO PART UHX OF ASME SECTION VIII DIVISION 1

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ASME
SETTING THE STANDARD

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FOREWORD

The purpose of this document is to justify and provide technical criteria for the rules of Part Unfired Heat Exchanger (UHX) of ASME Section VIII Division 1, 2013 Edition, devoted to the design of U-tube, Fixed and Floating head Tubesheet Heat Exchangers. The criteria document applies also to Paragraph 4.18 of Section VIII, Division 2, 2013 Edition, which is entirely based on Part UHX.

Confirmation and documentation of the basis for UHX-rules is important for the members of the ASME Subgroup on Heat Transfer Equipment to use as a future reference, for confirmation or comparisons of code requirements, and for code development. It will be a valuable reference for both early career and experienced engineers who are using the UHX rules and may become involved in code development of such rules in the future.

The analytical treatment of the fixed tubesheet heat exchangers is based on classical discontinuity analysis methods to determine the moments and forces that the tubesheet, tubes, shell and channel must resist. The treatment provides, at any radius of the perforated tubesheet, the deflection, the rotation, the bending and shear stresses and the axial stress in the tubes. A parametric study permits one to determine the maximum stresses in the tubesheet and in the tubes which are given in UHX-13. The Floating Tubesheet and U-tube Tubesheet heat exchangers are treated as simplified cases of fixed tubesheet heat exchangers. A check of the results obtained is provided by comparing Finite Element Analysis (FEA) results, Tubular Exchanger Manufacturers Association (TEMA) results, and the French pressure vessel code Code Français de Construction des Appareils à Pression (CODAP). Applying the appropriate simplifications, the classical formulas for circular plates subjected to pressure, have been obtained.

The author thanks the members of the peer review committee who sent many valuable comments and provided helpful consulting in the development of this Criteria Document. In particular Ramsey Mahadeen for his support and detailed reviews, Urey Miller for his help in stress classification considerations, Tony Norton for his comments on theoretical issues and performing FEA calculations, Guido Karcher for his support, Anne Chaudouet who spent so much time for checking the development of the formulas and Gabriel Aurioles who supplied the raw Excel spreadsheets and graphs for analysis and was very helpful for computer issues.

The author acknowledges Centre Technique des Industries Mécaniques (CETIM) for its support in the development of the Criteria Document appearing in PART 3, dedicated to fixed tubesheet heat exchangers. The author further acknowledges, with deep appreciation, the activities of ASME ST-LLC and ASME staff and volunteers who have provided valuable technical input, advice and assistance with review and editing of, and commenting on this document.

Established in 1880, the American Society of Mechanical Engineers (ASME) is a professional not-for-profit organization with more than 130,000 members and volunteers promoting the art, science and practice of mechanical and multidisciplinary engineering and allied sciences. ASME develops codes and standards that enhance public safety, and provides lifelong learning and technical exchange opportunities benefiting the engineering and technology community. Visit www.asme.org for more information.

ABBREVIATIONS AND ACRONYMS

ASME	American Society of Mechanical Engineers
CL	Clamped
EEC	Effective Elastic Constants
FEA	Finite Element Analysis
FL	Floating
HEs	Heat Exchanger(s)
LE	Ligament Efficiency
SG-HTE	Subgroup on Heat Transfer Equipment
SS	Simply Supported
ST	Stationary
TEMA	Tubular Exchanger Manufacturers Association
TSs	Tubesheet(s)
UHX	Unfired Heat Exchanger

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PART 1
INTRODUCTION

1 SCOPE

This criteria document covers the development of the tubesheet (TS) design rules for the U-tube, Fixed, and Floating Head TS Heat Exchangers (HE) configurations contained in Part UHX of Section VIII Division 1, 2013 Edition. It applies also to Paragraph 4.18 of Section VIII, Division 2 which is entirely based on Part UHX.

The free body diagram of the HE, the equilibrium and compatibility equations, the solution of resulting differential equations and all intermediate steps are provided to show the derivation of:

- the deflection and the rotation at any radius of the TS,
- the bending and shear stress at any radius of the TS,
- the axial stresses in the tubes at any radius of the TS,
- the axial stretch force acting in the shell,
- the axial displacement of the shell.

The document provides the technical basis of the following items:

- the required loading case combinations,
- the acceptance criteria for each TS configuration, as applicable,
- the TS characteristics including the Effective Elastic Constants,
- the TS extended as a flange.

The following effects are in addition to the above basic items:

- the effect of different shell material or thickness adjacent to the TS,
- the effect of plasticity at TS-shell-channel joint,
- the effect of radial differential thermal expansion between the TS and integral shell and channel,
- the tubesheet calculated as a simply supported TS.

2 HISTORICAL BACKGROUND

The first rules devoted to tubular HEs were developed by TEMA [CS-1] for the first time in 1941 to design U-tube and floating TSs. The design formula was based on the formula for circular plates subject to pressure:

$$T = F\frac{G}{2}\sqrt{\frac{P}{S}}$$

This semi-empirical formula does not account for the tubes that stiffen the TS, and for the holes that weaken it. Fixed TS HEs were covered in TEMA 1968 5th edition, based on Gardner's work.
Gardner [1][2] in 1948 for floating TSs and in 1952 for fixed TSs was the very first to set-up the basis of a more rational approach by taking into consideration the support afforded by the tubes and the weakening effect of the TS holes. This design method, which involves 15 parameters instead of 3 previously in TEMA, was adopted by TEMA in 1968 in its 5th edition and by the Stoomwezen [CS-2] in 1973.

Simultaneously, and independently, Miller [3] proposed a similar approach that was published in the British Code BS 1515 [CS-3] in 1965. These design rules have the drawback of considering the TS as either simply supported or clamped at its periphery, which compels the designer to make a more or less arbitrary choice between these two extreme cases.

Galletly [4] in 1959 overcame this issue by taking into account the degree of rotational restraint of the TS at its periphery by the shell and channel. This method was adopted by the French pressure vessel code Code Français de Construction des Appareils à Pression (CODAP) [CS-4] in 1982 for fixed and floating HEs. CODAP rules were adopted by the European Pressure Vessel Standard EN 13445 [CS-5] published in 2002.

Gardner [5] in 1969 improved his method for U-tube and floating HEs by considering the unperforated solid rim at the periphery of the TS. This design method was adopted by BS 5500 [CS-6], CODAP (for U-tube) and ISO [CS-7] in the late seventies and by ASME Section VIII (Appendix AA) in 1982.
Despite these improvements, TEMA rules have been extensively used throughout the world during the last six decades as they have the merit of long satisfactory industrial experience and simplicity. However due to that simplicity (the strengthening of the tubes is assumed to counterbalance the weakening effect of the tubesheet holes), they often lead to TS over-thickness if the strengthening effect controls or under-thickness if the weakening effect controls. Today these disadvantages increase as the chemical and power industries need larger exchangers operating at higher pressures and temperatures. For more details, see Osweiller [6].

In 1975 ASME Subcommittee VIII established a "Subgroup on Heat-Transfer Equipment (SG-HTE)" with the task of developing new rules for the design of TS HEs based on a more rigorous approach than TEMA. This was achieved by considering the perforated TS, the tubes acting as an elastic foundation, the unperforated rim and the connection of the TS with the shell and channel. The analytical treatment is based on Soler's book [6].

In 1992 ASME and CODAP decided to reconcile their rules (scope, TS configurations, loading cases notations, ligament efficiencies, effective elastic constants, design formulas), as they were based on the same approach. For more details, see Osweiller [7]. The 1st ASME draft was issued in 1985.
From 1992 to 2002 the ASME tubesheet rules have been published in non-mandatory Appendix AA of Section VIII-Div. 1 so that manufacturers can use them as an alternative to TEMA rules.

In 2003 these rules were upgraded to mandatory status and published in the 2003 Addenda as a new Part UHX of Section VIII Div. 1, UHX standing for Unfired Heat Exchanger. As of January 1, 2004 a designer does not have the option of using the TEMA rules if the HE needs to be U-stamped.

UHX design rules cover essentially the design by formula of the heat exchanger pressure containing components. For other aspects such as fabrication, tube-to-tubesheet joints, inspection, maintenance, repair, troubleshooting, etc. see Reference [8]. TEMA [CS-1] also covers similar aspects of design such as minimum component thickness (e.g., baffle plates, pass partitions, etc.), tube vibration, etc.

3 TYPES OF HEAT EXCHANGERS COVERED

UHX rules apply to the three typical types of tubular HEs:

- **U-Tube Heat Exchanger:** HE with one stationary TS attached to the shell and channel. The HE contains a bundle of U-tubes attached to the TS, as shown in Figure 1 sketch (a).
- **Fixed Tubesheet Heat Exchanger:** HE with two stationary TSs, each attached to the shell and channel. The HE contains a bundle of straight tubes connecting both TSs, as shown in Figure 1 sketch (b).
- **Floating Tubesheet Heat Exchanger*:*** HE with one stationary TS attached to the shell and channel, and one floating TS that can move axially. The HE contains a bundle of straight tubes connecting both TSs, as shown in Figure 1 sketch (c).

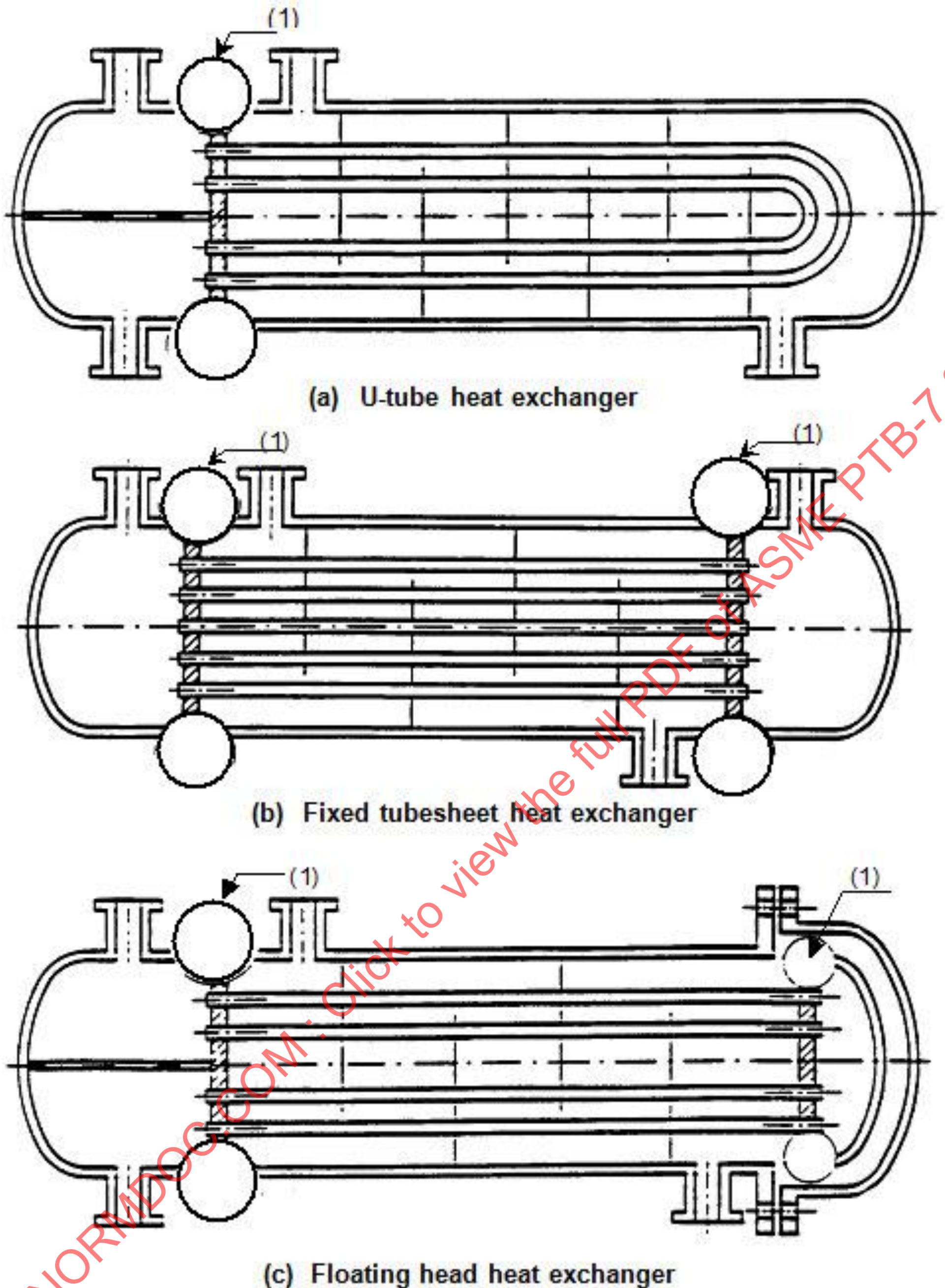


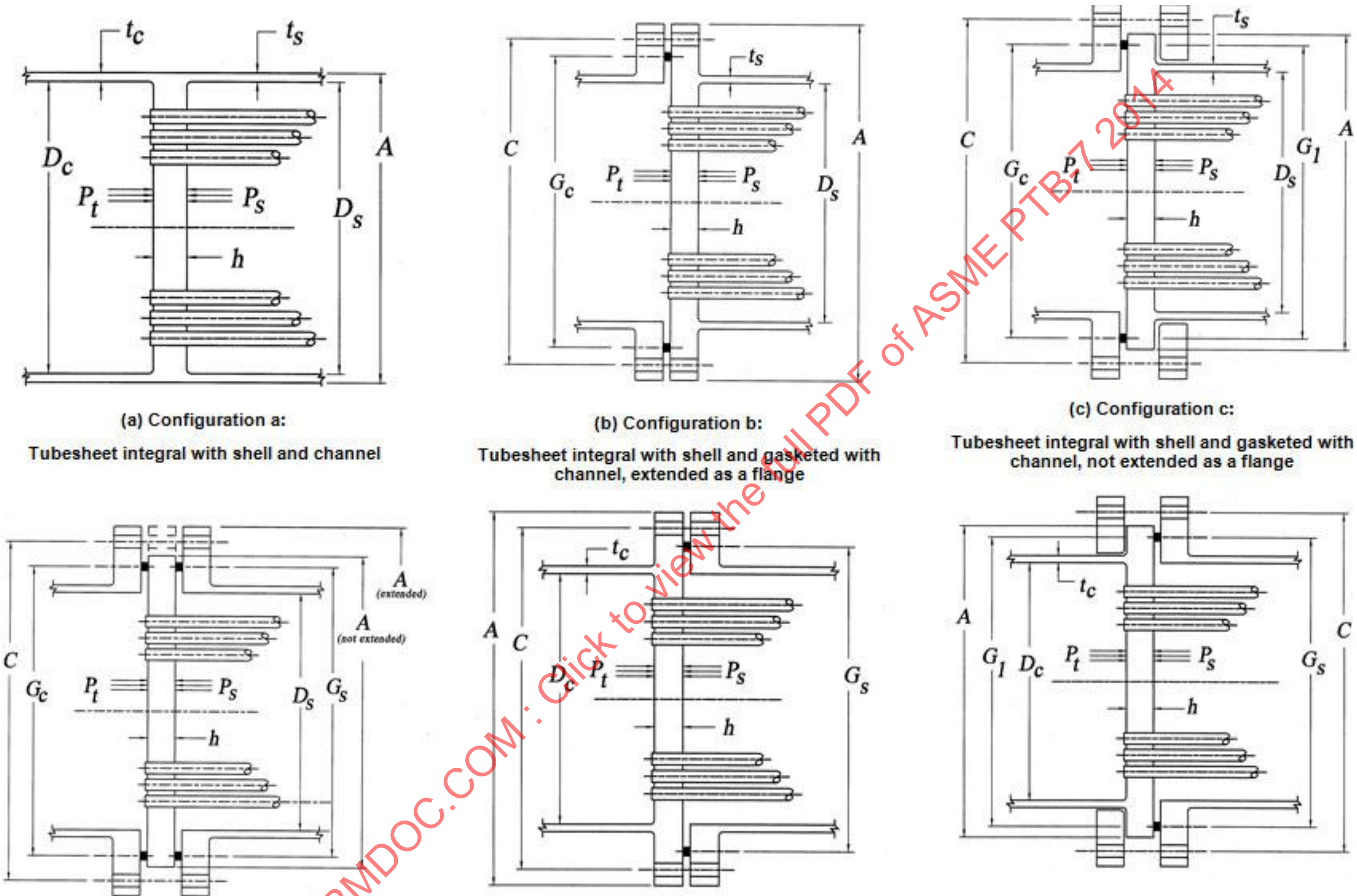
Figure 1 — Three Types of Tubesheet Heat Exchangers [1]

[1] Configurations of tubesheet – shell – channel connections are detailed in 3-2.

4 TYPES OF TS CONFIGURATIONS

The TS is attached to the shell and channel either by welding (integral TS) or by bolting (gasketed TS) according to 6 configurations encountered in the industry (see Figure 2):

- configuration a: tubesheet integral with shell and channel;
- configuration b: tubesheet integral with shell and gasketed with channel, extended as a flange;
- configuration c: tubesheet integral with shell and gasketed with channel, not extended as a flange;
- configuration d: tubesheet gasketed with shell and channel, extended as a flange or not
- configuration e: tubesheet gasketed with shell and integral with channel, extended as a flange;
- configuration f: tubesheet gasketed with shell and integral with channel, not extended as a flange.



(a) Configuration a:
Tubesheet integral with shell and channel

(b) Configuration b:
Tubesheet integral with shell and gasketed with channel, extended as a flange

(c) Configuration c:
Tubesheet integral with shell and gasketed with channel, not extended as a flange

(d) Configuration d:
Tubesheet gasketed with shell and channel

(e) Configuration e:
Tubesheet gasketed with shell and integral with channel, extended as a flange

(f) Configuration f:
Tubesheet gasketed with shell and integral with channel, not extended as a flange

Figure 2 — Tubesheet Configurations

5 LOADING CASES

The normal operating condition of the HE is achieved when the tube side pressure P_t and shell side pressure P_s act simultaneously. However, a loss of pressure or a loss of temperature is always possible. Accordingly, for safety reasons, the designer must always consider the cases where P_s=0 and P_t=0 for the normal operating conditions.

He must also consider the start-up conditions, the shutdown conditions and the upset conditions, if any, which may govern the design.

A TS HE is a statically indeterminate structure for which it is difficult to determine the most severe condition of coincident pressure and temperature. Thus, it is necessary to evaluate all the anticipated loading conditions mentioned above to ensure that the worst load combination has been considered in the design.

For each of these conditions, the following 3 pressure loading cases must be considered.

- Loading Case 1: Tube side pressure P_t acting only (P_s = 0).
- Loading Case 2: Shell side pressure P_s acting only (P_t = 0).
- Loading Case 3: Tube side pressure P_t and shell side pressure P_s acting simultaneously.

For fixed TS HEs, the axial differential thermal expansion between tubes and shell has to be considered and one set of arbitrary thermal loading cases must be added (loading cases 4, 5, 6, 7) as they act simultaneously with the pressure loading cases.

When vacuum exists, each loading case is considered with and without the vacuum.
These loading cases have been traditionally considered in TEMA, French code CODAP, European Standard EN 13445, and earlier editions of UHX.

The 2013 UHX Edition replaces these arbitrary thermal loading cases by actual loading cases accounting for the actual operating pressures and temperatures so that the designer can realistically determine the controlling conditions for each operating loading case considered, including, but not be limited to, normal operating, startup, shutdown, cleaning, and upset conditions. The pressure definitions have been changed to include maximum and minimum design and operating pressures that may be encountered in a particular design. These new loading cases are detailed in Section 3.3 of Part 3.

As the calculation procedure is iterative, a value h is assumed for the tubesheet thickness to calculate and check that the maximum stresses in tubesheet, tubes, shell, and channel are within the maximum permissible stress limits.

Because any increase of tubesheet thickness may lead to over-stress of the tubes, shell, or channel, a final check must be performed, using in the formulas the nominal thickness of tubesheet, tubes, shell, and channel, in both corroded and uncorroded conditions.

6 STRUCTURE OF PART UHX

- UHX-1 to UHX-8 provide general considerations (Scope, Material, Fabrication, Terminology,...) which are common to the three types of HEs.
- UHX-9 provides design rules for the TS flange extension
- UHX-10 (Conditions of Applicability) specifies under which conditions the rules are applicable
- UHX-11 (TS Characteristics) is also common to the three types of HEs and provides the design formula for the ligament efficiency and the effective elastic constants.
- UHX-12, UHX-13 and UHX-14 provide the design rules for U-tube, Fixed and Floating TS HEs.

  These three chapters are self-supporting and structured in the same way:
  (1) Scope
  (2) Conditions of Applicability
  (3) Notations
  (4) Design Considerations
  (5) Calculation Procedure

  Additional rules are provided to cover more specific calculations:
  (1) Effect of different shell material or thickness adjacent to the TS
  (2) Effect of plasticity at the tubesheet-shell-channel joint
  (3) Effect of radial thermal expansion adjacent to the TS
  (4) Calculation of the TS when considered as simply supported
  (5) Calculation of the TS flange extension

- UHX-15 to UHX-19 provide considerations on Tube-to-Tubesheet Welds, Expansion Bellows, Pressure Tests and Marking.
- ASME PTB-4-2013, ASME Section VIII – Division 1 Example Problem Manual (PTB-4), provides design examples for each type of HE.

7 STRUCTURE OF THE DOCUMENT

This document is structured in 6 PARTS.

- PART 1*:* Introduction (purpose, background, general issues)
- PART 2*:* Tubesheet Characteristics (ligament efficiencies, effective elastic constants)
- PART 3*:* Analytical treatment of Fixed TS HEs
- PART 4*:* Analytical treatment of Floating TS HEs
- PART 5*:* Analytical treatment of U-tube TS HEs
- PART 6*:* Conclusions

Each PART is independent with basically the same chapters: Scope, Historical Background, Notations, Configurations covered, Design assumptions, and Analytical treatment.

The order of the analytical treatment is based on the complexity of the HE model. The Fixed TS HE is treated first, because it is the most complex. The Floating TS HE treated second, since it is a simplified case of the Fixed TS. The U-Tube TS HE is treated last, since it is a simplified case of the Floating TS. The detailed structure is given in the Table of Contents.

This document provides the derivation of UHX design rules (UHX-1, UHX-9 to 14, UHX-17 and 20) and refers explicitly to these as necessary. Other UHX rules (UHX-2, 3 and 4, UHX-15 to 19), which are not linked to design formulas are not covered.

8 NOTATIONS

Notations are common to the three (3) types of HEs. They are detailed in PART 3, Section 3.2.

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Codes & Standards (CS)

[CS-1] TEMA: Standards of Tubular Exchangers Manufacturers Association – 9th Edition 2007 – Chapter R7 "TS"

[CS-2] STOOMWEZEN: Dutch Code for Pressure Vessels – 1985 Edition – Sheet D0403 "Wall thickness calculation of TSs"

[CS-3] BS 5515: Unfired Fusion Welded Pressure Vessels – 1965 Edition – Chapter 3.9 "Flat HE TSs"

[CS-4] CODAP: French Code for Unfired pressure Vessels – 2005 Edition – Section C7 "Design rules for HE TSs"

[CS-5] EN 13445: European Standard for Unfired Pressure Vessels – 2002 Edition – Chapter 13 "HE TSs"

[CS-6] BS 5500: Unfired Fusion Welded Pressure Vessels – 1976 Edition – Chapter 3.9 "Flat HE TSs"

[CS-7] ISO DIS 2694: Pressure Vessels – Draft 1973 Edition - Chapter 30 " Flat HE TSs"

PART 2
TUBESHEET CHARACTERISTICS

1 SCOPE (UHX-11.1)

PART 2 provides the technical basis for the determination of:

- the ligament efficiencies μ and μ^*
- the effective elastic constants E^*;

which are given in Section 11 of Part UHX.

These quantities are important as they enable the replacement of the actual perforated tubesheet (TS) by an equivalent solid plate, which is necessary to develop the analytical treatment.

2 NOTATIONS

Notations are taken from UHX-11.3 and are repeated here for convenience.

A_L	=	total area of untubed lanes = $U_{L1} L_{L1} + U_{L2} L_{L2} + \ldots$ limited to $4D_o p$
c_s	=	tubesheet corrosion allowance on the shell side
c_t	=	tubesheet corrosion allowance on the tube side
D_o	=	equivalent diameter of outer tube limit circle (see Figure 6)
d	=	diameter of tube hole
d_t	=	nominal outside diameter of tubes
d^*	=	effective tube hole diameter
E	=	modulus of elasticity for tubesheet material at the tubesheet design temperature
E_{tT}	=	modulus of elasticity for tube material at tubesheet design temperature
E^*	=	effective modulus of elasticity of tubesheet in perforated region
h	=	tubesheet thickness
h_g	=	tube side pass partition groove depth (see Figure 9)
h_g'	=	effective tube side pass partition groove depth
$L_{L1}, L_{L2}, \ldots$	=	length(s) of untubed lane(s) (see Figure 7)
ℓ_{tx}	=	expanded length of tube in tubesheet ($0 \le \ell_{tx} \le h$) (see Figure 8).
p	=	tube pitch
p^*	=	effective tube pitch
r_o	=	radius to outermost tube hole center (see Figure 6)
S	=	allowable stress for tubesheet material at tubesheet design temperature
S_{tT}	=	allowable stress for tube material at tubesheet design temperature
t_t	=	nominal tube wall thickness
$U_{L1}, U_{L2}, \ldots$	=	center-to-center distance between adjacent tube rows of untubed lane(s), limited to $4p$ (see Figure 7)
μ	=	basic ligament efficiency for shear
μ^*	=	effective ligament efficiency for bending
ν^*	=	effective Poisson's ratio in perforated region of the tubesheet
ρ	=	tube expansion depth ratio = ℓ_{tx}/h, $(0 \le \rho \le 1)$

3 DESIGN ASSUMPTIONS (UHX-11.2)

The perforated TS is assumed to be uniformly perforated in a triangular or square pattern.

4 LIGAMENT EFFICIENCIES (UHX-11.5.1)

4.1 Introduction

The TS treatment necessitates the replacement of the actual perforated TS by an equivalent unperforated solid plate of same diameter. The ligament efficiency accounts for the fact that the shear load and the bending moment calculated in this unperforated solid plate must be corrected as they apply only to the ligament located between two adjacent holes in the actual TS.

The ligament is represented by the hatched area in Figure 3. Its length varies from αα'=p-d to δδ'=p.

Its minimum length αα'=p-d leads to a basic ligament efficiency $\mu = \dfrac{p-d}{p} = 1 - \dfrac{d}{p}$

So to on the safe side, this formula is generally used to calculate the shear stress in the actual TS. This means that, for the example pattern (p=1.25 and d=1.0) μ= 0.2 , the shear stress in the actual TS will be 5 times the shear stress calculated in the equivalent solid plate.

Among all the ligaments located at radius r of the TS, one can reasonably assume that at least one of length αα' is radially oriented as shown in Figure 4. Therefore the radial bending moment M(r) calculated in the equivalent TS applies only to the ligament between the two holes, which means that the bending moment in the actual TS must be multiplied by the ratio ωω'/αα' or divided by the minimum bending ligament efficiency:

$$\mu^* = \frac{\alpha\alpha'}{\omega\omega'} = \frac{p-d}{p}$$

Its mean length ββ' leads to a higher ligament efficiency value:

$$\mu^* = \frac{pd - \pi d^2/4}{pd} = 1 - \frac{\pi}{4}\cdot\frac{d}{p} = 1 - 0.785\frac{d}{p}$$

For the example pattern above: μ*=0.372 , which means that the calculated bending moment in the actual TS would be almost half the value of that using the minimum ligament efficiency of 0.2 above. These two extreme examples show the significant impact of the ligament efficiency on the stresses obtained in the actual TS.

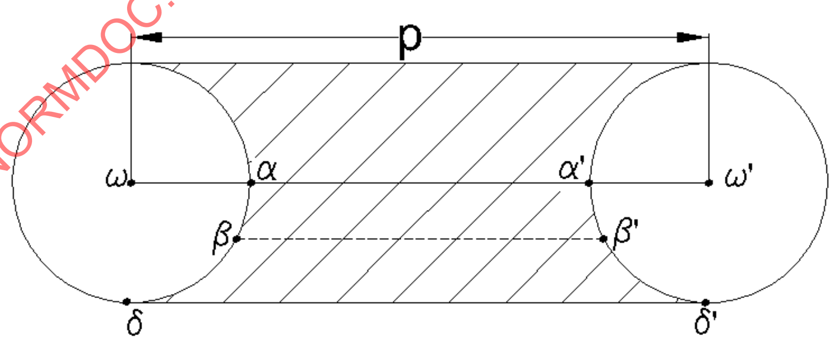


Figure 3 — Ligament Area in the Actual Tubesheet

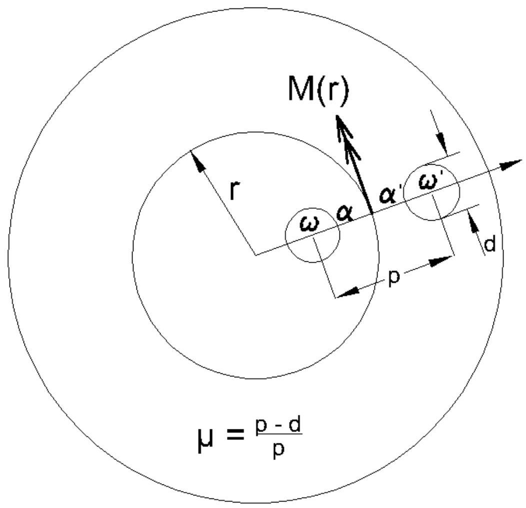


Figure 4 — Ligament Orientation in the Actual Tubesheet

4.2 Historical Background

Various formulas have been used for the determination of the bending LE in codes and standards.

- **BS5500 [CS-1], CODAP [CS-2], EN 13445 [CS-3], ISO [CS-4] and STOOMWEZEN [CS-5]** have used the minimum LE when the tubes are welded only: $\mu^* = \frac{p-d}{p}$.

  When the tubes are expanded throughout the full depth of the TS, experimental tests have shown that about half of the thickness participates to the TS strength, which leads to: $\mu^* = \frac{p-(d-t_t)}{p}$

- **TEMA [CS-6] uses the mean LE** based on the ratio of the hole area, $s_o = \pi d^2/4$, to the portion of TS area "s" pertaining to that hole, as shown in Figure 5: $\mu^* = 1 - \frac{s_o}{s}$ (μ* quoted η in TEMA).This leads to:

  For square pattern: $s = p^2$ and $\mu^* = 1 - \frac{\pi}{4}\left(\frac{d}{p}\right)^2 = 1 - \frac{0.785}{(p/d)^2}$

  For triangular pattern: $s = p^2 \sin(60°)$ and $\mu^* = 1 - \frac{\pi}{4}\frac{d^2}{p^2 sin(60°)} = 1 - \frac{0.907}{(p/d)^2}$

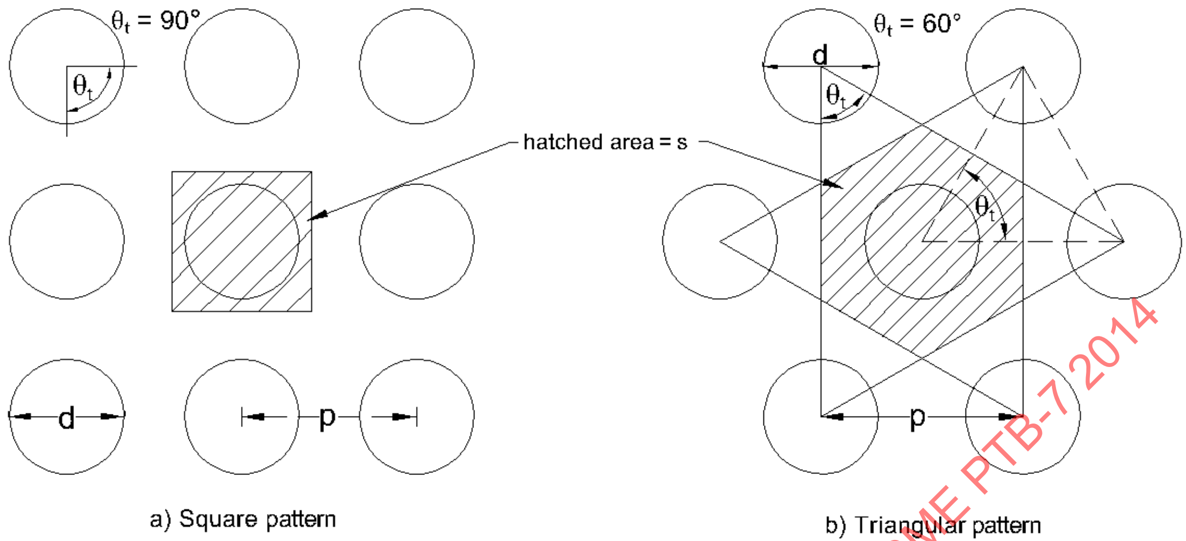


Figure 5 — Ligament Efficiency Used in TEMA

For shear, these codes and standards, use the minimum ligament efficiency: $\mu = \frac{p-d}{p}$

4.3 LE in Part UHX (UHX-11.5.1)

(a) **Equivalent diameter D_o** of the perforated TS is defined as the equivalent diameter of the outer tube limit circle, calculated from the radius r_o of the outermost tube hole center (see Figure 6):

$$\boxed{D_o = 2r_o + d_t}$$

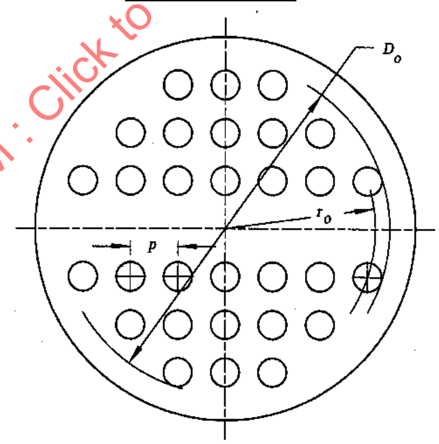


Figure 6 — TS Equivalent Diameter D_o

The diameter D_o corresponds to the similar concept used in TEMA for the equivalent diameter D_L used for the determination of shear stress and D_o and D_L will have about the same values.

(b) **Basic ligament efficiency for shear load**

Once the shear load has been determined in the equivalent TS, it must be corrected to account for the holes in the actual TS, by applying the minimum ligament efficiency μ:

$$\boxed{\mu = \frac{p-d}{p}}$$

(c) **Effective ligament efficiency for bending moment**

Once the bending moment has been determined in the equivalent TS, it must be corrected to account for the holes in the actual TS, by applying the bending effective ligament efficiency μ* defined as follows:

$$\mu^* = \frac{p^* - d^*}{p^*}$$

(d) **Effective pitch p***

- If the TS is uniformly perforated p*=p
- If the TS has an unperforated lane of area $A_L=U_L L_L$ as shown in Figure 7(a), the N_t tubes are redistributed so that the equivalent TS is uniformly perforated over the area $\pi \frac{D_o^2}{4}$ with an equivalent pitch p*. This is necessary as the analytical treatment is performed for a uniform array of tubes.

  Assuming that the portion of TS area pertaining to each hole is p^2, p* is obtained from the equation:

$$N_t p^{*2} \pi \frac{D_o^2}{4} = N_t p^2 \left(\pi \frac{D_o^2}{4} - A_L\right)$$

  which leads to:

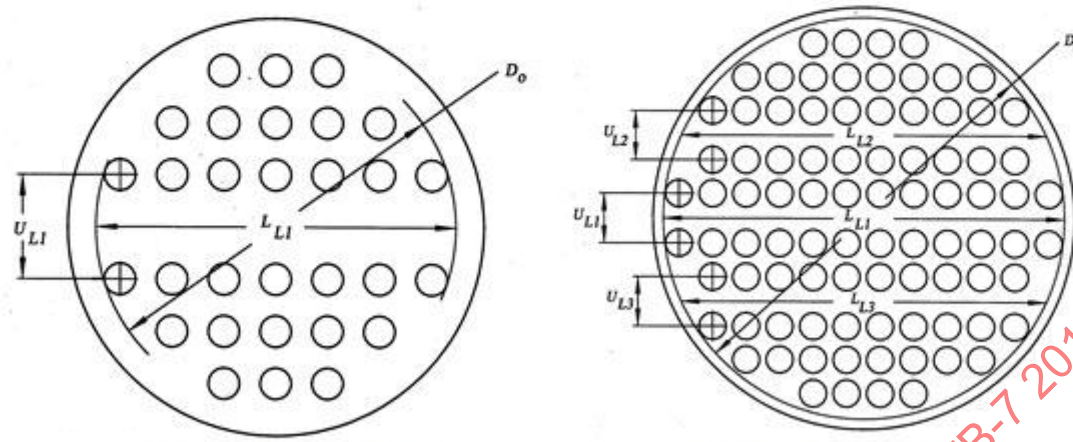
$$p^* = \frac{p}{\sqrt{1 - \frac{4A_L}{\pi D_o^2}}}$$

  So as not to deviate too much from the assumption of uniform pattern, the width U_L of the untubed lane in Figure 7(a) is limited to 4p over the diameter D_o, i.e. an area $4pD_o$, which leads to: $A_L \leq 4pD_o$. This condition of applicability appears explicitly in the definition of A_L in UHX-11.3. It has been set-up after discussions among ASME and CODAP experts, based on sound engineering practice, and does not have a theoretical basis.

  If there are several untubed lanes of widths U_{L1}, U_{L2}, U_{L3},... and lengths L_{L1}, L_{L2}, L_{L3},... as shown in Figure 7(b), the limit is still $4pD_o$. Accordingly, p* becomes:

$$p^* = \frac{p}{\sqrt{1 - \frac{4MIN\left[(A_L),(4pD_o)\right]}{\pi D_o^2}}}$$

  If there is no untubed lane, $A_L=0$ and p*=p.



(a) one unperforated lane $A_L = U_{L1} L_{L1}$

(b) several unperforated lanes $A_L = U_{L1} L_{L1} + U_{L2} L_{L2} + U_{L3} L_{L3}$

Figure 7 — TS with Unperforated Lanes

(e) **Effective tube hole diameter d***

- When the tubes are welded only: $d^*=d_t$
- When the tubes are expanded into the TS, feedback from HE manufacturers has shown that the bending strength of the TS is increased by the degree of tube expansion ρ (see Figure 8).

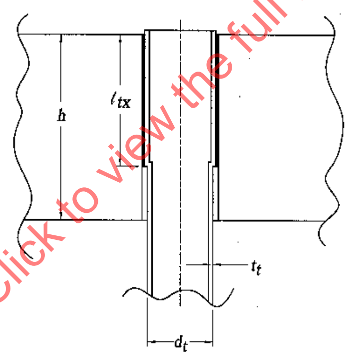


Figure 8 — Tube Expansion Depth Ratio $\rho=l_{t,x}/h$

The degree of increased strength is also dependent on the difference between the TS and tube material properties. Finally, the effective tube hole diameter is written:

$$d^* = d_t - 2t_t\left(\frac{E_{tT}}{E}\right)\left(\frac{S_{tT}}{S}\right)\rho$$

where ρ is the tube expansion ratio: $\rho = l_{t,x}/h$. This formula was proposed for the first time in the 1980 Edition of ASME Section VIII-Division 2.

- If the tubes are welded only: $\rho=0$ and $d^*=d_t$
- If the tubes are fully expanded: $\rho=1$ and $d^*=d_t-2t_t$ if the TS and tubes are made of the same materials.

The effective diameter cannot be less than the inside tube diameter d_t-2t_t, which leads to:

$$d^* = \text{MAX}\left\{\left[d_t - 2t_t\left(\frac{E_{tT}}{E}\right)\left(\frac{S_{tT}}{S}\right)\rho\right],\left[d_t - 2t_t\right]\right\}$$

Values of μ* are generally comprised between 0.25 and 0.4.

To illustrate the difference between the TEMA method and the Part UHX method for determining the bending ligament efficiency, consider a tube (d_t=1.0, t_t=0.0625, p=1.25) that is the same material as the tubesheet.

- If the tube is expanded throughout the full depth of the tubesheet, then $\mu^* = 0.304$
- If the tube is welded and not expanded at all, then $\mu^* = 0.20$.

However, the TEMA ligament efficiency is 0.420 and 0.498 for triangular and square pitch layouts respectively, regardless of whether the tubes are expanded or not.

The ligament efficiency has a direct bearing on the calculated tubesheet stress. A smaller ligament efficiency results in a larger predicted tubesheet stress and a larger ligament efficiency results in a smaller predicted tubesheet stress. Thus, as may be seen, if the same basic theory is used to determine the stress in a plate, then the TEMA ligament efficiency would result in a smaller calculated stress as compared to the ASME method, even when the full tube wall is considered. This difference is exacerbated when the tube is not expanded.

(f) **Effective tube side pass partition groove**

When there is no pass partition groove on tube side of a TS of nominal thickness h_n, the TS corroded thickness h_{min} is given by: $h_{min}=h_n-c_s-c_t=h$, as h is the corroded TS thickness obtained by calculation in Part UHX.

When there is a pass partition groove of depth h_g, it is assumed that the bottom of the groove does not corrode.

Two cases are possible:

- If $h_g<c_t$ (see Figure 9(a)): $h_{min}=h_n-c_s-c_t=h$
- If $h_g>c_t$ (see Figure 9(b)), a correction for the groove depth in excess of the tube side corrosion is necessary: $h_{min}=h_n-c_s-c_t-(h_g-c_t)=h-(h_g-c_t)$

In both cases h_{min} can be written: $h_{min}=h-h'_g$ which will be used to calculate the TS bending stress where:

$$\boxed{h'_g = \mathrm{MAX}\left[\left(h_g - c_t\right),\ (0)\right]}$$

The same formula is used in TEMA.

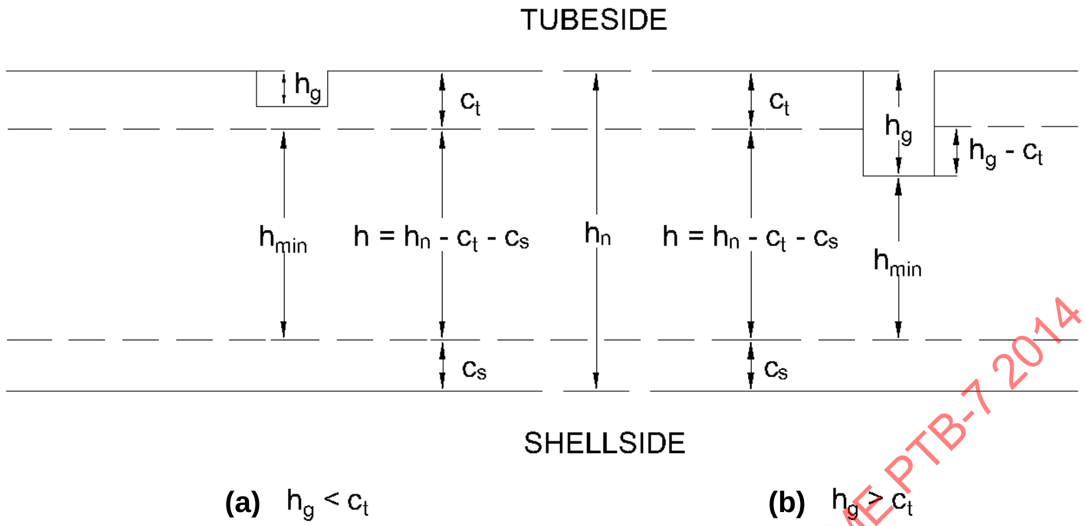


Figure 9 — Pass Partition Groove on Tubeside of the TS

5 EFFECTIVE ELASTIC CONSTANTS (UHX-11.5.2)

5.1 Introduction

The analytical treatment of TS HEs necessitates the replacement of the actual perforated TS Modulus of elasticity E and Poisson's ratio ν by an equivalent solid plate of Modulus of elasticity E* and Poisson's ratio ν*. These EECs E* and ν* are determined so that the equivalent TS has the same mechanical behavior as the actual TS when subjected to the same loading. Due to the weakening effect of the holes, E* is lower than E and E*/E is always lower than 1. Values of ν* may be higher or lower than ν.
The EECs depend on the ligament efficiency, μ, ratio h/p and pattern type (triangular or square). They must be as correctly evaluated as possible:

- If they are underestimated, the calculated TS stresses at the junction with shell and channel will be lower than reality.
- If they are overestimated, the calculated TS stresses close to the center of the TS will be lower than reality.

The consequence is that inaccurate estimates of the EECs result in inaccurate stress results.

5.2 Historical Background

During the last decades many authors (about 60 papers) have proposed experimental and theoretical methods to solve the problem. A detailed review of these works was published in 1989 in a JPVT paper by Osweiller [1]. A short synthesis is provided below.

a) Between 1948 and 1958 several authors (Gardner, Miller, Horvay, Duncan, Salerno and Mahoney, ...) proposed various methods for the determination of EECs. These methods had no sound basis and leading to a great disparity of results, PVRC decided in 1960 to undertake theoretical and experimental investigations in order to determine more accurate values for the EEC.

b) In 1960 Sampson [2] undertook experimental tests on plastic plates using photo-elastic techniques for in-plane and bending loadings.

- For in-plane loading, values of E*/E and ν* are independent of the TS thickness.
- For bending loading, values of E*/E and ν* vary significantly with the TS thickness when h≤2p.

When h>2p this variation is very slow, and as the plates gets thicker, the bending values approach the plane stress values.

It appears that that h=2p is a transition zone between thin and thick perforated plates. These results were confirmed by Leven [3] in 1960.

c) In 1962 O'Donnell & Langer [4] made a synthesis of these results and proposed a curve for in-plane and bending loading that enables the determination of E*/E and ν* as a function of the ligament efficiency for thick plates (h≥2p) perforated with triangular pitch. This curve was adopted by ASME Section III in 1966 and later by Section VIII-Div. 2.

d) In 1963 and later, new theoretical methods were developed on powerful computers to enable the determination of EECs for triangular patterns and square patterns. Square patterns were not previously covered. These methods are based on doubly periodic stress distribution theory induced in an infinite plate evenly perforated in two directions and loaded by in-plane or bending stress. Two techniques have been -used:

- The "direct technique" developed by Meijers [5] in 1969 for thin plates loaded in bending or plane-stress. Grigoljuk and Fil'shtinski [11] obtained the same results using a similar method (see Annex A, Table 4)
  In 1985, Meijers [12] improved his method by proposing for the determination of E*/E and ν*:

  - An asymptotic solution for thin plates in bending (h/p→0) and for thick plates (h/p→∞), consistent with the 1969 results
  - An interpolated solution for the intermediate range (0<h/p→0), substantiated by FEA calculations.

  See Table 1 for triangular pitch.

These values were adopted by many pressure vessel codes (BS 5500, CODAP, EN 13445, ISO, STOOMWEZEN) for treating thin and thick plates loaded in bending.

Table 1 — Values for E*/E and ν∗ for Triangular Pattern from Meijers [12]

μ	h/P	0	0.25	0.5	1	2	4	∞
0.25	E^*_Δ/E	0.348	0.297	0.261	0.230	0.215	0.208	0.205
	ν^*_Δ	0.032	0.171	0.273	0.359	0.399	0.421	0.427
0.33	E^*_Δ/E	0.452	0.410	0.365	0.340	0.319	0.313	0.310
	ν^*_Δ	0.073	0.159	0.250	0.302	0.346	0.357	0.364

- **The "indirect technique"** developed by Bailey and Hicks [6] in 1960 and Slot & O'Donnell [7] in 1971 for thick plates.
  These two techniques led to very good agreement both for triangular and square patterns with discrepancies lower than 0.1 % (see Annex A, Table 4). For more details, see Ref. [1].

e) These theoretical results have been corroborated by many experimental investigations undertaken by Duncan and Upfold [8] in 1963 and O'Donnell [9][10] in 1967 and 1973 and by F.E.M. calculations from Tran-Huu-Hanh in 1971 and Roberts in 1975.

5.3 The Square Pattern Problem

Contrary to the triangular pattern, the square pattern is characterized by an anisotropic behavior which has been enlightened both by theoretical and experimental investigations. Values of EECs E* and ν∗ are different in the pitch direction P and diagonal direction D (See Figure 10). Anisotropy is more marked for low values of ligament efficiency $0.2 \le \mu* \le 0.4$ than for high values approaching 1(which corresponds to an unperforated plate), as shown by Slot and O'Donnell [7] (see Table 2).

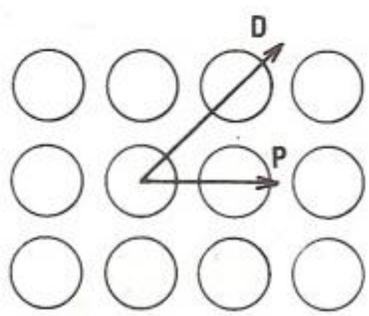


Figure 10 — Pitch and Diagonal Directions for Square Pattern

Table 2 — Values of E*/E and ν* for Square Pattern in Pitch and Diagonal Directions from Slot and O'Donnell [7]

μ	Pitch direction		Diagonal direction		"Isotropic" value	
	$\frac{E_p^*}{E}$	ν_p^*	$\frac{E_d^*}{E}$	ν_d^*	$\frac{E_\square^*}{E}$	$\nu_\square^*$
0.2	0.311	0.122	0.123	0.654	0.235	0.337
0.4	0.525	0.216	0.380	0.433	0.459	0.316
0.6	0.734	0.275	0.681	0.328	0.708	0.301
0.8	0.918	0.297	0.914	0.301	0.916	0.299

This anisotropy has been confirmed theoretically by O'Donnell [9] and experimentally by Bayley & Hicks [6] and O'Donnell [10].

Due to this anisotropy, the equivalent plate cannot be treated with the classical isotropic solution, like in the triangular case. The anisotropic solution should be used.

When the plate is clamped or simply supported, the anisotropic circular plate deflection is given by formulaswhich can be compared to the classical isotropic formulae. From that comparison, equivalent "isotropic" values for E^*_{square} and ν^*_{square} have been determined by O'Donnell [10], which enables the application of the classical isotropic equations to the equivalent solid plate.

$$E^*_{square} = E^*_p \frac{1-\upsilon^*_{square}}{1-\upsilon^*_p} \qquad \upsilon^*_{square} = \frac{4}{\dfrac{3+\upsilon^*_p}{1+\upsilon^*_p} + \dfrac{1-\upsilon^*_d}{1+\upsilon^*_d}} - 1$$

These formulas have been used for the square pattern to calculate the "isotropic" EECs E^*_{square} and ν^*_{square} from the anisotropic values proposed by various authors, as shown in Table 2.

5.4 Synthesis of Results

In 1989, Osweiller [1] made a synthesis of all these experimental and theoretical results for triangular and square patterns which are presented in a graphical form in Figure 55 of Annex A. They give the values of E*/E and ν* as a function of the ligament efficiency μ, for various ratios h/p.

This figure shows that experimental values are available for μ ranging from 0.1 to 0.5, whereas theoretical values cover the full range of μ.

5.5 Determination of EECs for the Full Range of μ* (0.1≤μ*≤1.0)

From this synthesis of results, three ranges of TS thickness were set-up for the determination of E*/E and ν*:

- **for thin plates (h/p≤0.1)** values are taken from theoretical values of Meijers [5][12] and combined with experimental values
- **for thick plates (h/p≥2.0)** values are taken from theoretical values of Slot & O'Donnell [7] and combined with experimental values.
- **for the intermediate range (0.1<h/p<2.0)** values are taken from theoretical values of Meijers [5][12] and combined with experimental values of Sampson [2] and O'Donnell [10].

Numerical values and resulting curves are given respectively in Sections 3 and 2 of Annex B. These curves were initially used in CODAP in 1985 and in Nonmandatory Appendix AA of Section VIII-Div. 1, which later became Part UHX.

Note: Experimental and theoretical values of EECs have been obtained from perforated plates for which the ligament efficiency is μ=(p-d)/p. For TSs, the stiffening effect of the tubes mentioned in Section 4.3(e) IV-3c may also be accounted for in the determination of the EECs. Accordingly, the EECs are determined using the bending ligament efficiency μ=(p*-d*)/p*.*

In 1991 Woody Caldwell, as a member of the ASME SG-HTE, provided polynomial approximations for the entire range of these curves (0≤μ*≤1) by using polynomials of degree 4 for E*/E and of degree 7 for ν∗ (see Section 4 of Annex B).

Section VIII Div.2 has retained these curves for the range 0.05≤μ*≤1.0 for the stress analysis of perforated plates (Annex 5.E of 2013 edition).

5.6 Determination of EECs for the UHX Rules (UHX-11.5.2)

Numerical values obtained from the polynomials developed for Appendix AA polynomials were not accurate enough for curve h/p=2.0 for low values of μ*. The discrepancy was about 6% on E*/E for μ*= 0.15 and could reach 15% for μ*=0.1. This caused significant effects on calculated tubesheet stresses. Accordingly, the precision for these polynomials was improved by Osweiller [13] in 1994 by:

- using more points (35 instead of 21)
- exploring several polynomials degrees (degrees 3, 4 and 5 were tested)
- reducing the range of μ* from 0.1 to 0.6 (0.1≤μ*≤0.6), based on the fact that the ligament efficiency of HEs is always between these two limits.

The precision targeted for these polynomials was 1%, which corresponds to the reading error of the curves. Polynomials of degree 4 were finally retained, both for E*/E and for ν∗, which leads to a discrepancy of about 0.2%, with a maximum of 0.77% for the curve h/p=2.0.

Figure 11 for Triangular patterns and Figure 12 for Square patterns provide the curves and polynomials for calculating the EECs E*/E and ν∗ for the reduced ligament efficiency range 0.1≤μ*≤0.6 and for fixed values of the ratio h/p:

- **for E*/E: h/p= 0.1, 0.25, 0.5, 2.0.**
  Note: For h/p=0.25 no curve was available for E/E for triangular pattern, due to lack of results. This curve has been added by interpolation between the two adjacent curves relative to h/p=0.1 and h/p=0.6.*
- **for ν∗: h/p= 0.1, 0.15, 0.25, 0.5, 1.0, 2.0.**

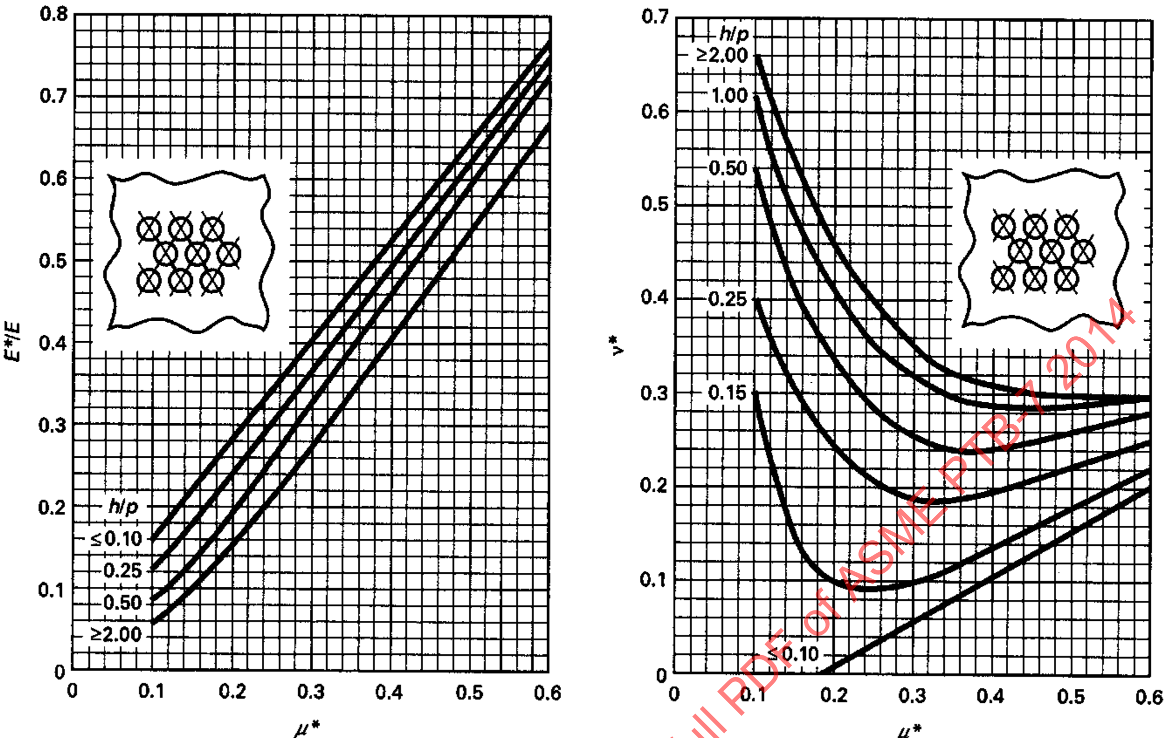
Numerical values are available in Section 3 of Annex B. *Note: The polynomials cannot be used for ligament efficiencies outside the range 0.1≤μ*≤0.6, as shown by* Figure 13. *In such a case E*/E and ν∗ should be determined per Section 5.5 above.*

5.7 Conclusion

The perforated tubesheet plate is now replaced by an equivalent solid plate of:

- diameter D_o determined from Section 4.3(a),
- equivalent modulus of elasticity E* and Poisson's ratio ν∗ determined from Section 5.6.

The unperforated TS rim extends from diameter D_o to diameter D_s with the basic modulus of elasticity E and Poisson's ratio ν.



(a) E^*/E (Equilateral Triangular Pattern)

(b) ν^* (Equilateral Triangular Pattern)

(a) Equilateral Triangular Pattern: $E^*/E = \alpha_0 + \alpha_1\mu^* + \alpha_2\mu^{*2} + \alpha_3\mu^{*3} + \alpha_4\mu^{*4}$

h/p	α_0	α_1	α_2	α_3	α_4
0.10	0.0353	1.2502	−0.0491	0.3604	−0.6100
0.25	0.0135	0.9910	1.0080	−1.0498	0.0184
0.50	0.0054	0.5279	3.0461	−4.3657	1.9435
2.00	−0.0029	0.2126	3.9906	−6.1730	3.4307

(b) Equilateral Triangular Pattern: $\nu^* = \beta_0 + \beta_1\mu^* + \beta_2\mu^{*2} + \beta_3\mu^{*3} + \beta_4\mu^{*4}$

h/p	β_0	β_1	β_2	β_3	β_4
0.10	−0.0958	0.6209	−0.8683	2.1099	−1.6831
0.15	0.8897	−9.0855	36.1435	−59.5425	35.8223
0.25	0.7439	−4.4989	12.5779	−14.2092	5.7822
0.50	0.9100	−4.8901	12.4325	−12.7039	4.4298
1.00	0.9923	−4.8759	12.3572	−13.7214	5.7629
2.0	0.9966	−4.1978	9.0478	−7.9955	2.2398

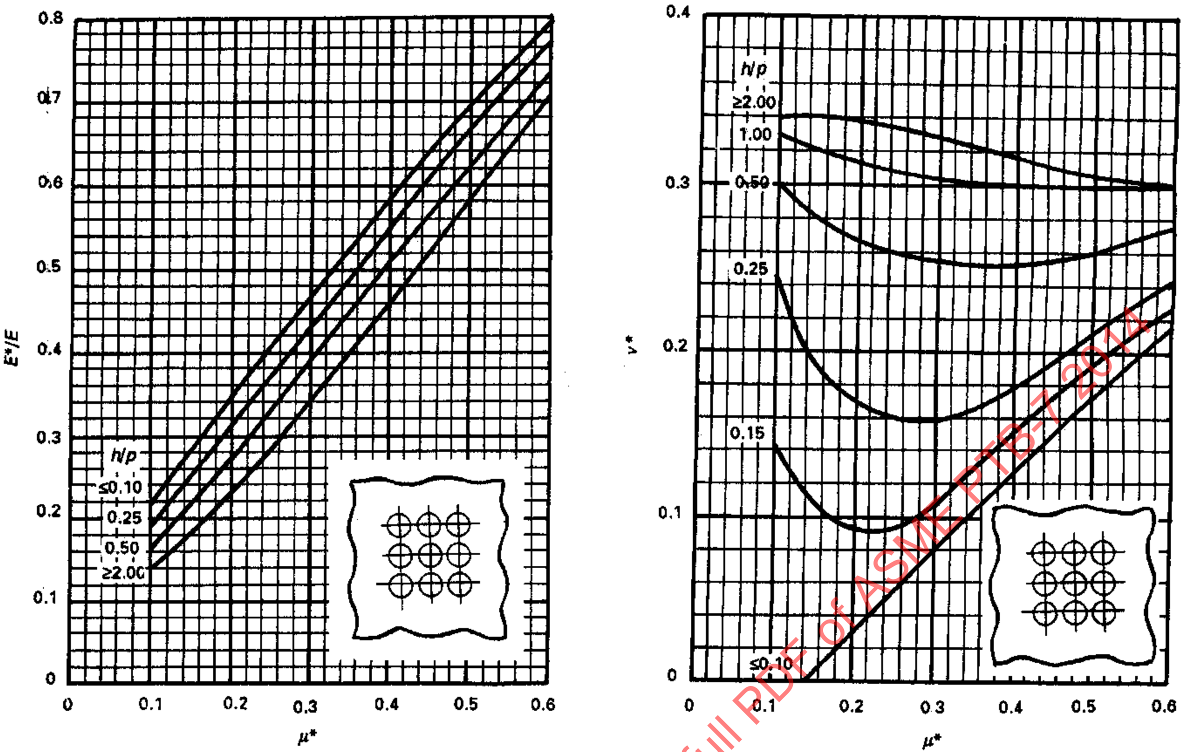
GENERAL NOTES:

(a) The polynomial equations given in the tabular part of this Figure can be used in lieu of the curves.

(b) For both parts (a) and (b) in the tabular part of this Figure, these coefficients are only valid for $0.1 \le \mu^* \le 0.6$.

(c) For both parts (a) and (b) in the tabular part of this Figure: for values of h/p lower than 0.1, use $h/p = 0.1$; for values of h/p higher than 2.0, use $h/p = 2.0$.

Figure 11 — Curves and Tables for the Determination of E*/E and ν* (Triangular Pattern)



(a) E*/E (Square Pattern)

(b) ν* (Square Pattern)

(a) Square Pattern: $E^*/E = \alpha_0 + \alpha_1\mu^* + \alpha_2\mu^{*2} + \alpha_3\mu^{*3} + \alpha_4\mu^{*4}$

h/p	α_0	α_1	α_2	α_3	α_4
0.10	0.0676	1.5756	−1.2119	1.7715	−1.2628
0.25	0.0250	1.9251	−3.5230	6.9830	−5.0017
0.50	0.0394	1.3024	−1.1041	2.8714	−2.3994
2.00	0.0372	1.0314	−0.6402	2.6201	−2.1929

(b) Square Pattern: $\nu^* = \beta_0 + \beta_1\mu^* + \beta_2\mu^{*2} + \beta_3\mu^{*3} + \beta_4\mu^{*4}$

h/p	β_0	β_1	β_2	β_3	β_4
0.10	−0.0791	0.6008	−0.3468	0.4858	−0.3606
0.15	0.3345	−2.8420	10.9709	−15.8994	8.3516
0.25	0.4296	−2.6350	8.6864	−11.5227	5.8544
0.50	0.3636	−0.8057	2.0463	−2.2902	1.1862
1.00	0.3527	−0.2842	0.4354	−0.0901	−0.1590
2.00	0.3341	0.1260	−0.6920	0.6877	−0.0600

GENERAL NOTES:

(a) The polynomial equations given in the tabular part of this Figure can be used in lieu of the curves.

(b) For both parts (a) and (b) in the tabular part of this Figure, these coefficients are only valid for $0.1 \le \mu^* \le 0.6$.

(c) For both parts (a) and (b) in the tabular part of this Figure: for values of h/p lower than 0.1, use $h/p = 0.1$; for values of h/p higher than 2.0, use $h/p = 2.0$.

Figure 12 — Curves and Tables for the Determination of E*/E and ν* (Square Pattern)

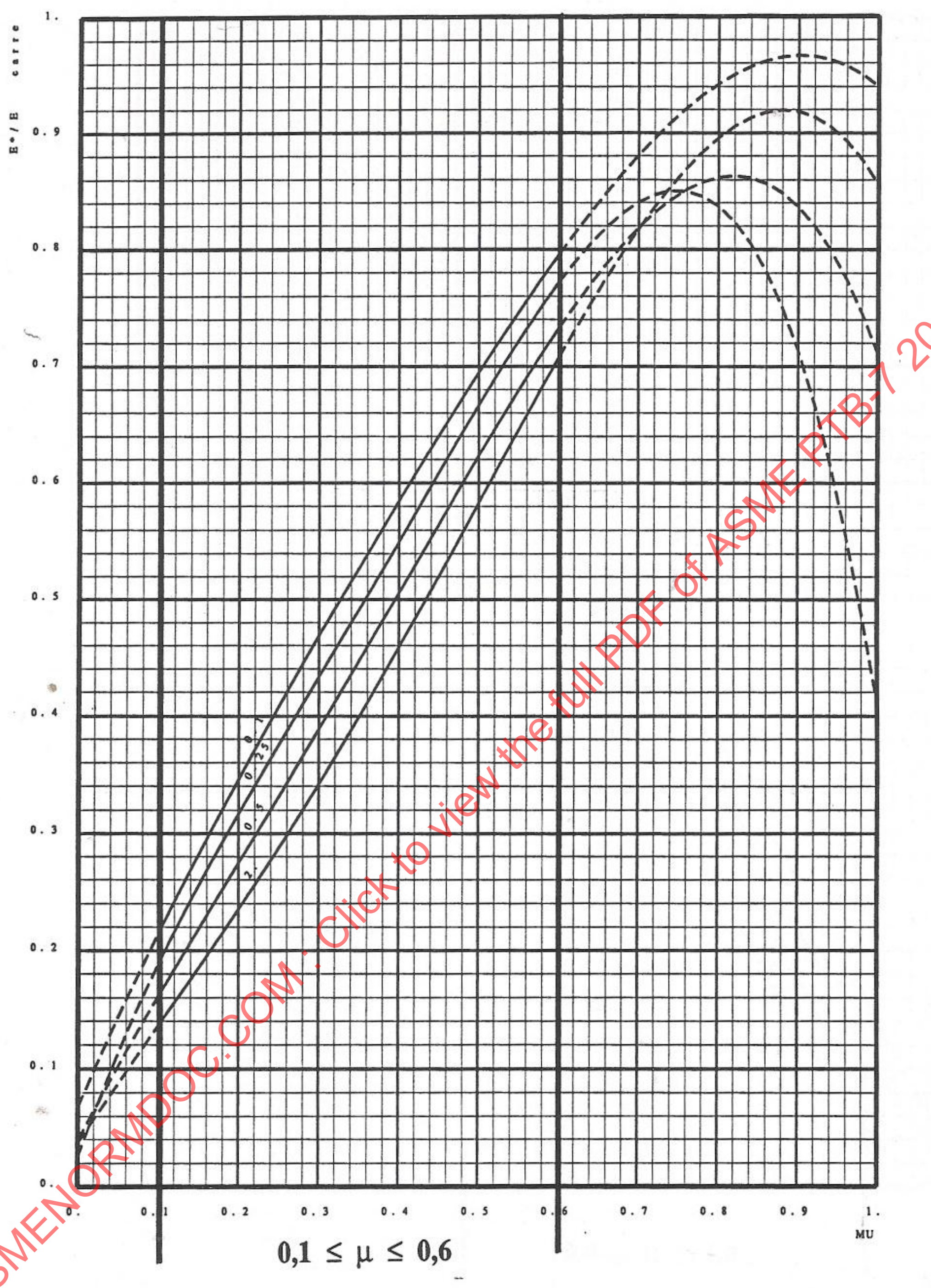


Figure 13 — Curves E*/E for Square Pattern Obtained from Polynomial Approximation Given in Figure 12

(Only valid for 0.1≤μ*≤0.6) from [13]

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Codes & Standards (CS)

[CS-1] BS 5500: Unfired Fusion Welded Pressure Vessels – Chapter 3.9 “Flat HE TSs”

[CS-2] CODAP: French Code for Unfired Pressure Vessels – Chapter C7 “Design rules for HE TSs”

[CS-3] EN 13445: European Standard for Unfired Pressure Vessels – Chapter 13 “HE TSs”

[CS-4] ISO DIS 2694: Pressure Vessels – Chapter 30 “ Flat HE TSs”

[CS-5] STOOMWEZEN: Dutch Code for Pressure Vessels – Chapters D0403 “Wall thickness calculation of TSs”

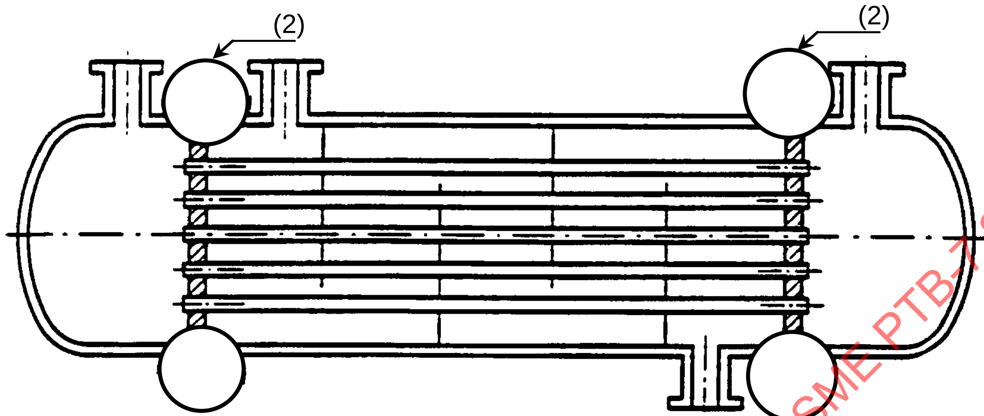
[CS-6] TEMA: Standards of Tubular Exchangers Manufacturers Association – Chapter R7 “TS”

PART 3
ANALYTICAL TREATMENT OF FIXED TUBESHEET HEAT EXCHANGERS

1 SCOPE (UHX-13.1)

PART 3, devoted to Fixed TS HEs (Figure 14), provides the technical basis for the determination of

- the displacement and loads acting on the TS, tubes, shell and channel,
- the stresses in these four components and their relationships with the design rules of UHX-13



[2]

Figure 14 — Fixed Tubesheet Heat Exchanger

[2] see TS configurations in Figure 15

2 HISTORICAL BACKGROUND

In the past decades many authors have proposed theoretical methods for the design of fixed TS HEs. The most important contributions are provided below. A more detailed list of technical papers can be found in Ref. [1] .

Gardner [2] in 1952 was the very first to develop an analytical approach by taking into consideration the support afforded by the tubes and the weakening effect of the TS holes. The TS is considered as either simply supported or clamped at its periphery to simulate the rotational restraint afforded by the shell and the channel, which compels the designer to make a more or less arbitrary choice between these two extreme cases.

TEMA adopted this method in its 5th edition published in 1968. K.A.G. Miller [3] at the same time, proposed a similar approach that was published in the British Code BS 1515 in 1965.

Galletly [4] in 1959 improved these design methods by accounting for the degree of rotational restraint of the TS at its periphery by the shell and the channel. This method was adopted by the French Pressure Vessel Code CODAP in 1982 and by the European Pressure Vessel Standard EN13445 in 2002.
Other authors (Yi Yan Yu [5] [6] , Boon and Walsh [7] , Hayashi [8]) have developed more refined methods accounting for the membrane loads acting at mid-surface of the TS, the unperforated rim, the TS-shell-channel connection, and the bending effect of the tubes which reinforces the strength of the TS. Since a solution using these methods requires the use of a computer they were not adopted by Codes. Soler [9] in 1984 developed a similar method accounting for the unperforated rim and the TS-shell-channel connection. Thanks to a parametric study, it does not need the use of a computer.

The method was adopted by the SG-HTE in the 80's, put into a code format, and published for the first time in 1995 in Nonmandatory Appendix AA of Section VIII Division 1. In 2003 it was published in a new Part UHX of Section VIII Division 1 "Rules for Shell and Tubes Heat Exchangers" which became mandatory in 2004.

This criteria document provides the technical basis of Part UHX-13 of Section VIII Div. 1, 2007 Edition [10] , including the 2008 and 2009 Addenda. A few items (analysis of cylindrical and spherical shells, allowable stress limits) are taken from Appendix 4 of Section VIII Div. 2, 2004 Edition [11] .

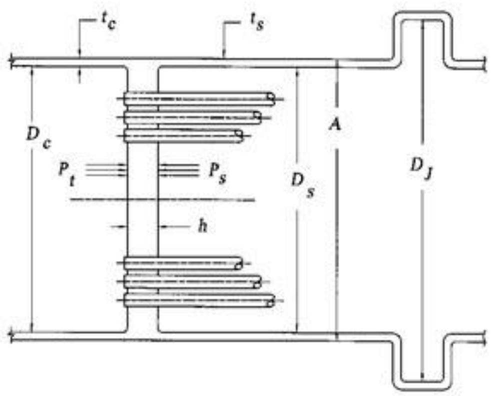
3 GENERAL

3.1 TS Configurations (UHX-13.1)

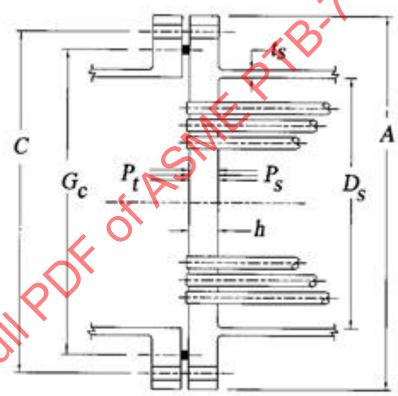
The TS is attached to the shell and the channel by welding (integral TS) or by bolting (gasketed TS) in accordance with the following 4 configurations (see Figure 15):

- configuration a: tubesheet integral with shell and channel;
- configuration b: tubesheet integral with shell and gasketed with channel, extended as a flange;
- configuration c: tubesheet integral with shell and gasketed with channel, not extended as a flange;
- configuration d: tubesheet gasketed with shell and channel, extended as a flange or not extended.

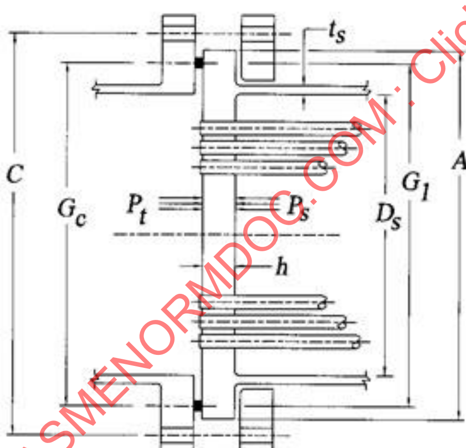
An expansion joint can be set-up on the shell as shown on Figure 15 configuration a.



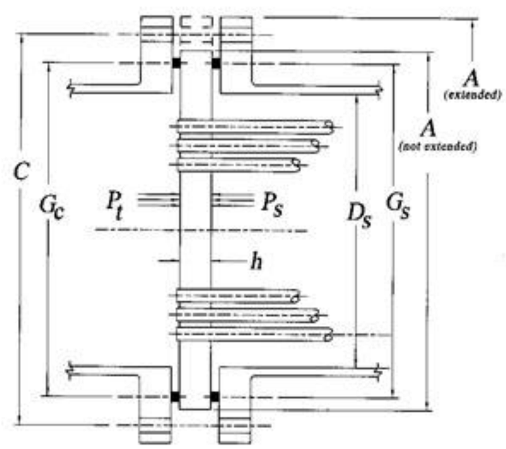
(a) Configuration a:
Tubesheet integral with shell and channel
gasketed with channel



(b) Configuration b:
Tubesheet integral with shell and
extended as a flange



(c) Configuration c:
Tubesheet integral with shell and gasketed with channel,
not extended as a flange



Configuration d:
Tubesheet gasketed with shell and channel

Figure 15 — Tubesheet Configurations

3.2 Notations (UHX-13.3)

(a) Data for the design of the HE are as follows.

Symbols D_o, E^*, h'_g, μ, μ^* and ν^* are defined in Section 2 of PART 2.

A	=	outside diameter of tubesheet
a_c	=	radial channel dimension
a_o	=	equivalent radius of outer tube limit circle
a_s	=	radial shell dimension
C	=	bolt circle diameter
D_c	=	inside channel diameter
D_J	=	inside diameter of the expansion joint at its convolution height
D_s	=	inside shell diameter
d_t	=	nominal outside diameter of tubes
E	=	modulus of elasticity for tubesheet material at T
E_c	=	modulus of elasticity for channel material at T_c
E_s	=	modulus of elasticity for shell material at T_s
E_t	=	modulus of elasticity for tube material at T_t
G_c	=	diameter of channel gasket load reaction
G_s	=	diameter of shell gasket load reaction
G_1	=	midpoint of contact between flange and tubesheet
h	=	tubesheet thickness
K_J	=	axial rigidity of expansion joint, total force/elongation
L_t	=	tube length between outer tubesheet faces
N_t	=	number of tubes
P_e	=	effective pressure acting on tubesheet
P_s	=	shell side design or operating pressure, as applicable. For shell side vacuum use a negative value for P_s.
P_t	=	tube side design or operating pressure, as applicable. For tube side vacuum use a negative value for P_t. *Notation P_c, instead of P_t, is used throughout the analytical development so as to maintain the symmetry of the equations involving the shell (subscript s) and the channel (subscript c).*
S	=	allowable stress for tubesheet material at T
S_c	=	allowable stress for channel material at T_c
S_s	=	allowable stress for shell material at T_s
S_t	=	allowable stress for tube material at T_t
$S_{y,c}$	=	yield strength for channel material at T_c
$S_{y,s}$	=	yield strength for shell material at T_s
$S_{y,t}$	=	yield strength for tube material at T_t
S_{PS}	=	allowable primary plus secondary stress for tubesheet material at T
$S_{PS,c}$	=	allowable primary plus secondary stress for channel material at T_c
$S_{PS,s}$	=	allowable primary plus secondary stress for shell material at T_s
T	=	tubesheet design temperature
T_a	=	ambient temperature, 70ºF (20ºC)
T_c	=	channel design temperature
T_s	=	shell design temperature
T_t	=	tube design temperature
$T_{s,m}$	=	mean shell metal temperature along shell length
$T_{t,m}$	=	mean tube metal temperature along tube length
t_c	=	channel thickness
t_s	=	shell thickness

t_t	=	nominal tube wall thickness
W_s , W_c	=	shell or channel flange design bolt load for the gasket seating condition
W_{m1s}, W_{m1c}	=	shell or channel flange design bolt load for the operating condition
W_{max}	=	MAX [(W_c), (W_s)]
W^*	=	tubesheet effective bolt load determined in accordance with UHX-8
$\alpha_{s,m}$	=	mean coefficient of thermal expansion of shell material at $T_{s,m}$
$\alpha_{t,m}$	=	mean coefficient of thermal expansion of tube material at $T_{t,m}$
γ	=	axial differential thermal expansion between tubes and shell
ν	=	Poisson's ratio of tubesheet material
ν_c	=	Poisson's ratio of channel material
ν_s	=	Poisson's ratio of shell material
ν_t	=	Poisson's ratio of tube material

(b) **Design coefficients** (UHX-13.5.1 to 4)
The following coefficients, specific to each component of the HE, will be used in the analytical treatment.

1) **Perforated TS**

Equivalent diameter of outer tube limit circle (see Section 4.3(a) of PART 2): $D_o = 2 r_o + d_t$

Equivalent radius of outer tube limit circle: $a_o = \dfrac{D_o}{2}$

TS coefficients:

- Shell side: $x_s = 1 - N_t \left(\dfrac{d_t}{2a_o}\right)^2$; $1 - x_s = N_t \left(\dfrac{d_t}{2\,a_o}\right)^2$
- Tube side: $x_t = 1 - N_t \left(\dfrac{d_t - 2t_t}{2a_o}\right)^2$; $1 - x_t = N_t \left(\dfrac{d_t - 2t_t}{2\,a_o}\right)^2$
- $x_t - x_s = N_t \left(\dfrac{d_t^2 - (d_t - 2t_t)^2}{4\,a_o^2}\right) = \dfrac{N_t \cdot s_t}{\pi\,a_o^2} = \dfrac{N_t \cdot k_t}{E_t} \cdot \dfrac{l}{\pi\,a_o^2} = \dfrac{N_t \cdot K_t}{E_t} \cdot \dfrac{L}{\pi\,a_o^2}$ [III.2.b1]
- Ligament efficiency: $\mu^* = \dfrac{p^* - d^*}{p^*}$

  Effective tube hole diameter d* and effective pitch p* are defined in Section 4.3(d) and (c) of PART 2.
- Effective elastic constants E* and ν* are given in Section 5.6 of PART 2 as a function of μ^* and h/p (triangular or square pitch).
- Bending stiffness: $D^* = \dfrac{E^* \cdot h^3}{12\left(1 - \nu^{*2}\right)}$
- Effective tube side pass partition groove depth given in Section 4.3(f) of PART 2: h'_g
- Effective pressure acting on tubesheet: P_e

2) **Tube Bundle**

Tube cross-sectional area:

$$s_t = \frac{\pi}{4}\left[d_t^2 - (d_t - 2t_t)^2\right] = \pi\, t_t\,(d_t - t_t) = \frac{\pi\, a_0^2}{N_t}(x_t - x_s)$$

Axial stiffness K_t of one tube: $K_t = \frac{E_t\, s_t}{L} = \frac{\pi\, t_t\,(d_t - t_t)\,E_t}{L}$

Axial stiffness k_t of one half tube of length l=L/2: $k_t = \frac{E_t\, s_t}{l} = \frac{2\pi\, t_t\,(d_t - t_t)\,E_t}{L} = 2\,K_t$

Effective elastic foundation modulus equivalent to the half tube bundle:

$$k_w = \frac{N_t \cdot k_t}{\pi\, a_o^2} = \frac{2\,N_t \cdot K_t}{\pi\, a_o^2} = \frac{2\,N_t \cdot E_t \cdot t_t\,(d_t - t_t)}{L\, a_o^2} = \frac{2\,E_t}{L}(x_t - x_s) = \frac{E_t}{l}(x_t - x_s)$$

$$k = \sqrt[4]{\frac{k_w}{D^*}} \quad ; \quad x = k\,r \quad ; \quad 0 \le r \le a_o \quad \Rightarrow \quad 0 \le x \le k\,a_o \qquad k a_o = X_a$$

Axial stiffness ratio tubes/TS:

$$X_a = k\,a_o = \sqrt[4]{\frac{k_w}{D^*}}\,a_o = \left[24\,\left(1-\nu^{*2}\right)\,N_t\,\frac{E_t\;t_t\;(d_t - t_t)\;a_o^2}{E^*\;L\;h^3}\right]^{1/4}$$

3) **Shell**

Radial shell dimension: a_s $\quad \rho_s = \frac{a_s}{a_o}$

Integral configurations (a, b and c): $a_s = D_s / 2$
Gasketed configuration (d): $a_s = G_s / 2$

Mean shell radius: $a_s' = \frac{D_s + t_s}{2}$

Shell cross-sectional area: $s_s = \pi\, t_s\,(D_s + t_s)$

Axial stiffness K_s of the shell of length L: $K_s = \frac{E_s\, s_s}{L} = \frac{\pi\, t_s\,(D_s + t_s)\,E_s}{L}$

Axial stiffness k'_s of the half shell of length l=L/2: $k_s' = \frac{E_s\, s_s}{l} = \frac{2\pi\, t_s\,(D_s + t_s)\,E_s}{L} = 2\,K_s$

Axial stiffness ratio shell and tube bundle: $K_{s,t} = \frac{K_s}{N_t \cdot K_t} = \frac{t_s\,(D_s + t_s)\,E_s}{N_t \cdot t_t\,(d_t - t_t)\,E_t}$

Shell coefficient: $\beta_s = \frac{\sqrt[4]{12\,(1 - \nu_s^2)}}{\sqrt{(D_s + t_s)\; t_s}}$

Bending stiffness: $k_s = \beta_s\,\frac{E_s \cdot t_s^3}{6\,(1 - \nu_s^2)}$

4) **Channel**

Radial channel dimension: a_c $\quad \rho_c = \frac{a_c}{a_o}$

Integral configuration (a): $a_c = D_c / 2$
Gasketed configurations (b, c and d): $a_c = G_c / 2$

Mean channel radius: $a_s^{'} = \dfrac{D_c + t_c}{2}$

Channel coefficient: $\beta_c = \dfrac{\sqrt[4]{12\left(1-\nu_c^2\right)}}{\sqrt{\left(D_c + t_c\right)t_c}}$

Bending stiffness: $k_c = \beta_c \cdot \dfrac{E_c \cdot t_c^3}{6\left(1-\nu_c^2\right)}$

5) **Axial differential thermal expansion between the tubes and the shell:**

$$\gamma = \left[\alpha_{t,m}(T_{t,m} - T_a) - \alpha_{s,m}(T_{s,m} - T_a)\right]L$$

6) **Expansion joint**

D_J = inside diameter of the expansion joint at its convolution height
K_J = axial rigidity of the expansion joint, total force/elongation

J = ratio of expansion joint to shell axial rigidity (J = 1.0 if no joint); $J = \dfrac{K_J}{K_s + K_J}$

7) **Unperforated rim**

D_o = internal diameter
A = external diameter
Diameter ratio: $K = A / D_o$

3.3 Loading Cases (UHX-13.4)

The normal operating condition of the HE is achieved when the tube side pressure P_t, shell side pressure P_s and axial differential thermal expansion between tubes and shell γ act simultaneously. However, a loss of pressure or a loss of temperature is always possible. Accordingly, for safety reasons, the designer must always consider the cases where P_s=0 or P_t=0 with and without thermal expansion for the normal operating condition(s).

The designer must also consider the startup condition(s), the shutdown condition(s) and the upset condition(s), if any, which may govern the design.

A fixed TS HE is a statically indeterminate structure for which it is difficult to determine the most severe condition of coincident pressure, temperature and differential thermal expansion. Thus, it is necessary to evaluate all the anticipated loading conditions mentioned above to ensure that the worst load combination has been considered in the design.

For each of these conditions, ASME, TEMA, and CODAP used to consider the following loading cases.

(a) Pressure only loading cases

- Loading Case 1: Tube side pressure P_t acting only.
- Loading Case 2: Shell side pressure P_s acting only.
- Loading Case 3: Tube side pressure P_t and shell side pressure P_s acting simultaneously.

(b) Pressure and Thermal loading cases

- Loading Case 4: Differential thermal expansion acting only (P_t = 0, P_s = 0).
- Loading Case 5: Tube side pressure P_t acting only, with differential thermal expansion.
- Loading Case 6: Shell side pressure P_s acting only, with differential thermal expansion.
- Loading Case 7: Tube side pressure P_t and shell side pressure P_s acting simultaneously, with differential thermal expansion.

ASME 2013 Edition provides the detail of the pressure "design loading cases" and "operating (thermal) loading cases" to be considered for each operating condition specified by the user (normal operating conditions, startup conditions, the shutdown conditions, etc.). For the pressure loading cases, a table (table UHX-13.4-1) provides the values to be used for the design pressures P_s and P_t in the formulas, accounting for their maximum and minimum values.

For the operating (thermal) loading cases a second table (table UHX-13.4-2) provides the values to be used for the operating pressures P_s and P_t in the formulas for each operating condition considered. So, this new concept allows the operating pressures (instead of the design pressures to use for operating (thermal) loadings.)

As the calculation procedure is iterative, a value *h* is assumed for the tubesheet thickness to calculate and check that the maximum stresses in tubesheet, tubes, shell, and channel are within the maximum permissible stress limits.

Because any increase of tubesheet thickness may lead to overstresses in the tubes, shell, or channel, a final check must be performed, using in the formulas the nominal thickness of tubesheet, tubes, shell, and channel, in both corroded and uncorroded conditions.

3.4 Design Assumptions (UHX-13.2)

A fixed TS HE is a complex structure and several assumptions are necessary to derive a 'design by rules' method. Most of them could be eliminated, but the analytical treatment would lead to 'design by analysis' method requiring the use of a computer.

The design assumptions are as follows.

(a) HE
- The analytical treatment is based on the theory of elasticity applied to the thin shells of revolution.
- The HE is axi-symmetrical.
- The HE is a symmetrical unit with identical TSs so as to analyze a half-unit.

(b) TSs
- The two tubesheets are circular and identical (same diameter, uniform thickness, material, temperature and edge conditions)
- The tubesheets are uniformly perforated over a nominally circular area, in either equilateral triangular or square patterns. This implies that the TSs are fully tubed (no large untubed window)
- Radial displacement at the mid-surface of the TS is ignored
- Temperature gradient through the TS thickness is ignored
- Shear deformation and transverse normal strain in TS are ignored
- The unperforated rim of the TS is treated as a rigid ring without distortion of the cross section

(c) Tubes
- Tubes are assumed identical, straight and at same temperature
- Tubes are uniformly distributed in sufficient density to play the role of an elastic foundation for the TS
- The effect of the rotational stiffness of the tubes is ignored

(d) Shell and channel
- Shell and channel are cylindrical with uniform diameters and thicknesses
- If the channel head is hemispherical, it must be attached directly to the TS, without any cylindrical section between the head and the TS.
- Shell and channel centerlines are the same.

(e) Weights and pressures drops
- Weights and pressures drops are ignored
- Pressures P_s and P_t are assumed uniform

3.5 Basis of Analytical Treatment

3.5.1 General

The design of a fixed TS HEs is complex as the two TSs are connected to each other both through the tube bundle and the shell, and subjected to different temperatures which generate an axial differential thermal expansion. Accordingly the structure is statically indeterminate. Many geometrical, mechanical, thermal and material properties are involved in the design as shown in Section 3.2(a) which lists the extensive input data.

The analysis includes the effects of the shell and tube side pressures, the axial stiffening effect of the tubes, the stiffening effect of the unperforated ring at the tubesheet edge, and the stiffening effect of the integrally attached channel or shell to the tubesheet. For a tubesheet that is extended as a flange to which a channel or shell is to be bolted, the bolt load causes an additional moment in the tubesheet which is included in the total stress in the tubesheet in addition to the moments caused by pressure.

The analysis is based on classical discontinuity analysis methods to determine the moments and forces that the tubesheet, tubes, shell and channel must resist. These components are treated using the theory of elasticity applied to the thin shells of revolution.

Because the heat exchanger is assumed to be symmetric about a plane midway between the two tubesheets, only half of the heat exchanger is treated. The main steps of the analytical treatment are as follows.

(a) The tubesheet is disconnected from the shell and channel. Shear load V_a and moment M_a are applied at the tubesheet edge as shown in Figure 16.
(b) The perforated tubesheet is replaced by an equivalent solid circular plate of diameter D_o and effective elastic constants E^* (effective modulus of elasticity) and ν^* (effective Poisson's Ratio) depending on the ligament efficiency μ^* of the tubesheet. This equivalent solid plate is treated by the theory of thin circular plates subjected to pressures P_s and P_t and relevant applied loads to determine the maximum stresses.
(c) The unperforated tubesheet rim is treated as a rigid ring whose cross section does not change under loading.
(d) The tubes are replaced by an equivalent elastic foundation of modulus k_w.
(e) Connection of the tubesheet with shell and channel accounts for the edge displacements and rotations of the 3 components.
(f) The shell and channel are treated by the elastic theory of thin shells of revolution subjected to shell side and tube side pressures P_s and P_t and edge loads to determine the maximum stresses.
(g) The maximum stresses in tubesheet, tubes, shell and channel are determined and limited to the appropriate allowable stress-based classifications of Section VIII Division 2 PART 4.

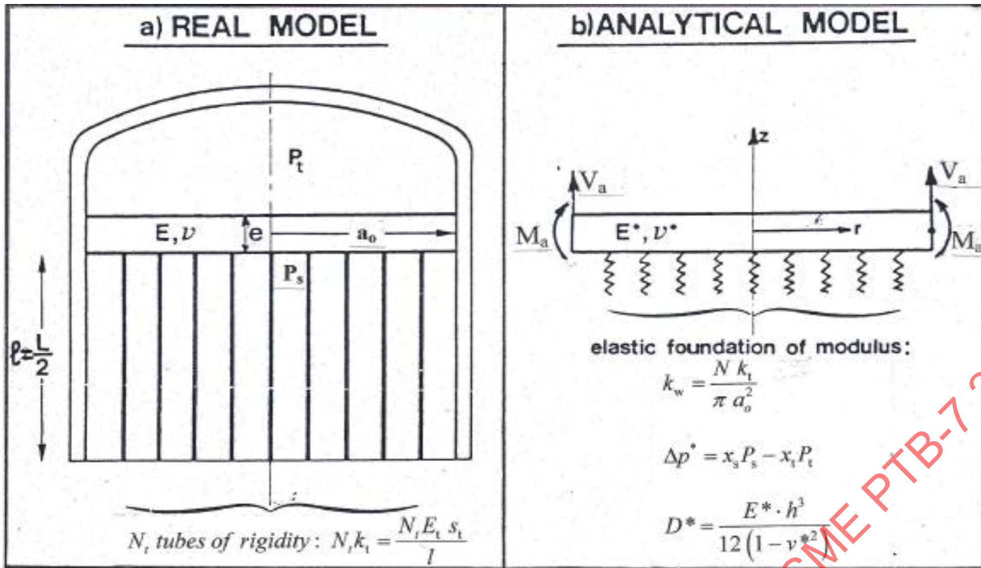


Figure 16 — Analytical Model Used in Design Method

3.5.2 Free Body Diagram

Figure 17 shows, for a tubesheet integral both sides (configuration a), the free body diagram of the component parts (perforated tubesheet, unperforated tubesheet rim, shell, channel). The figure details the relevant discontinuity forces (V_a, V_s, Q_s, V_c, Q_c) and moments (M_a, M_s, M_c, M_R) applied on each component, together with edge displacements.

In this figure, forces (V_a, V_s, Q_s, V_c, Q_c) and moments (M_a, M_s, M_c, M_R) are per unit of circumferential length. The following subscripts are used:

- s for shell,
- c for channel,
- R for unperforated rim

No subscript is used for the perforated TS.

Notation P_c instead of P_t (tube side pressure) is used throughout the analytical development so as to maintain the symmetry of the equations involving the shell (subscript s) and the channel (subscript c).

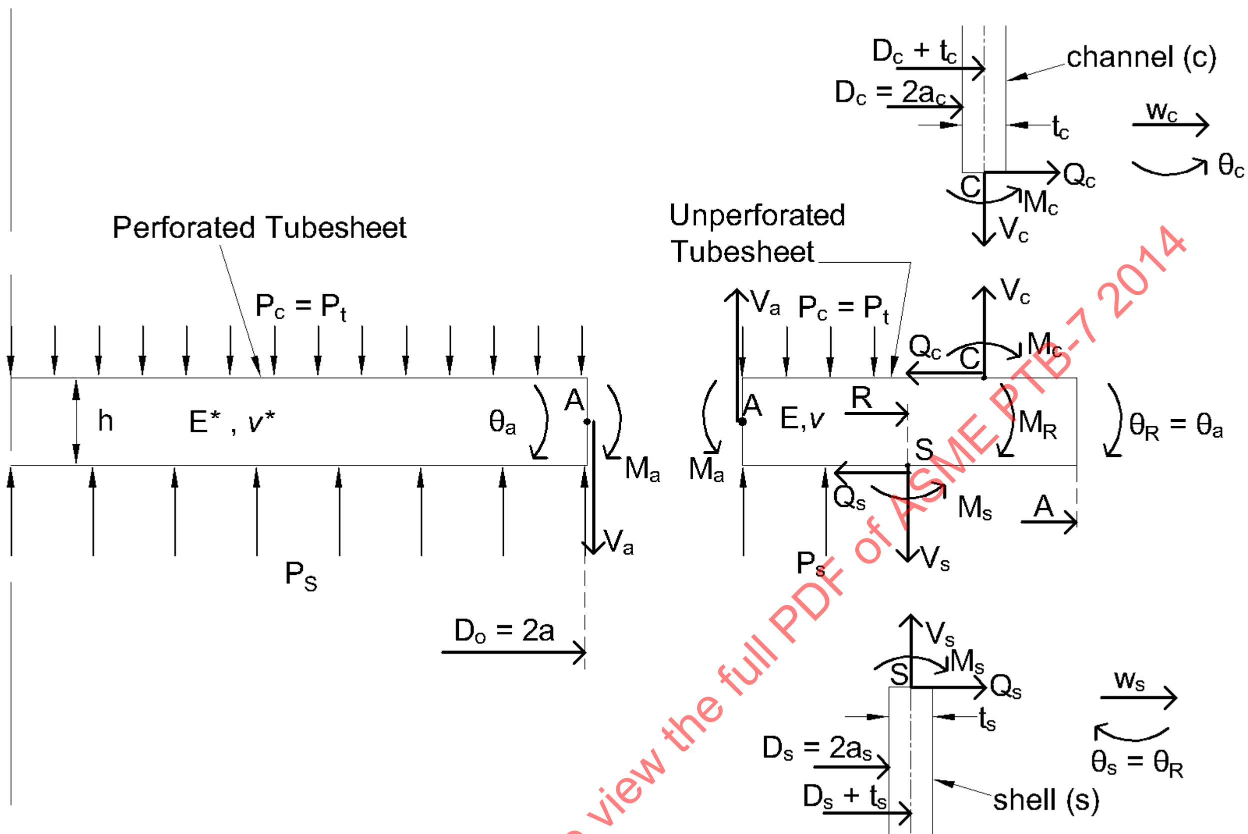


Figure 17 — Free Body Diagram of the Analytical Model

4 AXIAL DISPLACEMENTS AND FORCES ACTING ON THE TUBES AND ON THE SHELL

4.1 Axial Displacement and Force Acting on the Tubes (Figure 18)

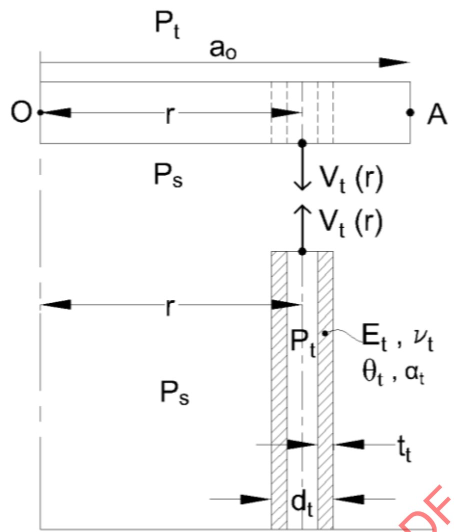


Figure 18 — Axial Displacement of Tubes

(a) Axial Displacement of the tubes due to axial force $V_t(r)$ acting on the tube row at radius r:

$$\delta_t(V_t)=\frac{V_t(r)}{k_t}=\frac{V_t(r)\,l}{E_t\,s_t}=\frac{l}{\pi\,E_t\,(d_t-t_t)\,t_t}V_t(r)$$

(b) Axial Displacement of the tubes due to temperature θ_t of tubes:

$$\delta_t(\theta_t)=\alpha_{t,m}\,\theta_t\,l \qquad \theta_t=T_{t,m}-T_a$$

(c) Axial Displacement of the tubes due to Poisson's ratio ν_t of tubes (Annex C):

$$\delta_t(\nu_t)=-\frac{2}{k_w}\left[(1-x_t)\,P_t-(1-x_s)\,P_s\right]\nu_t$$

(d) Total axial displacement of the tube row at radius r:

$$\delta_{t,Total}(r)=\underbrace{\left[\delta_t(V_t)\right]}_{\substack{\text{tube displct. due}\\ \text{to axial force}\\ \text{(unknown)}}}+\underbrace{\left[\delta_t(\theta_t)+\delta_t(\nu_t)\right]}_{\substack{\text{free displct.}\\ \text{of tube}\\ \text{(known)}}} \qquad \text{[IV.1d]}$$

(e) axial force acting on each tube at radius r:

$$V_t(r)=k_t\left[\delta_t(V_t)\right]=k_t\left[\delta_{t,Total}(r)-\delta_t(\theta_t)-\delta_t(\nu_t)\right]$$

(f) net effective pressure acting on the TS due to each tube at radius r of TS area $\pi a_0^2/N_t$:

$$q_t(r)=\frac{-V_t(r)}{\pi a_0^2/N_t}=-\frac{N_t k_t\,\delta_t(V_t)}{\pi a_0^2}=-\underbrace{\frac{N_t k_t}{\pi a_0^2}}_{k_w}\left[\delta_{t,Total}(r)-\delta_t(\theta_t)-\delta_t(\nu_t)\right] \qquad k_w=\frac{N_t\,k_t}{\pi\,a_0^2}$$

$$\boxed{q_t(r)=-k_w\left[\delta_t(V_t)+\delta_t(\theta_t)+\delta_t(\nu_t)-\delta_t(\theta_t)-\delta_t(\nu_t)\right]}$$

4.2 Axial Displacement and Force Acting on the Shell

***(a)* Axial displacement of the shell and expansion joint due to axial force acting on the shell**

- due to axial force V_s acting on the shell:

$$\delta_s^{'}(V_s) = \frac{V_s\ 2\pi\ a_s^{'}}{k_s^{'}} = \frac{V_s\ 2\pi\ a_s^{'}\ l}{E_s\ 2\pi\ a_s^{'} t_s} = \frac{l}{E_s\ t_s} V_s \qquad V_s \text{ is per unit of length}$$

$$a_s^{'} = mean\ shell\ radius = \frac{D_s + t_s}{2}$$

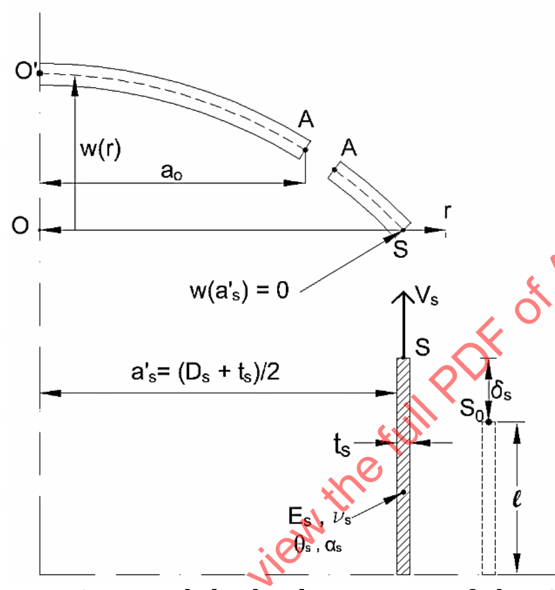


Figure 19 — Axial Displacement of the Shell

- due to axial force V_s acting on the expansion joint J:

$$\delta_J^{'}(V_s) = \frac{V_s\ 2\ \pi\ a_s^{'}}{2\ K_J} \qquad 2K_J = \text{ axial rigidity of half expansion joint}$$

- axial displacement of shell and expansion joint:

$$\delta_s(V_s) = \delta_s^{'}(V_s) + \delta_J^{'}(V_s) = \frac{V_s\ 2\ \pi\ a_s^{'}}{k_s^{'}}\left[1 + \frac{k_s^{'}}{2\ K_J}\right] = \frac{V_s\ 2\ \pi\ a_s^{'}}{k_s^{'}\ J} = \frac{\delta_s^{'}(V_s)}{J} = \frac{l}{J\ E_s\ t_s} V_s$$

$$J = \frac{1}{1 + k_s^{'} / (2\ K_j)} \qquad k_s^{'} = 2\ K_s \qquad \boxed{J = \frac{K_J}{K_J + K_s}}$$

Verification: axial rigidity k_s^* *shell and expansion joint:* $\frac{1}{k_s^*} = \frac{1}{k_s^{'}} + \frac{1}{2\ k_J} = \frac{1}{k_s^{'}}\left(1 + \frac{k_s^{'}}{2\ k_J}\right) = \frac{1}{J\ k_s^{'}}$

$$k_s^* = J\ k_s^{'} \qquad \delta_s(V_s) = \frac{V_s\ 2\ \pi\ a_s^{'}}{k_s^*} = \frac{V_s\ 2\ \pi\ a_s^{'}}{k_s^{'}} = \frac{\delta_s^{'}(V_s)}{J}$$

***(b)* Axial displacement of the shell due to temperature θ_s of shell:**

$$\delta_s(\theta_s) = \alpha_{s,m}\ \theta_s\ l \qquad \theta_s = T_{s,m} - T_a$$

***(c)* Axial displacement of the shell due to Poisson's ratio ν_s of shell** (Annex C):

$$\delta_s(\nu_s) = -\frac{\pi}{2\,k_s'}\left(P_s\,D_s^2\right)\nu_s$$

(d) **Axial displacement of the shell due to pressure P_s acting on bellows joint side walls J** (Annex D):

$$\delta_s(J) = P_s\,\frac{\pi\left(D_J^2 - D_s^2\right)/2}{4\times 2K_J} = \frac{\pi}{8}\,\frac{D_J^2 - D_s^2}{2K_J}\,P_s$$

(e) **Total axial displacement of shell:**

$$\delta_{s,\text{Total}} = \underbrace{\delta_s(V_s)}_{\substack{\text{shell displct. due}\\ \text{to axial force}\\ \text{(unknown)}}} + \underbrace{\delta_s(\theta_s) + \delta_s(\nu_s) + \delta_s(J)}_{\substack{\text{free displct.}\\ \text{of shell}\\ \text{(known)}}} \qquad \text{[IV.2e]}$$

(f) **Axial force acting on the shell:**

$$\delta_s(V_s) = \delta_{s,\text{Total}} - \delta_s(\theta_s) - \delta_s(\nu_s) - \delta_s(J) = \frac{l}{J\,E_s\,t_s}V_s$$

$$V_s = \frac{J\,E_s\,t_s}{l}\left[\delta_{s,\text{Total}} - \delta_s(\theta_s) - \delta_s(\nu_s) - \delta_s(J)\right]$$

(g) **Axial displacement of tubes at radius r:**

$$\delta_{t,\text{Total}}(r) = \delta_{s,\text{Total}} + w(r) = \delta_s(V_s) + \delta_s(\theta_s) + \delta_s(\nu_s) + \delta_s(J) + w(r)$$

where w(r) is the TS deflection at radius r (see Figure 21)

[IV.1d]: $\delta_{t,\text{Total}}(r) = \delta_t(V_t) + \delta_t(\theta_t) + \delta_t(\nu_t)$

$$\boxed{\delta_t(V_t) = \delta_s(V_s) + w(r) + \underbrace{\left[\delta_s(\theta_s) - \delta_t(\theta_t)\right]}_{-\gamma/2} + \left[\delta_s(\nu_s) - \delta_t(\nu_t)\right] + \left[\delta_s(J)\right]}$$

(h) **TS deflection at radius r:**

$$w(r) = \left[\delta_t(V_t) - \delta_s(V_s)\right] + \left[\underbrace{\delta_t(\theta_t) - \delta_s(\theta_s)}_{\gamma/2}\right] + \left[\delta_t(\nu_t) - \delta_s(\nu_s)\right] - \left[\delta_s(J)\right]$$

5 DEFLECTION AND LOADS ACTING ON THE TUBESHEET

5.1 Equivalent Plate Resting on an Elastic Foundation

(a) **Net effective pressure acting on the TS**

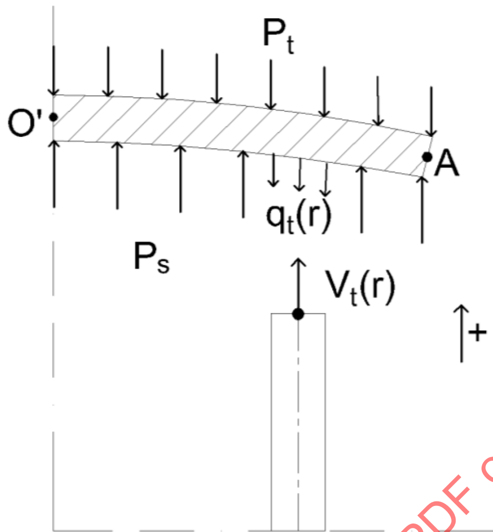


Figure 20 — Loads Acting on the TS

due to tubes: $q_t(r) = -k_w\left[\delta_t(V_t)+\delta_t(\theta_t)+\delta_t(v_t)-\delta_t(\theta_t)-\delta_t(v_t)\right]$

Tubes act as an elastic foundation of equivalent modulus given by the axial rigidity of the half-bundle per unit of TS area:

$$\boxed{k_w = \frac{N_t\, k_t}{\pi\, a_o^2}}$$

$$q_t(r) = -\frac{V_t(r)}{\pi\, a_o^2 / N_t} = -k_w\left[\delta_s(V_s)+w(r)-\delta(\theta)+\delta_s(v_s)-\delta_t(v_t)+\delta_J(P_s)\right]$$

due to pressures P_s and P_t acting on the equivalent plate (see Annex E):

$$q_P = x_s\, P_s - x_t\, P_t = \Delta p^*$$

net effective pressure:

$$q(r) = q_P + q_t(r) = \Delta p^* - \frac{V_t(r)}{\pi\, a_o^2 / N_t} \qquad \text{[V.1a]}$$

$$q(r) = q_P + q_t(r) = \underbrace{\Delta p^* - k_w\left[\left[\delta_s(\theta_s)-\delta_t(\theta_t)\right]+\left[\delta_s(v_s)-\delta_t(v_t)\right]+\left[\delta_J(P_s)\right]+\delta_s(V_s)\right]}_{Q} - k_w w(r)$$

- $$\boxed{Q = \Delta p^* + k_w\left[\left[\delta_t(\theta_t)-\delta_t(\theta_t)\right]+\left[\delta_t(v_t)-\delta_s(v_s)\right]-\left[\delta_J(P_s)\right]-\left[\delta_s(V_s)\right]\right]} \qquad \text{[V.1a']}$$

In this equation, the displacement $\delta_s(V_s)$ of the shell subjected to axial force V_s is unknown. Other quantities are known for a given HE.

- Annex C: $k_w\ \delta_t(v_t) = -2\, v_t\left[P_t(1-x_t) - P_s(1-x_s)\right]$

$$k_w\,\delta_s(\nu_s) = -\frac{2\,\nu_s}{k_{s,t}}\left(\frac{D_s}{D_o}\right)^2 P_s$$

- Annex D:

$$k_w\delta_J(P_s) = \frac{\pi}{16}\frac{k_w}{K_J}\left(D_j^2 - D_s^2\right)P_s = \frac{\pi}{16}\frac{N_t\,k_t}{\pi\,a_o^2\,K_J}\left(D_J^2 - D_s^2\right)P_s = \frac{N_t\,k_t}{4\,K_J}\frac{D_J^2 - D_s^2}{D_o^2}P_s$$

$$\boxed{q(r) = Q - k_w\,w(r)}$$

(b) **deflection of TS** (Figure 21)

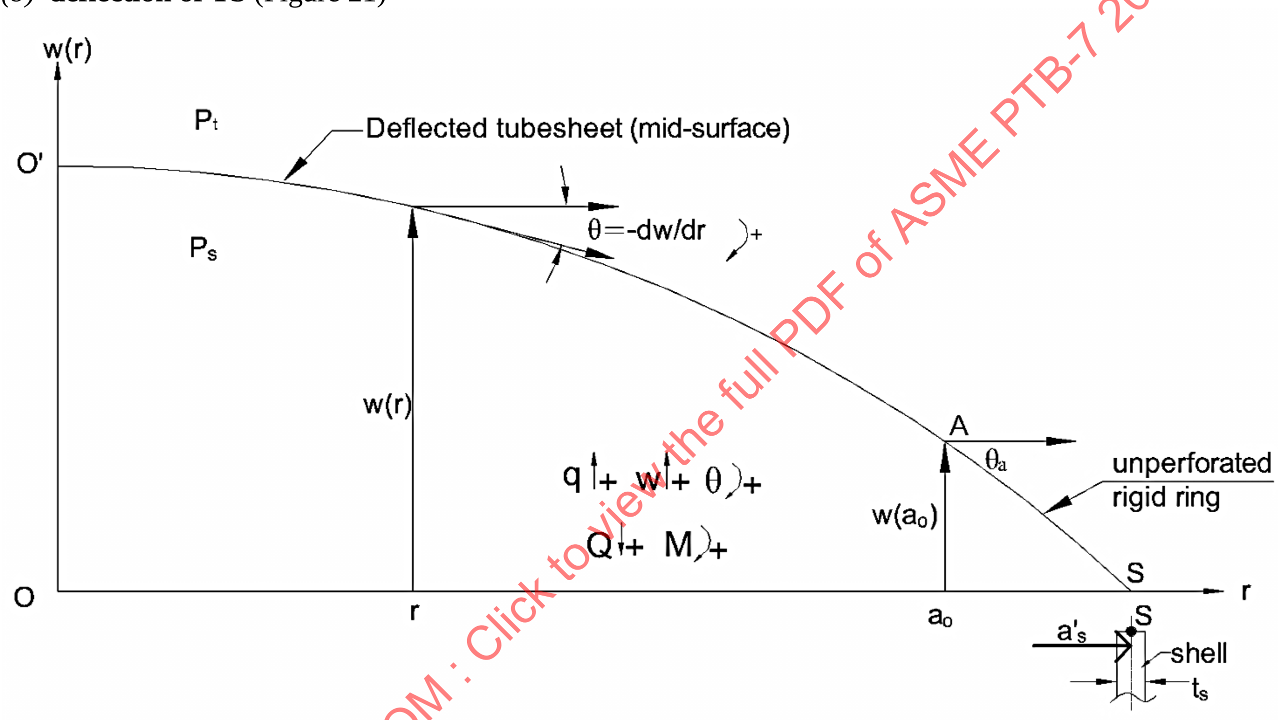


Figure 21 — TS Displacement

The TS is replaced by an equivalent solid plate:

- of equivalent radius a_o
- having effective elastic constants E* and ν*.
- With bending rigidity: $$D^* = \frac{E^*\,h^3}{12\left(1-\nu^{*2}\right)}$$
- resting on an elastic foundation of modulus: $$k_w = \frac{N\,k_t}{\pi\,a_o^2}$$
- subjected to a net effective pressure q(r)
- subjected at its periphery to edge loads V_a and M_a

The deflection of such a plate is governed by a differential equation of 4th order:

$$\frac{d^4w}{dr^4} + \frac{2}{r}\frac{d^3w}{dr^3} - \frac{1}{r^2}\frac{d^2w}{dr^2} + \frac{1}{r^3}\frac{dw}{dr} = \frac{q(r)}{D^*} = \frac{Q - k_w\,w(r)}{D^*}$$

whose solution is written (Annex F): $w(x) = A\,ber(x) + B\,bei(x) + \dfrac{Q}{k_w}$

where:

- x is a new dimensionless integration variable which is introduced to solve the differential equation $x = k\,r = \sqrt[4]{\dfrac{k_w}{D^*}}\,r$.

  At the TS periphery (r =a_o): $X_a = k\,a_o \qquad 0 \le r \le a_o \;\Rightarrow\; 0 \le x \le X_a$
- ber(x) and bei(x) are Kelvin functions of order 0, which will be noted respectively berx and beix in the following. Accordingly: $\boxed{w(x) = A\,berx + B\,beix + \dfrac{Q}{k_w}}$
- A and B are integration constants to be determined by boundary conditions at TS periphery ($x = X_a$)

At the TS periphery (r =a_o): $\boxed{X_a = k\,a_o = \sqrt[4]{\dfrac{k_w}{D*}}\,a_o = \left[24\,\left(1-\nu*^2\right) N_t \dfrac{E_t\,t_t\,(d_t - t_t)\,a_o^2}{E*\,L\,h^3}\right]^{1/4}}$

net effective pressure: $q(w) = -k_w\,\left(A\,berx + B\,beix\right)$

(c) ***the rotation,*** using the sign conventions shown in Figure 21 (θ>0 clockwise), is written:

$$\theta(r) = -\frac{dw}{dr} = -k\,\frac{dw}{dx}$$

$$\theta(x) = -k\left[A\,ber'x + B\,bei'x\right]$$

Note: For x=0 θ(0)=0: the rotation at the center of the TS is 0 as expected.

(d) The shear force writes:

$$Q_r(r) = \frac{1}{r}\int_0^r \rho\,q(\rho)\,d\rho \qquad 0 \le \rho \le r \qquad x = k\,r$$

$$= \frac{1}{r}\int_0^r -k_w\,\rho\left[A\,ber(k\rho) + B\,bei(k\rho)\right]d\rho \qquad y = k\rho \qquad 0 < y \le x$$

$$= -\frac{1}{r}\frac{k_w}{k^2}\left[A\int_0^x y\,bery\,dy + B\int_0^x ybeiy\,dy\right]$$

$$y\,bery = +\,y\,bei''y + bei'y = +\left(y\,bei'y\right)' \Rightarrow \int y\,bery\,dy = +\,y\,bei'y$$

$$y\,beiy = -\,y\,ber''y + ber'y = -\left(y\,ber'y\right)' \Rightarrow \int y\,beiy\,dy = -\,y\,ber'y$$

$$Q_r(r) = -\frac{1}{r}\frac{k_w}{k^2}\left[A\,x\,bei'x - B\,x\,ber'x\right]$$

$$Q_r(r) = -\frac{k_w}{k}\left(A\,bei'x - B\,ber'x\right)$$

(e) ***radial bending moment*** is written:

$$M_r(r) = -D^*\left[\frac{d^2w}{dr^2} + \frac{\nu^*}{r}\frac{dw}{dr}\right] \qquad x = k\,r$$

$$M_r(x) = -D^* k^2 \left[A\, ber'' x + B\, bei'' x \right] + \frac{\nu^*}{x}\left[A\, ber' x + B\, bei' x \right]$$

with: $ber'' x = -beix - \dfrac{ber' x}{x}$ $\qquad bei'' x = +berx - \dfrac{bei' x}{x}$

$$M_r(x) = -D^* k^2 \left[-A \left(\underbrace{beix + \frac{1-\nu^*}{x} ber' x}_{\Psi_1(x)} \right) + B \left(\underbrace{berx - \frac{1-\nu^*}{x} bei' x}_{\Psi_2(x)} \right) \right]$$

$$M_r(x) = -D^* k^2 \left[-A\,\Psi_1(x) + B\,\Psi_2(x) \right]$$

with: $\begin{cases} \Psi_1(x) = beix + \dfrac{1-\nu^*}{x} ber' x & \qquad \Psi_1 = \Psi_1(X_a) \\ \Psi_2(x) = berx + \dfrac{1-\nu^*}{x} bei' x & \qquad \Psi_2 = \Psi_2(X_a) \end{cases}$

(f) ***circumferential bending moment*** is written:

$$M_\theta = -D^* \left[\nu^* \frac{d^2 w}{dr^2} + \frac{1}{r}\frac{dw}{dr} \right]$$

The same calculation leads to:

$$M_\theta(x) = -D^* k^2 \left[-A\,\overline{\Psi_1(x)} + B\,\overline{\Psi_2(x)} \right]$$

with: $\overline{\Psi_1(x)} = \nu^*\, beix - \dfrac{1-\nu^*}{x} ber' x \qquad \overline{\Psi_2(x)} = \nu^*\, berx + \dfrac{1-\nu^x}{x} bei' x$

This moment does not generally control the TS design since:

$$\left| M_r(x) \right| > \left| M_\theta(x) \right|$$

5.2 Determination of Integration Constants A and B

(a) A and B are determined from the boundary conditions at periphery of TS:

$$Q_r(a_o) = V_a \qquad M_r(a_o) = M_a$$

where V_a et M_a are the loads applied at periphery of TS (point A in Figure 21).

$$\begin{cases} Q_r(X_a) = -\dfrac{k_w}{k}\left[A\, bei'(X_a) - B\, ber'(X_a) \right] & \Rightarrow A\, bei' - B\, ber' = -\dfrac{k}{k_w} V_a \\ M_r(X_a) = -D^* k^2 \left[-A\,\Psi_1(X_a) + B\,\Psi_2(X_a) \right] & \Rightarrow A\,\Psi_1 - B\,\Psi_2 = \dfrac{M_a}{D^* k^2} = \dfrac{k^2}{k_w} M_a \end{cases}$$

$$M_r(X_a) = -D^* k^2 \left[-A\,\Psi_1(X_a) + B\,\Psi_2(X_a) \right] \Rightarrow A\,\Psi_1 - B\,\Psi_2 = \frac{M_a}{D^* k^2} = \frac{k^2}{k_w} M_a$$

$$ber = ber(X_a) \quad bei = bei(X_a) \quad ber' = ber'(X_a) \quad bei' = bei'(X_a) \quad \Psi_1 = \Psi_1(X_a)$$

$$\Psi_2 = \Psi_2(X_a)$$

(b) Integration constants A and B are obtained from Section 5.2a:

$$\begin{cases} A = -\dfrac{k}{k_w} \dfrac{(k\, ber')\, M_a + \Psi_2 V_a}{Z_a} \\ B = -\dfrac{k}{k_w} \dfrac{(k\, bei')\, M_a + \Psi_1 V_a}{Z_a} \end{cases} \quad \text{with: } Z_a = -\Psi_1\, ber' + \Psi_2\, bei'$$

(c) The 3rd boundary condition at the periphery of the unperforated rim, at its connection with the shell of mean radius a_s' (point *S* in Figure 22) is written: $w(a_s') = 0$

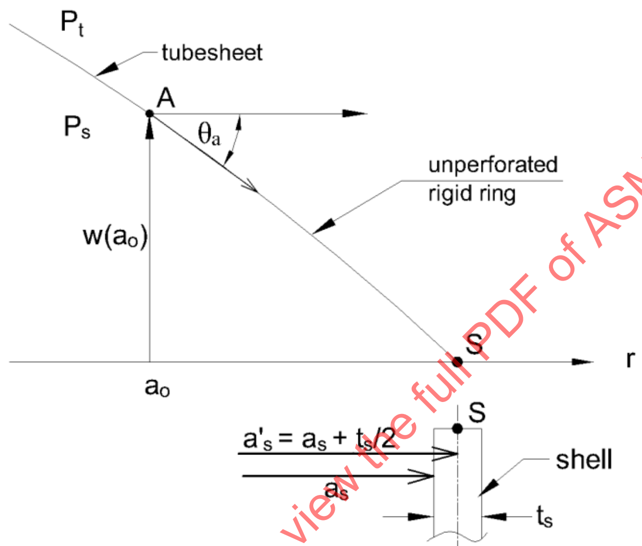


Figure 22 —TS Displacement of the Unperforated Ring and Connection to Shell

If the shell is integral, the TS rotation at point A is written:

$$\theta_a = -\frac{dw}{dr} = -\frac{\Delta w}{\Delta r} = -\frac{w(a_s') - w(a_o)}{a_s' - a_o} = \frac{w(a_o)}{a_s' - a_o}$$

where $\Delta w = w(a_s') - w(a_o)$ as the unperforated rim is considered as rigid.

$$w(a_o) = \theta_a (a_s' - a_o) = a_o\, \theta_a \left(\frac{a_s'}{a_o} - 1 \right)$$

Neglecting shell thickness: $a_s' = a_s$ $\boxed{w(a_o) = a_o\, \theta_a\, (\rho_s - 1)}$ [V.2c] with $\rho_s = \dfrac{a_s}{a_o} = \dfrac{D_s}{2a_o}$

If the shell is extended as a flange (configuration d): $\rho_s = \dfrac{G_s}{2\, a_o} = \dfrac{a_s}{a_o}$

(d) Substituting the expressions of A and B in V.1b enables one to determine $w(x), q(x), \theta(x), Q(x), M(x)$ as functions of x, depending on V_a and M_a which are still unknown.

5.3 Deflection

$$w(x)=\frac{Q}{k_w}-\frac{k}{k_w}\frac{\left(k\, ber'\, M_a+\Psi_2 V_a\right) ber\, x+\left(k\, bei'\, M_a+\Psi_1 V_a\right) bei\, x}{Z_a}$$

$$=\frac{Q}{k_w}-\frac{1}{D^*}\frac{a_o^3}{X_a^3}\left[k\, M_a\,\frac{ber'\, ber\, x+bei'\, bei\, x}{Z_a}+V_a\,\frac{\Psi_2\, ber\, x+\Psi_1\, bei\, x}{Z_a}\right]$$

$$\boxed{w(x)=\frac{Q}{k_w}-\frac{a_o^2}{D^*}\left[M_a\, Z_w(x)+(a_o V_a)\, Z_d(x)\right]} \qquad \text{[V.3]}$$

$$Z_w(x)=\frac{ber'\, ber\, x+bei'\, bei\, x}{X_a^2\, Z_a} \qquad Z_d(x)=\frac{\psi_2\, ber\, x+\Psi_1\, bei\, x}{X_a^3\, Z_a}$$

Note: Alan Soler's formula is incorrect: $Z_v(x)$ *is used instead of* $Z_w(x)$. *However the UHX rule is correct as it only uses* $Z_w(X_a)$ *in its formulas and it can be easily shown (see Annex G) that*

$$Z_w(X_a)=Z_v(X_a)$$

5.4 Net Effective Pressure

$$q(x)=Q-k_w\, w(x)$$

$$\boxed{q(x)=\frac{a_o^2\, k_w}{D^*}\left[M_a\, Z_w(x)+a_o\, V_a\, Z_d(x)\right]} \qquad \text{[V.4]}$$

5.5 Rotation

$$\theta(x)=-\frac{dw}{dr}=-\frac{dw}{dx}\frac{dx}{dr}=-k\frac{dw}{dx}=k\frac{a_o^2}{D^*}\left[M_A\, Z_w'(x)+(a_o V_a)\, Z_d'(x)\right]$$

$$\theta(x)=\frac{a_o}{D^*}\left[M_a\underbrace{\frac{ber'\, ber'x+bei'\, bei'x}{X_a\, Z_a}}_{Z_m(x)}+a_o\, V_a\underbrace{\frac{\Psi_2\, ber'x+\Psi_1\, bei'x}{X_a^2\, Z_a}}_{Z_v(x)}\right]$$

$$\boxed{\theta(x)=\frac{a_o}{D^*}\left[M_a\, Z_m(x)+a_o\, V_a\, Z_v(x)\right]} \qquad \text{[V.5]}$$

$$Z_m(x)=\frac{ber'\, ber'x+bei'\, bei'x}{X_a\, Z_a} \qquad Z_v(x)=\frac{\Psi_2\, ber'x+\Psi_1\, bei'x}{X_a^2\, Z_a}$$

Note: for x=0: $\left\{\begin{matrix} Z_m(x)=0 \\ Z_v(x)=0 \end{matrix}\right\}$ *θ(0)=0: the rotation at TS center is 0 as expected.*

5.6 Shear Force

$$Q_r(x) = \frac{k\,ber'\,M_a + \Psi_2\,V_a}{Z_a}\,bei'x - \frac{k\,bei'\,M_a + \Psi_1\,V_a}{Z_a}\,ber'x \qquad k = \frac{X_a}{a_o}$$

$$Q_r(x) = \frac{1}{a_o}\left[M_a \underbrace{\frac{ber'\,bei'x - bei'\,ber'x}{Z_a}\,X_a}_{Q_\alpha(x)} + a_o\,V_a \underbrace{\frac{\Psi_2\,bei'x - \Psi_1\,ber'(x)}{Z_a}}_{Q_\beta(x)} \right]$$

$$\boxed{Q_r(x) = \frac{1}{a_o}\left[M_a\,Q_\alpha(x) + (a_o\,V_a)\,Q_\beta(x)\right]} \qquad [V.6]$$

$$Q_\alpha(x) = \frac{ber'\,bei'x - bei'\,ber'x}{Z_a}\,X_a \qquad Q_\beta(x) = \frac{\psi_2\,bei'x - \psi_1\,ber'x}{Z_a}$$

Note: for $x = X_a$: $\begin{Bmatrix} Q_\alpha(X_a) = 0 \\ Q_\beta(X_a) = 1 \end{Bmatrix}$ $\quad Q_r(X_a) = \frac{1}{a_o}\,a_o\,V_a \;\Rightarrow\; Q(X_a) = V_a$

The boundary condition $Q_r(X_a) = V_a$ *is satisfied.*

5.7 Bending Moment

$$M_r(x) = \frac{D^* k^3}{k_w}\left[-\left(\frac{k\,ber'\,M_a + \Psi_2 V_a}{Z_a}\right)\Psi_1(x) + \left(\frac{k\,bei'\,M_a + \Psi_1 V_a}{Z_a}\right)\Psi_2(x)\right]$$

$$M_r(x) = M_a \underbrace{\frac{bei'\Psi_2(x) - ber'\Psi_1(x)}{Z_a}}_{Q_m(x)} + a_o V_a \underbrace{\frac{\Psi_1 \Psi_2(x) - \Psi_2 \Psi_1(x)}{X_a Z_a}}_{Q_v(x)}$$

$$\boxed{M_r(x) = M_a\, Q_m(x) + a_o\, V_a\, Q_v(x)} \qquad \text{[V.7]}$$

$$Q_m(x) = \frac{bei'\,\Psi_2(x) - ber'\,\Psi_1(x)}{Z_a} \qquad Q_v(x) = \frac{\Psi_1 \Psi_2(x) - \Psi_2 \Psi_1(x)}{X_a Z_a}$$

Note: for $x = X_a$ $\begin{Bmatrix} Q_m(X_a) = 1 \\ Q_v(X_a) = 0 \end{Bmatrix} \Rightarrow$ *$M_r(X_a)=M_a$: the boundary condition $M(X_a)=M_a$ is satisfied.*

5.8 Conclusion

1) Coefficients $Z_w(x), Z_d(x)$; $Z_m(x), Z_v(x)$; $Q_\alpha(x), Q_\beta(x)$; $Q_m(x), Q_v(x)$ are combinations of Kelvin functions. They are known for a given HE and for a given value of x varying from 0 at the TS center to X_a at the TS periphery. These coefficients are defined in Table UHX-13.1 of UHX-13. Annex G provides their expressions for x=0 and $x=X_a$.
2) Quantities $w(x), q(x), \theta(x), Q_r(x), M_r(x)$ depend on moment M_a and force V_a acting at TS periphery.
3) Edge loads at TS-shell-connection are still to be determined from the boundary conditions at TS periphery (see Section 6 hereafter).

6 TREATMENT OF THE UNPERFORATED RIM

6.1 Edge Loads Applied on Shell and Channel at their Connection to the TS

The following equations are developed for integral shell and channel.

(a) **Shell subjected to edge loads Q_s and M_s at its connection to the rim** (see Figure 17)
Radial displacement w_s and rotation θ_s are detailed in Annex H. Loads are per unit of length of circumference.

Radial displacement: $w_s = \frac{Q_s}{\beta_s^2 k_s} + \frac{M_s}{\beta_s k_s} + \delta_s P_s$ rotation: $\theta_s = \frac{Q_s}{\beta_s k_s} + \frac{2 M_s}{k_s}$

Where:

$\beta_s = \frac{\sqrt[4]{12\left(1-\nu_s^2\right)}}{\sqrt{\left(D_s + t_s\right) t_s}}$ is the shell coefficient.

$k_s = \beta_s \frac{E_s t_s^3}{6\left(1-\nu_s^2\right)}$ is the bending stiffness coefficient of the shell.

$\delta_s = \frac{D_s^2}{4 E_s t_s}\left(1 - \frac{D_s}{D_s + t_s}\frac{\nu_s}{2}\right)$ is the coefficient due to pressure acting on the shell.

Note: In UHX, t_s has been neglected compared to D_s and the formula is written:

$$\delta_s = \frac{D_s^2}{4 E_s t_s}\left(1 - \frac{\nu_s}{2}\right)$$

Equations for M_s and Q_s write: $\begin{cases} M_s = k_s \theta_s - \beta_s k_s w_s + \beta_s k_s \delta_s P_s \\ Q_s = -\beta_s k_s \theta_s + 2\beta_s^2 k_s w_s - 2\beta_s k_s \delta_s P_s \end{cases}$

Neglecting the radial displacement at mid-surface of the ring, compatibility of displacements of the ring and the shell is written (see Annex I): $w_s = -\frac{h}{2}\theta_s$ and loads M_s and Q_s become, using:

$t_s' = h \beta_s$:

$$\left.\begin{aligned} M_s &= +k_s\left(1 + \frac{t_s'}{2}\right)\theta_s + \beta_s k_s \delta_s P_s \\ Q_s &= -\beta_s k_s\left(1 + t_s'\right)\theta_s - 2\beta_s^2 k_s \delta_s P_s \end{aligned}\right\} \quad \text{[VI.1a]} \quad \text{for an integral shell (configurations a, b)}$$

When the shell is not integral with the TS (configuration d), k_s=0 and δs=0 lead to: $M_s = 0$ and $Q_s = 0$.

Note: These formulas are valid for a shell of sufficient length. Annex J provides the minimum length above which these formulas can be applied.

(b) **Channel subjected to edge loads Q_c and M_c** at its connection to the rim (see Figure 17).
Note: To ensure the symmetry of the equations, notation P_c=P_t is used.

Replacing subscript s by subscript c: $\begin{cases} w_c = \dfrac{Q_c}{\beta_c^2\, k_c} + \dfrac{M_c}{\beta_c\, k_c} + \delta_c\, P_c \\ \theta_c = \dfrac{Q_c}{\beta_c\, k_c} + \dfrac{2M_c}{k_c} \end{cases}$ for an integral channel (configuration a)

$$\boxed{\beta_c = \frac{\sqrt[4]{12\left(1-\nu_c^2\right)}}{\sqrt{\left(D_c + t_c\right)t_c}}}$$ channel coefficient.

$$\boxed{k_c = \beta_c \frac{E_c\, t_c^3}{6\left(1-\nu_c^2\right)}}$$ bending stiffness coefficient of channel

$$\boxed{\delta_c = \frac{D_c^2}{4\, E_c\, t_c}\left(1 - \frac{D_c}{D_c + t_c}\frac{\nu_c}{2}\right)}$$ coefficient due to pressure acting on channel.

Equations for M_c and Q_c write $\begin{cases} M_c = +k_c\, \theta_c - \beta_c\, k_c\, w_c + \beta_c\, k_c\, \delta_c\, P_c \\ Q_c = -\beta_c\, k_c\, \theta_c + 2\beta_c^2\, k_c\, w_c - 2\beta_c\, k_c\, \delta_c\, P_c \end{cases}$

compatibility of displacements: $w_c = -\frac{h}{2}\theta_c$ lead to equations for M_c and Q_c, using $t_c^{'} = h\,\beta_c$:

$$\begin{cases} \boxed{M_c = +k_c\left(1 + \dfrac{t_c^{'}}{2}\right)\theta_c + \beta_c\, k_c\, \delta_c\, P_c} \\ \boxed{Q_c = -\beta_c\, k_c\left(1 + t_c^{'}\right)\theta_c - 2\beta_c^2\, k_c\, \delta_c\, P_c} \end{cases}$$ [VI.1b] for an integral channel (configuration a)

When the channel is not integral with the TS (configurations b, c, d), $k_c = 0$ and $\delta_c = 0$ lead to: $M_c = 0$ and $Q_c = 0$

Note 1: These formulas are valid for a channel of sufficient length. Annex J provides the minimum length above which these formulas can be applied.

Note 2: These formulas are valid for a cylindrical channel. If the channel is hemispherical, it must be attached directly to the TS (configurations a, b or c), without any cylindrical section between the head and the TS. Annex K provides the relevant formulas for that case. Only coefficient δ_c is affected:

$$\boxed{\delta_c = \frac{D_c^2}{4\, E_c\, t_c}\left(\frac{1}{2} - \frac{D_c}{D_c + t_c}\frac{\nu_c}{2}\right)}$$

Note 3: In UHX, t_c has been neglected compared to D_c and the formula is written:

$$\boxed{\delta_c = \frac{D_c^2}{4\, E_c\, t_c}\left(\frac{1}{2} - \frac{\nu_c}{2}\right)}$$

6.2 Equilibrium of the Unperforated Rim

(a) **due to axial loads**

$$\begin{cases} a_s = D_s / 2 \\ a_s^{'} = a_s + \dfrac{t_s}{2} \end{cases} \qquad \begin{cases} a_c = D_c / 2 \\ a_c^{'} = a_c + \dfrac{t_c}{2} \end{cases}$$

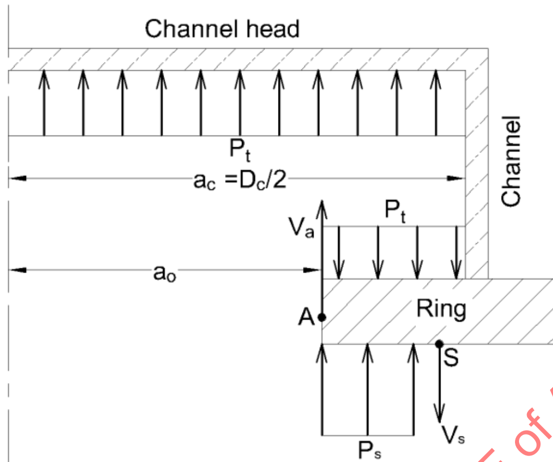


Figure 23 — Ring Equilibrium of the TS

The axial equilibrium of the ring is written: (see Figure 23):

$$2\,\pi\,a_s^{'}\,V_s + \pi\left(a_c^2 - a_o^2\right)P_c = 2\,\pi\,a_c^{'}\,V_c + 2\,\pi\,a_o\,V_a + \pi\left(a_s^2 - a_o^2\right)P_s \qquad \text{[VI.2a-1]}$$

where:

- V_a = axial edge load acting at connection of ring with equivalent plate is still to be determined
- V_s = axial force acting in the shell will be obtained from above equation, once V_a will have been determined. V_s positive when the shell is in tension.
- V_c = axial force acting in the ST channel is known.

$$2\,\pi\,a_c^{'}\,V_c = \pi\,a_c^2\,P_c \quad \Rightarrow \quad a_c^{'}\,V_c = \frac{a_c^2}{2}\,P_c \qquad \text{[VI.2a-2]}$$

Axial equilibrium of the ring is written:

$$\boxed{a_s^{'}\,V_s = a_o\,V_a + \frac{a_o^2}{2}\,P_c + \frac{a_s^2 - a_o^2}{2}\,P_s} \qquad \text{[VI.2a-3]}$$

Note: For U-tube and immersed floating head HEs:

$$2\pi a_s^{'} V_s = \pi a_s^2 P_s \Rightarrow a_s^{'} V_s = \frac{a_s^2}{2} P_s \Rightarrow a_o V_a = \frac{a_o^2}{2}\left(P_s - P_c\right) \quad V_a \textit{ is known.}$$

(b) **due to applied moments**

Equilibrium of moments applied to the ring relative to the axis located at radius a_o enables to determine the moment M_R (see Figure 17).

R = radius at center of ring = $\dfrac{A + 2a_o}{4}$

$$\left. \begin{aligned} RM_R = -\left[a_o\,M_a\right] + \left[a_c^{'}\,M_c - a_c^{'}\,Q_c\,\frac{h}{2}\right] + \left[M\left(P_c\right) - a_c^{'}\,V_c\left(a_c^{'} - a_o\right)\right] \\ - \left[a_s^{'}\,M_s - a_s^{'}\,Q_s\,\frac{h}{2}\right] + \left[M\left(P_s\right) - a_s^{'}\,V_s\left(a_s^{'} - a_o\right)\right] \end{aligned} \right\} \qquad \text{[VI.2b]}$$

$M(P_c) =$ moment due to pressure P_c acting on the ring $= \left(a_c^2 - a_o^2\right)\left(\dfrac{a_c + a_o}{2} - a_o\right)\dfrac{P_c}{2}$

$M(P_s) =$ moment due to pressure P_s acting on the ring $= \left(a_s^2 - a_o^2\right)\left(\dfrac{a_s + a_o}{2} - a_o\right)\dfrac{P_s}{2}$

Replacing terms of equation [VI.2b] by their expressions, one obtains for configuration a (see Annex L):

$$\left.\begin{aligned} R\,M_R = & -a_o\,M_a + a_o^2\,V_a\left(\rho_s - 1\right) + P_s\frac{a_o^3}{4}\left[\left(\rho_s^2 - 1\right)\left(\rho_s - 1\right)\right] - P_c\frac{a_o^3}{4}\left[\left(\rho_c - 1\right)\left(\rho_c^2 + 1\right) - 2\left(\rho_s - 1\right)\right] \\ & -\left[a_s'\,k_s\left(1 + t_s' + \frac{t_s'^2}{2}\right) + a_c'\,k_c\left(1 + t_c' + \frac{t_c'^2}{2}\right)\right]\theta_a + a_o\left[\omega_c\,P_c - \omega_s\,P_s\right] \end{aligned}\right\} \quad \text{[VI.2b']}$$

$$\boxed{\omega_s = \rho_s\,\beta_s\,k_s\,\delta_s\left(1 + h\,\beta_s\right)} \qquad \boxed{\omega_c = \rho_c\,\beta_c\,k_c\,\delta_c\left(1 + h\,\beta_c\right)} \quad \text{[VI.2b'']}$$

Annex L provides the relevant modifications to cover the three other configurations b, c and d.

(c) **Rotation of rigid ring**

$K = \dfrac{A}{D_o} \qquad \theta_R = \dfrac{12}{E\,h^3}\dfrac{R\,M_R}{Ln\,K}$ leads to, with $\theta_R = \theta_a$:

$$R\,M_R = \left[\frac{E\,h^3}{12}Ln\,K\right]\theta_R = \left[\frac{E\,h^3}{12}Ln\,K\right]\theta_a$$

Replacing RM_R by its expression [VI.2b'] leads to:

$$\left.\begin{aligned} & \overbrace{\left[\frac{E\,h^3}{12}Ln\,K + a_s'\,k_s\left(1 + t_s' + \frac{t_s'^2}{2}\right) + a_c'\,k_c\left(1 + t_c' + \frac{t_c'^2}{2}\right)\right]}^{C1}\theta_a = -a_o\,M_a + a_o^2\,V_a\left(\rho_s - 1\right) \\ & + P_s\underbrace{\left\{\frac{a_o^3}{4}\left[\left(\rho_s^2 - 1\right)\left(\rho_s - 1\right)\right] - a_o\,\omega_s\right\}}_{C2} - P_c\underbrace{\left\{\frac{a_o^3}{4}\left[\left(\rho_c - 1\right)\left(\rho_c^2 + 1\right) - 2\left(\rho_s - 1\right)\right] - a_o\,\omega_c\right\}}_{C3} \end{aligned}\right\}$$

$$C_1 = \frac{h^3}{12}\left[\underbrace{\frac{6}{h^3}(D_s + t_s)\,k_s\left(1 + t_s' + \frac{t_s'^2}{2}\right)}_{\lambda_s} + \underbrace{\frac{6}{h^3}(D_c + t_c)\,k_c\left(1 + t_c' + \frac{t_c'^2}{2}\right)}_{\lambda_c} + E\,Ln\,K\right] = \frac{h^3}{12}\left[\lambda_s + \lambda_c + E\,Ln\,K\right]$$

$$\boxed{\lambda_s = \frac{6}{h^3}(D_s + t_s)\,k_s\left(1 + t_s' + \frac{t_s'^2}{2}\right)} \qquad \boxed{\lambda_c = \frac{6}{h^3}(D_c + t_c)\,k_c\left(1 + t_c' + \frac{t_c'^2}{2}\right)}$$

Note: if $\lambda_s + \lambda_c$ is high (>3) the TS is considered clamped

if $\lambda_s + \lambda_c$ is low (<1) the TS is considered simply supported

$$C2= a_o \left\{ a_o^2 \left[\frac{(\rho_s^2-1)(\rho_s-1)}{4} \right] - \omega_s \right\} = a_o\, \omega_s^*$$

$$\boxed{\omega_s^* = a_o^2 \left[\frac{(\rho_s^2-1)(\rho_s-1)}{4} \right] - \omega_s}$$

$$C3= a_o \left\{ a_o^2 \left[\frac{(\rho_c^2+1)(\rho_c-1)}{4} - \frac{\rho_s-1}{2} \right] - \omega_c \right\} = a_o\, \omega_c^*$$

$$\boxed{\omega_c^* = a_o^2 \left[\frac{(\rho_c^2+1)(\rho_c-1)}{4} - \frac{\rho_s-1}{2} \right] - \omega_c}$$

where ω_s and ω_c are given by [VI.2b''].

When the shell and channel are integral with the TS (configuration a), edge loads M_a and V_a, which are still unknown, are linked by equation:

$$\frac{h^3}{12}\left[\lambda_s + \lambda_c + E\, Ln\, K \right] \theta_a = -a_o\, M_a + a_o^2\, V_a\, (\rho_s - 1) + a_o \left(\omega_s^*\, P_s - \omega_c^*\, P_c \right)$$

(Configuration a)

(d) **The generic equation covering the 4 configurations a, b, c and d is written**, accounting for the results obtained at Annex L for configurations b, c and d:

$$\boxed{\frac{h^3}{12}\left[\lambda_s + \lambda_c + E\, Ln\, K \right] \theta_a = -a_o\, M_a + a_o^2\, V_a\, (\rho_s - 1) + a_o \left(\omega_s^*\, P_s - \omega_c^*\, P_c \right) + \frac{a_o}{2\pi}\left[W_c\, \gamma_{bc} - W_s\, \gamma_{bs} \right] \qquad \text{[VI.2d]}}$$

where:

λ_s, λ_c and ω_s, ω_c are coefficients obtained above. They are known for a given HE.

W_s and W_c are bolt loads applied on shell and channel when the TS is gasketed (configurations b, c, d). Coefficients γ_{bs} and γ_{bc} are defined as follows:

- Configuration a: k_s given in VI.1a $\quad k_c$ given in VI.1b $\quad \gamma_{bs} = 0 \quad \gamma_{bc} = 0$
- Configuration b: k_s given in VI.1a $\quad k_c = 0 \quad \gamma_{bs} = 0 \quad \gamma_{bc} = \dfrac{G_c - C_c}{D_o}$
- Configuration c: k_s given in VI.1a $\quad k_c = 0 \quad \gamma_{bs} = 0 \quad \gamma_{bc} = \dfrac{G_c - G_1}{D_o}$
- Configuration d: $k_s = 0 \quad k_c = 0 \quad \gamma_{bs} = \dfrac{G_s - C_s}{D_o} \quad \gamma_{bc} = \dfrac{G_c - C_c}{D_o}$

UHX-13 covers the usual case where $C_s = C_c = C$, which leads to:

$$\gamma_b = 0 \quad \textit{for configuration a},$$
$$\gamma_b = \frac{G_c - C}{D_o} \quad \textit{for configuration b},$$
$$\gamma_b = \frac{G_c - G_1}{D_o} \quad \textit{for configuration c},$$
$$\gamma_b = \frac{G_c - G_s}{D_o} \quad \textit{for configuration d}$$

$$\frac{h^3}{12}\left[\lambda_s + \lambda_c + E\, Ln\, K\right]\theta_a = -a_o\, M_a + a_o^2\, V_a\left(\rho_s - 1\right) + a_o\left(\omega_s^*\, P_s - \omega_c^*\, P_c\right) + \frac{a_o}{2\pi}\gamma_b\left[W_c - W_s\right]$$

6.3 Edge Loads V_a and M_a Applied to the Tubesheet

(a) **Determination of M_a**

Quantities θ_a, V_a and M_a at periphery of TS, linked by relation [VI.2d] above, are still unknown. Equation [V.5] enables to determine:

$$\theta_R = \theta_a = \theta\left(X_a\right) = \frac{a_o}{D^*}\left[M_a\, Z_m + \left(a_o\, V_a\right) Z_v\right]$$

which leads to a 1st relationship between V_a and M_a:

$$\frac{h^3}{12}\left[\lambda_s + \lambda_c + E\, Ln\, K\right]\frac{a_o}{D^*}\left[M_a\, Z_m + \left(a_o\, V_a\right) Z_v\right] = -a_o\, M_a + a_o^2\, V_a\left(\rho_s - 1\right) + a_o\left(\omega_s^*\, P_s - \omega_c^*\, P_c\right) + \frac{a_o}{2\pi}\left[W_c\,\gamma_{bc} - W_s\,\gamma_{bs}\right]$$

$$\Phi = \frac{h^3}{12}\frac{1}{D^*}\left[\lambda_s + \lambda_c + E\, Ln\, K\right] = \frac{h^3}{12}\frac{12\left(1-\nu^{*2}\right)}{E^*\, h^3}\left[\lambda_s + \lambda_c + E\, Ln\, K\right]$$

$$\Phi = \frac{\left(1-\nu^{*2}\right)}{E^*}\left[\lambda_s + \lambda_c + E\, Ln\, K\right] \qquad \text{[VI.3]}$$

Note: Φ denotes the degree of restrain of the TS by the shell and channel

- *if Φ is high (>4) the TS can be considered as clamped ($\lambda_s + \lambda_c$ high)*
- *if Φ is low (<1) the TS can be considered as simply supported ($\lambda_s + \lambda_c$ low)*

Using coefficient F relative to U–tube HEs used in UHX-12.5.5:

$$F = \frac{1-\nu^*}{E^*}\left[\lambda_s + \lambda_c + ELnK\right] \qquad \Phi = (1+\upsilon^*)F$$

$$\Phi\left[M_a\, Z_m + \left(a_o\, V_a\right) Z_v\right] = -M_a + a_o\, V_a\left(\rho_s - 1\right) + \left(\omega_s^*\, P_s - \omega_c^*\, P_c\right) + \frac{1}{2\pi}\left[W_c\,\gamma_{bc} - W_s\,\gamma_{bs}\right] \qquad \text{[VI.3a']}$$

$$M_a\left(1+\Phi Z_m\right)=a_o V_a\left[\left(\rho_s-1\right)-\Phi Z_v\right]+\left[\left(\omega_s^* P_s-\omega_c^* P_c\right)+\left(\frac{W_c \gamma_{bc}-W_s \gamma_{bs}}{2\pi}\right)\right]$$

$$M_a=\left(a_o V_a\right)\underbrace{\frac{\left(\rho_s-1\right)-\Phi Z_v}{1+\Phi Z_m}}_{Q_1}+\underbrace{\frac{\left(\omega_s^* P_s-\omega_c^* P_c\right)+\left(W_c\frac{\gamma_{bc}}{2\pi}-W_s\frac{\gamma_{bs}}{2\pi}\right)}{1+\Phi Z_m}}_{Q_2} \quad \text{[VI.3a]}$$

$$M_a=\left(a_o V_a\right)Q_1+Q_2$$

$$Q_1=\left(a_o V_a\right)\frac{\left(\rho_s-1\right)-\Phi Z_v}{1+\Phi Z_m} \qquad Q_2=\frac{\left(\omega_s^* P_s-\omega_c^* P_c\right)+\left(W_c\frac{\gamma_{bc}}{2\pi}-W_s\frac{\gamma_{bs}}{2\pi}\right)}{1+\Phi Z_m}$$

For the usual case (see VI.2d) covered by UHX-13 ($C_s=C_c=C$ and $W_s=W_c=W^*$):

$$Q_2=\frac{\left(\omega_s^* P_s-\omega_c^* P_c\right)+W^*\frac{\gamma_b}{2\pi}}{1+\Phi Z_m}$$

Note: In M_a formula:

- *term $\left(a_o V_a\right)Q_1$ represents the moment due to the equivalent pressure P_e acting on the TS.*
- *term Q_2 is the sum of the moments acting on the TS, due to pressures P_s, P_c and bolt load W.*

(b) **2nd relationship between V_a and M_a**

A 2nd relationship between V_a and M_a is obtained from boundary condition [V.2c]:

$$w_a=a_o\,\theta_a\left(\rho_s-1\right)$$

Replacing w_a and θ_a by their expressions [V.3] and [V.5], [V.2c] is written:

$$\frac{Q}{k_w}-\frac{a_o^2}{D^*}\left[M_a Z_m+\left(a_o V_a\right)Z_d\right]=\frac{a_o^2}{D^*}\left[M_a Z_m+\left(a_o V_a\right)Z_v\right]\left(\rho_s-1\right)$$

$$\frac{Q}{k_w}=\frac{a_o^2}{D^*}\left\{M_a\left[\underbrace{Z_w+\left(\rho_s-1\right)Z_m}_{\rho_1}\right]+\left(a_o V_a\right)\left[\underbrace{Z_d+\left(\rho_s-1\right)Z_v}_{\rho_2}\right]\right\}$$

$$\frac{Q}{k_w}=\frac{a_o^2}{D^*}\left[M_a\,\rho_1+\left(a_o V_a\right)\rho_2\right] \quad \text{[VI.3b]} \qquad \begin{cases}\rho_1=Z_w+\left(\rho_s-1\right)Z_m\\ \rho_2=Z_d+\left(\rho_s-1\right)Z_v\end{cases}$$

where Q is given by [V.1a']:

$$Q=\Delta p^*+k_w\left[\underbrace{\left[\delta_t\left(\theta_t\right)-\delta_s\left(\theta_s\right)\right]+\left[\delta_t\left(v_t\right)-\delta_s\left(v_s\right)\right]-\left[\delta_J\left(P_s\right)\right]}_{\delta}-\delta_s\left(V_s\right)\right]$$

$$\delta = \left[\delta_t(\theta_t) - \delta_s(\theta_s)\right] + \left[\delta_t(\nu_t) - \delta_s(\nu_s)\right] - \left[\delta_J(P_s)\right]$$

In this equation, all quantities are known except $\delta_s(V_s) = \frac{2\pi\, a_s^{'}}{J\, k_s^{'}} V_s$:

With [VI.2a-3]: $a_s^{'}\, V_s = a_o\, V_a + \frac{a_o^2}{2} P_c + \frac{a_s^2 - a_o^2}{2} P_s$

$$\frac{Q}{k_w} = \frac{\Delta p^*}{k_w} + \delta - \frac{2\pi}{J\, k_s^{'}}\left[(a_o\, V_a) + \frac{a_o^2}{2} P_c + \frac{a_s^2 - a_o^2}{2} P_s\right]$$

(c) Determination of V_a

Introducing Q/k_w in [VI.3b] leads to a 2nd relationship between V_a and M_a, already linked by relationship [VI.3a] $M_a = (a_o\, V_a)\, Q_1 + Q_2$

which enables to determine V_a:

$$\frac{Q}{k_w} = \frac{\Delta p^*}{k_w} + \delta - \frac{2\pi}{J\, k_s^{'}}\left[(a_o\, V_a) + \frac{a_o^2}{2} P_c + \frac{a_s^2 - a_o^2}{2} P_s\right] = \frac{a_o^2}{D^*}\left\{\left[(a_o\, V_a)\, Q_1 + Q_2\right]\rho_1 + \left[(a_o\, V_a)\,\rho_2\right]\right\}$$

$$a_o\, V_a\left[1 + J\frac{k_s^{'}\, a_o^2}{2\pi\, D^*}(\rho_1\, Q_1 + \rho_2)\right] = \frac{J\, k_s^{'}}{2\pi}\left[\frac{\Delta p^*}{k_w} + \delta\right] - \frac{a_o^2}{2} P_c - \frac{a_s^2 - a_o^2}{2} P_s - \frac{J\, k_s^{'}}{2\pi}\frac{a_o^2}{D^*}\rho_1\, Q_2$$

- $k^4 = \frac{k_w}{D^*} = \frac{N_t\, k_t}{\pi\, a_o^2\, D^*} \Rightarrow \frac{1}{\pi\, D^*} = \frac{k^4\, a_o^2}{N_t\, k_t} \Rightarrow \frac{k_s^{'}\, a_o^2}{2\pi\, D^*} = \frac{k_s^{'}\, a_o^4\, k^4}{2 N_t\, k_t} = K_{s,t}\frac{X_a^4}{2}$
- $(\rho_1\, Q_1 + \rho_2) = Z_m\, Q_1 + (\rho_s - 1)\, Q_1\, Z_m + Z_d + (\rho_s - 1)\, Z_v = (Z_w\, Q_1 + Z_d) + (\rho_s - 1)(Z_w\, Q_1 + Z_v)$
- $J\frac{k_s^{'}\, a_o^2}{2\pi\, D^*}(\rho_1\, Q_1 + \rho_2) = J K_{s,t}\left[\underbrace{(Z_w\, Q_1 + Z_d)\frac{X_a^4}{2}}_{Q_{z1}} + (\rho_s - 1)\underbrace{(Z_w\, Q_1 + Z_v)\frac{X_a^4}{2}}_{Q_{z2}}\right] = J K_{s,t}\, F_q^{'}$

$$\boxed{Q_{Z1} = (Z_w\, Q_1 + Z_d)\frac{X_a^4}{2}} \qquad \boxed{Q_{Z2} = (Z_m\, Q_1 + Z_v)\frac{X_a^4}{2}} \qquad F_q^{'} = Q_{z1} + (\rho_s - 1)\, Q_{z2}$$

$$a_o\, V_a\left(1 + J K_{s,t}\, F_q^{'}\right) = J\frac{k_s^{'}}{2\pi}\left[\left(\frac{\Delta p^*}{k_w} + \delta\right) - \frac{a_o^2}{D^*}\rho_1\, Q_2\right] - \frac{a_o^2}{2} P_c - \frac{a_s^2 - a_o^2}{2} P_s \qquad \text{[VI.3c]}$$

For a given HE all above quantities are known, which enable to calculate V_a and thus M_a:

$$\boxed{a_o\, V_a = \frac{J\frac{k_s^{'}}{2\pi}\left[\left(\frac{\Delta p^*}{k_w} + \delta\right) - \frac{a_o^2}{D^*}\rho_1\, Q_2\right] - \frac{a_o^2}{2} P_c - \frac{a_s^2 - a_o^2}{2} P_s}{\left(1 + J\, K_{s,t}\, F_q^{'}\right)}} \qquad \text{[VI.3c']}$$

M_a is given by [VI.3a]: $\boxed{M_a = (a_o\, V_a)\, Q_1 + Q_2}$

7 EQUIVALENT PRESSURE ACTING ON TUBESHEET

7.1 Definition

(a) A circular plate under uniform pressures P_s and P_t is subjected to a differential pressure $P_e = P_s - P_t$

The axial force V_a at periphery is determined from the plate equilibrium:

$$2\pi\, a_o\, V_a = P_e\, \pi\, a_o^2 \quad \Rightarrow \quad a_o\, V_a = \frac{a_o^2\, P_e}{2} \qquad P_e = \frac{2\, V_a}{a_o} \qquad \text{[VII.1]}$$

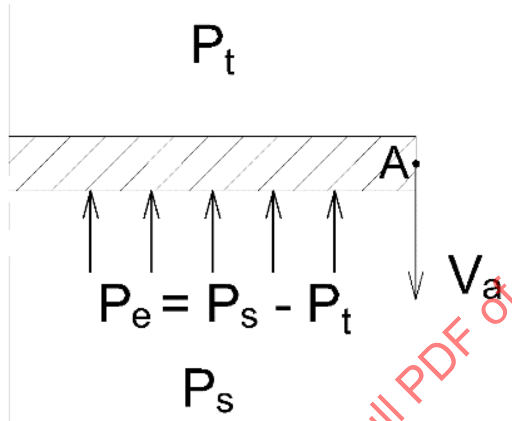


Figure 24 — Equivalent Pressure and Axial Force Acting on Plate

(b) The uniform pressure acting on the equivalent solid plate is written: $P_e = \dfrac{2\, V_a}{a_o}$

This equivalent pressure is due to the various loads acting on the TS:

- Differential pressure: $\Delta p^* = x_s\, P_s - x_t\, P_t$
- Axial differential thermal expansion tubes-shell: $\delta(\theta) = \dfrac{\gamma}{2}$
- Force due to Poisson's ratio of tubes: $\delta_t(\nu_t) = -\dfrac{2}{k_w}\left[(1-x_t)\, P_t - (1-x_s)\, P_s\right]\nu_t$
- Force due to Poisson's ratio of shell: $\delta_s(\nu_s) = -\dfrac{\pi}{2k_s'}\left[P_s\, D_s^2\right]\nu_s$
- Force due to shell pressure acting on the joint: $\delta_J(P_s) = \dfrac{\pi}{16}\dfrac{D_J^2 - D_s^2}{K_J} P_s$

(see Annex C, Annex D, and Annex E)

Note: Equilibrium of the equivalent solid plate is written (see [VI.2a-3]):

$$a_o\, V_a = \frac{a_o^2}{2}(P_s - P_t) - \frac{a_s^2}{2} P_s + a_s'\, V_s$$

Equilibrium of a circular plate is written (see [VII.1]): $a_o\, V_a = \dfrac{a_o^2}{2}(P_s - P_t)$

7.2 Determination of P_e

Equivalent pressure $P_{\mathrm{e}} = \dfrac{2\,V_{\mathrm{a}}}{a_{\mathrm{o}}}$ can be obtained directly from V_a formula [VI.3c]:

$$a_o\, V_{\mathrm{a}} \left(1 + J\, K_{\mathrm{s,t}}\, F_{\mathrm{q}}^{'}\right) = J\, \frac{k_{\mathrm{s}}^{'}}{2\pi} \left[\left(\frac{\Delta p^*}{k_{\mathrm{w}}} + \delta \right) - \frac{a_o^2}{D^*}\, \rho_1\, Q_2 \right] - \frac{a_o^2}{2}\, P_t - \frac{a_{\mathrm{s}}^2 - a_o^2}{2}\, P_{\mathrm{s}}$$

All terms appearing in this formula are known for a given HE. However, like TEMA and CODAP did, it is better to write P_e formula in a format which makes appear explicitly the contributions of pressures P_s and P_t, acting directly on the equivalent plate, loads acting on the unperforated rim, bolt load W* when the TS is gasketed, and differential thermal expansion tubes-shell. Following manipulations are necessary to achieve this goal.

$$\left(1 + J\, K_{\mathrm{s,t}}\, F_{\mathrm{q}}^{'}\right) P_{\mathrm{e}} = \frac{J\, k_{\mathrm{s}}^{'}}{\pi\, a_o^2} \left[\frac{1}{k_w} \left(\Delta p^* + k_w\, \delta \right) - \frac{a_o^2}{D^*}\, \rho_1\, Q_2 \right] - P_{\mathrm{t}} - \left(\rho_{\mathrm{s}}^2 - 1 \right) P_{\mathrm{s}}$$

with:

$$\frac{k_{\mathrm{s}}^{'}}{\pi\, a_o^2\, k_{\mathrm{w}}} = \frac{k_{\mathrm{s}}^{'}}{N_t\, k_{\mathrm{t}}} = K_{\mathrm{s,t}} \qquad \frac{k_{\mathrm{s}}^{'}}{\pi\, D^*} = \frac{X_{\mathrm{a}}^4}{a_o^2}\, K_{\mathrm{s,t}} \qquad \delta = \left[\delta_t(\theta_t) - \delta_s(\theta_s) \right] + \left[\delta_{\mathrm{t}}(\nu_{\mathrm{t}}) - \delta_{\mathrm{s}}(\nu_{\mathrm{s}}) \right] - \left[\delta_{\mathrm{J}}(P_{\mathrm{s}}) \right]$$

- $$\frac{k_{\mathrm{s}}^{'}}{\pi a_o^2 k_w} \left[\Delta p^* + k_w \delta \right] = K_{\mathrm{s,t}} \left[(x_{\mathrm{s}} P_{\mathrm{s}} - x_{\mathrm{t}} P_t) + k_{\mathrm{w}}\, \frac{\gamma}{2} - 2(1 - x_{\mathrm{t}})\, \nu_{\mathrm{t}} P_t + 2(1 - x_s)\, \nu_{\mathrm{t}} P_{\mathrm{s}} - \frac{\pi\, k_{\mathrm{w}}}{2 k_s^{'}}\, P_{\mathrm{s}} D_{\mathrm{s}}^2 \nu_{\mathrm{s}} - \frac{\pi\, k_w}{16}\, \frac{D_{\mathrm{J}}^2 - D_{\mathrm{s}}^2}{K_{\mathrm{J}}} \right]$$

  $$\frac{\pi\, k_{\mathrm{w}}}{2\, k_s^{'}}\, P_{\mathrm{s}}\, D_{\mathrm{s}}^2\, \nu_{\mathrm{s}} = \frac{\pi\, N_t\, K_{\mathrm{t}}}{2\, k_{\mathrm{s}}^{'}\, \pi\, a_o^2}\, D_s^2\, \nu_{\mathrm{s}}\, P_{\mathrm{s}} = \frac{2}{K_{s,t}} \left(\frac{D_{\mathrm{s}}}{D_o} \right)^2 \nu_{\mathrm{s}}\, P_{\mathrm{s}}$$

  $$\frac{\pi\, k_{\mathrm{w}}}{16\, K_{\mathrm{J}}} \left(D_{\mathrm{J}}^2 - D_{\mathrm{s}}^2 \right) = \frac{k_{\mathrm{s}}^{'}}{a_o^2\, K_{\mathrm{s,t}}\, 16\, K_{\mathrm{J}}} \left(D_{\mathrm{J}}^2 - D_{\mathrm{s}}^2 \right) = \frac{1}{2\, K_{\mathrm{s,t}}}\, \frac{k_{\mathrm{s}}^{'}}{2\, K_{\mathrm{J}}}\, \frac{D_{\mathrm{J}}^2 - D_{\mathrm{s}}^2}{4\, a_o^2} = \frac{1}{2\, K_{\mathrm{s,t}}}\, \frac{1 - J}{J}\, \frac{D_{\mathrm{J}}^2 - D_{\mathrm{s}}^2}{D_o^2}$$

- $$-\frac{k_{\mathrm{s}}^{'}}{\pi\, D^*}\, \rho_1\, Q_2 = -K_{\mathrm{s,t}}\, \frac{1}{a_o^2} \underbrace{\left[\frac{Z_{\mathrm{w}} + (\rho_{\mathrm{s}} - 1)\, Z_{\mathrm{m}}}{1 + \Phi\, Z_{\mathrm{m}}}\, X_{\mathrm{a}}^4 \right]}_{U} \left[\left(\omega_{\mathrm{s}}^*\, P_{\mathrm{s}} - \omega_{\mathrm{c}}^*\, P_{\mathrm{t}} \right) + \frac{\gamma_{\mathrm{b}}}{2\,\pi}\, W^* \right]$$

  W is given by [VI.2d]

  $$= K_{\mathrm{s,t}} \left\{ -\frac{U}{a_o^2} \left[\omega_{\mathrm{s}}^*\, P_{\mathrm{s}} - \omega_{\mathrm{c}}^*\, P_{\mathrm{t}} \right] - \frac{U}{a_o^2} \left[\frac{\gamma_{\mathrm{b}}}{2\,\pi}\, W^* \right] \right\} \qquad \text{where} \qquad \boxed{U = \frac{Z_{\mathrm{w}} + (\rho_{\mathrm{s}} - 1)\, Z_{\mathrm{m}}}{1 + \Phi\, Z_{\mathrm{m}}}\, X_{\mathrm{a}}^4}$$

  1st term accounts for the effect of moments due to pressures P_s and P_t acting on the unperforated rim:

  $$\boxed{P_{\mathrm{rim}} = -\frac{U}{a_o^2} \left[\omega_{\mathrm{s}}^*\, P_{\mathrm{s}} - \omega_{\mathrm{c}}^*\, P_{\mathrm{t}} \right]}$$

  2nd term accounts for the effective bolting load W* acting on the TS (configurations b, c, d)

  $$\boxed{P_{\mathrm{w}} = -\frac{U}{a_o^2}\, \frac{\gamma_{\mathrm{b}}}{2\,\pi}\, W^*}$$

  Thus $\quad -\dfrac{k_{\mathrm{s}}^{'}}{\pi\, D^*}\, \rho_1\, Q_2 = K_{\mathrm{s,t}} \left[P_{rim} + P_w \right]$

$$\left(1+J\,K_{s,t}\,F_q^{'}\right)P_e = J\,K_{s,t}\left\{\begin{array}{l} P_s\left[x_s+2\left(1-x_s\right)\nu_t+\dfrac{2}{K_{s,t}}\left(\dfrac{D_s}{D_o}\right)^2\nu_s-\dfrac{\rho_s^2-1}{J\,K_{s,t}}-\dfrac{1-J}{2\,J\,K_{s,t}}\dfrac{D_J^2-D_s^2}{D_o^2}\right] \\ -P_t\left[x_t+2\left(1-x_t\right)\nu_t+\dfrac{1}{J\,K_{s,t}}\right]+\gamma\left[\dfrac{k_w}{2}\right]+\left[P_w\right]+\left[P_{rim}\right]\end{array}\right\}$$

$$P_e=\frac{J\,K_{s,t}}{1+J\,K_{s,t}\,F_q^{'}}\left[P_s^{'}-P_t^{'}+P_\gamma+P_w+P_{rim}\right] \quad \text{[VII.2]} \qquad F_q^{'}=Q_{z1}\left(\rho_s-1\right)Q_{z2}$$

In this formula:

$P_s^{'}$ **accounts for the loads due pressure P_s acting on:**

- the perforated TS,
- the external wall of the tubes (effect of Poisson's ratio ν_t),
- the external wall of the shell (effect of Poisson's ratio ν_s),
- the unperforated rim,
- the sidewall of the joint.

$$P_s^{'}=P_s\left[x_s+2\left(1-x_s\right)\nu_t+\frac{2}{K_{s,t}}\left(\frac{D_s}{D_o}\right)^2\nu_s-\frac{\rho_s^2-1}{J\,K_{s,t}}-\frac{1-J}{2\,J\,K_{s,t}}\frac{D_J^2-D_s^2}{D_o^2}\right]$$

$P_t^{'}$ **accounts for the loads due pressure P_t acting on**:

- the perforated TS,
- the internal wall of the tubes (effect of Poisson's ratio ν_t),
- the unperforated rim.

$$P_t^{'}=P_t\left[x_t+2\left(1-x_t\right)\nu_t+\frac{1}{J\,K_{s,t}}\right]$$

P_γ **accounts for the loads due to differential thermal expansion between tubes and shell γ**.

$$P_\gamma=\gamma\left[\frac{k_w}{2}\right]=\left[\frac{N_t\,K_t}{\pi\,a_o^2}\right]\gamma$$

P_W **accounts for the effect bolting load W* acting on the TS** (configurations b, c, d)

$$P_w=-\frac{U}{a_o^2}\frac{\gamma_b}{2\,\pi}W^*$$

P_{rim} **accounts for the effect of moments due to pressures P_s and P_t acting on the unperforated rim:**

$$P_{rim}=-\frac{U}{a_o^2}\left[\omega_s^*\,P_s-\omega_c^*\,P_t\right]$$

Note 1: Different expression of the equivalent pressure. *P_e can be formulated differently if it is needed to set J equal to 0 to simulate a shell of rigidity 0. Introducing the numerator $JK_{s,t}$ inside the parenthesis leads to:*

$$\boxed{P_e = \frac{1}{1+J\ K_{s,t}\ F_q^{'}}\left[P_{s1}^{'} - P_{t1}^{'} + P_{\gamma 1} + P_{w1} + P_{rim1}\right]}$$

with: $$P_{s1}^{'} = P_s\left[\left(JK_{s,t}\right)x_s + (JK_{s,t})2(1-x_s)\nu_t + (J)2\left(\frac{D_s}{D_o}\right)^2\nu_s - (\rho_s^2-1) - \frac{(1-J)}{2}\frac{\left[D_J^2 - D_s^2\right]}{D_o^2}\right]$$

$$P_{t1}^{'} = P_t\left[\left(JK_{s,t}\right)x_t + \left(JK_{s,t}\right)2\left(1-x_t\right)\nu_t + 1\right]$$

$$P_{\gamma 1} = JK_{s,t}\left[\frac{N_t K_t}{\pi\ a_o^2}\gamma\right] \qquad P_{W1} = JK_{s,t}\left[-\frac{U}{a_o^2}\frac{\gamma_b}{2\pi}W^*\right] \qquad P_{rim} = JK_{s,t}\left[-\frac{U}{a_o^2}\left(\omega_s^* P_s - \omega_c^* P_t\right)\right]$$

This is the way TEMA formula is presented.

Note 2: Externally sealed HE. *Setting the bellows rigidity to 0 simulates the case of a HE with an externally sealed floating head which will be covered in PART 4. K_J=0 and D_J=0 lead to:*

$$J = \frac{K_J}{K_s + K_J} = 0 \qquad JK_{s,t}=0 \qquad P_{s1}^{'} = \left[-(\rho_s^2-1)\right]P_s = (1-\rho_s^2)P_s \qquad P_{t1}^{'} = [1]P_t = P_t$$

$$P_{\gamma 1} = JK_{s,t}\left[\frac{N_t K_t}{\pi\ a_o^2}\gamma\right] = 0 \quad P_{W1} = JK_{s,t}\left[-\frac{U}{a_o^2}\frac{\gamma_b}{2\pi}W^*\right] = 0 \quad P_{rim1} = JK_{s,t}\left[-\frac{U}{a_o^2}\left(\omega_s^* P_s - \omega_c^* P_t\right)\right] = 0$$

$$\boxed{P_e = P_{s1}^{'} - P_{t1}^{'} = (1-\rho_s^2)P_s - P_t}$$ *which is the UHX-14 formula for an externally sealed floating head.*

Note 3: Direct determination of the equivalent pressure. *Equivalent pressures P'_s , P'_t and P'_γ can be obtained directly by examining the loads applied on the TS as shown in Annex M.*

8 STRESSES IN THE HEAT-EXCHANGER COMPONENTS

Deformations (w, θ) and loads (q, Q_r, M_r) applied in the TS have been determined in Sections 5.3 to 5.7 as a function of the TS radius. These formulas are general as they are valid whether $P_e \neq 0$ or Pe=0. They depend on loads V_a and M_a acting at TS periphery, given by equations [VI.3c'] and [VI.3a]:

$$a_o V_a = \frac{a_o^2}{2} P_e$$ with P_e given by [VII.2] $$M_a = (a_o V_a) Q_1 + Q_2$$

The formulas used in UHX-13 are presented in a different way.

- **If $V_a \neq 0$** ($P_e \neq 0$): $$\frac{M_a}{a_o V_a} = Q_1 + \frac{Q_2}{a_o V_a} = Q_3 \qquad M_a = (a_o V_a) Q_3 \qquad Q_3 = Q_1 + \frac{2Q_2}{a_o^2 P_e} \quad \text{[VIII]}$$

  The advantage of this presentation is that it enables to provide directly the maximum of the TS bending stress and the maximum of the tube stresses, thanks to a parametric study using X_a and Q_3.
- **If $V_a = 0$** ($P_e = 0$): Q_3 becomes infinity and equation [VIII] is no longer valid.

  Equation [VI.3a] shows that M_a is written: $M_a = Q_2$

  See Annex N for relevant equations to be applied in this case.

The following Sections provide the deformations (w, θ) and loads (q, Q_r, M_r) applied in the TS, using both the general formulas and the UHX-13 formulas

Maximum stresses calculated in the TS, tubes, shell and channel must remain below allowable stress limits which are defined in Section 9.

8.1 TS Net Effective Pressure

[V.4] $$q(x) = \frac{a_o^2 k_w}{D^*}\left[M_a Z_w(x) + (a_o V_a) Z_d(x)\right] = \frac{X_a^4}{a_o^2}(a_o V_a)\left[\frac{M_a}{(a_o V_a)} Z_w(x) + Z_d(x)\right]$$

$$q(x) = \frac{X_a^4}{a_o^2}(a_o V_a)\left[Q_3 Z_w(x) + Z_d(x)\right] = \underbrace{\frac{X_a^4}{2}\left[Q_3 Z_w(x) + Z_d(x)\right]}_{F_t(x)} P_e = F_t(x) P_e \quad \text{[VIII.1]}$$

As shown by Figure 21, a positive value of q denotes a pressure upward.

8.2 TS Axial Displacement

[V.3] $$w(x) = \frac{Q}{k_w} - \frac{q(x)}{k_w} = \frac{Q}{k_w} - \frac{1}{k_w}\underbrace{\frac{X_a^4}{2}\left[Q_3 Z_w(x) + Z_d(x)\right]}_{F_t(x)}$$

with [VI.3b]: $$\frac{Q}{k_w} = \frac{a_o^2}{D^*}(a_o V_a)\left[Q_3 \rho_1 + \rho_2\right]$$

$$w(x) = \frac{Q - F_t(x) P_e}{k_w} \qquad F_t(x) = \frac{X_a^4}{2}\left[Q_3 Z_w(x) + Z_d(x)\right]$$

Developing ρ_1, ρ_2, V_a, Q and $F_t(x)$, w(x) is written:

$$w(x)=\frac{a_o^4 P_e}{2D^*}\left\{\left[\left(Q_3 Z_w+Z_d\right)-\left(Q_3 Z_w(x)+Z_d(x)\right)\right]+\left[\left(\rho_s-1\right)\left(Q_3 Z_m+Z_v\right)\right]\right\}$$

The minimum and maximum values of w(x) when x varies from 0 to X_a are obtained from the minimum and maximum values $F_{t,min}$ and $F_{t,max}$ of $F_t(x)$.

$F_t(x)$ depends on parameters X_a and Q_3 which are known for a given HE. X_a is usually comprised between 1 and 20, and Q_3 between -0.8 and +0.8. A parametric study has been performed using 20 values for X_a (X_a= 1 ; 2 ; 3…… ; 20) and 17 values of Q_3 (Q_3= -0.8 ; -0.7 ;….-0.1 ; 0.0 ; +0.1 ;….. ; +0.7 ; +0.8), which enables to determine, for each couple [X_a , Q_3]:

$$F_{t,min} = \text{MIN}\left[F_t(x)\right] \quad \text{and} \quad F_{t,max} = \text{MAX}\left[F_t(x)\right]$$

Annex O provides for $1 \le X_a \le 20$ and $-0.8 \le Q_3 \le +0.8$:

- values and graphs of Ft(x) for $0 < x < X_a$
- values and graphs of $F_{t,min}$ and $F_{t,max}$
- locations of the minimum and maximum of $F_t(x)$: x_{min} and x_{max}

The minimum and maximum of w(x) are given by:

$$w_1=\frac{1}{k_w}\left[\frac{X_a^4}{2}\left(\rho_1\cdot Q_3+\rho_2\right)-F_{t,\min}\right]\cdot P_e \quad \text{and} \quad w_2=\frac{1}{k_w}\left[\frac{X_a^4}{2}\left(\rho_1\cdot Q_3+\rho_2\right)-F_{t,\max}\right]\cdot P_e$$

As shown by Figure 21, a positive value of w denotes a deflection upward.

The maximum tubesheet deflection is given by:

$$w_{\max}=MAX\left[\left|w_1\right|,\left|w_2\right|\right]$$

8.3 TS Rotation

[V.5] $$\theta(x)=\frac{a_o}{D^*}\left[M_a Z_m(x)+(a_o V_a) Z_v(x)\right]=\frac{a_o}{D^*}(a_o V_a)\left[Q_3 Z_m(x)+Z_v(x)\right]$$

$$\theta(x)=\frac{a_o^3 P_e}{2D^*}\left[Q_3 Z_m(x)+Z_v(x)\right]=\frac{X_a^3}{k^3\, 2D^*}\left[Q_3 Z_m(x)+Z_v(x)\right]P_e=\frac{k}{k_w}\underbrace{\frac{X_a^3}{2}\left[Q_3 Z_m(x)+Z_v(x)\right]}_{F_\theta(x)}P_e$$

$$\theta(x)=\frac{k}{k_w}F_\theta(x)\,P_e \qquad F_\theta(x)=\frac{X_a^3}{2}\left[Q_3 Z_m(x)+Z_v(x)\right]$$

Note: for x=0: $\left\{\begin{matrix} Z_m(x)=0 \\ Z_v(x)=0 \end{matrix}\right\}$ *θ(0)=0: the rotation at TS center is 0 as expected.*

As shown by Figure 21, a positive value of θ denotes a rotation clockwise.

8.4 Stresses in the Tubesheet

(a) **Bending stress**

[V.7]

$$M_r(x) = M_a\, Q_m(x) + (a_o\, V_a)\, Q_v(x) = (a_o\, V_a)\left[Q_3\, Q_m(x) + Q_v(x)\right] = a_o^2\, P_e \underbrace{\left[\frac{Q_3\, Q_m(x) + Q_v(x)}{2}\right]}_{F_m(x)}$$

$$M_r(x) = a_o^2\, P_e\, F_m(x) \quad F_m(x) = \frac{Q_3\, Q_m(x) + Q_v(x)}{2} \quad \text{with:} \begin{cases} Q_m(x) = \dfrac{bei'\,\Psi_2(x) - ber'\,\Psi_1(x)}{Z_a} \\ Q_v(x) = \dfrac{\Psi_1\,\Psi_2(x) - \Psi_2\,\Psi_1(x)}{X_a\, Z_a} \end{cases}$$

Note: At TS periphery(x=X_a): $\left.\begin{matrix} Q_m(X_a) = 1 \\ Q_v(X_a) = 0 \end{matrix}\right\} F_m(X_a) = \dfrac{Q_3}{2} \Rightarrow M_r(X_a) = M_a$ *:*

the boundary condition is satisfied.

The bending stress in the TS is given by: $\sigma_r(x) = \dfrac{6\, M_r(x)}{\mu^*\, h^2} = \dfrac{6\, a_o^2\, P_e\, F_m(x)}{\mu^*\, h^2}$

$$\boxed{\sigma_r(x) = \frac{1.5\, F_m(x)}{\mu^*}\left(\frac{2\, a_o}{h}\right)^2 P_e} \qquad \boxed{F_m(x) = \frac{Q_3\, Q_m(x) + Q_v(x)}{2}}$$

As shown by Figure 21, a positive value of $\sigma_r(x)$ denotes a bending moment clockwise.

The maximum value of $\sigma_r(x)$ when x varies from 0 to X_a is obtained from the maximum value $F_{m,max}$ of $F_m(x)$. $F_m(x)$ depends on parameters X_a and Q_3 which are known for a given HE. A parametric study has been performed which enables to determine, for each couple [X_a , Q_3]: $F_{m,,max}$ = MAX [$F_m(x)$].
Figure 25 shows the bending stress distribution throughout the TS for Q_3=0.0 and X_a=1, 3, 5, 7, 10 and 15.

Annex P provides for $1 \le X_a \le 20$ and $-0.8 \le Q_3 \le +0.8$:

- values and graphs of $F_m(x)$ for $0 \le x \le X_a$
- values and graphs of $F_{m,max}$
- location of the maximum of $F_m(x)$: x_{max}

The maximum stress is obtained for: $F_m = \left|F_{m,max}\right| = MAX\left[\left|F_m(x)\right|\right]$

The maximum bending stress in the TS is written: $\boxed{\sigma = \dfrac{1.5\, F_m}{\mu^*}\left(\dfrac{2\, a_o}{h}\right)^2 P_e}$ $\boxed{F_m = MAX\left[\left|F_m(x)\right|\right]}$

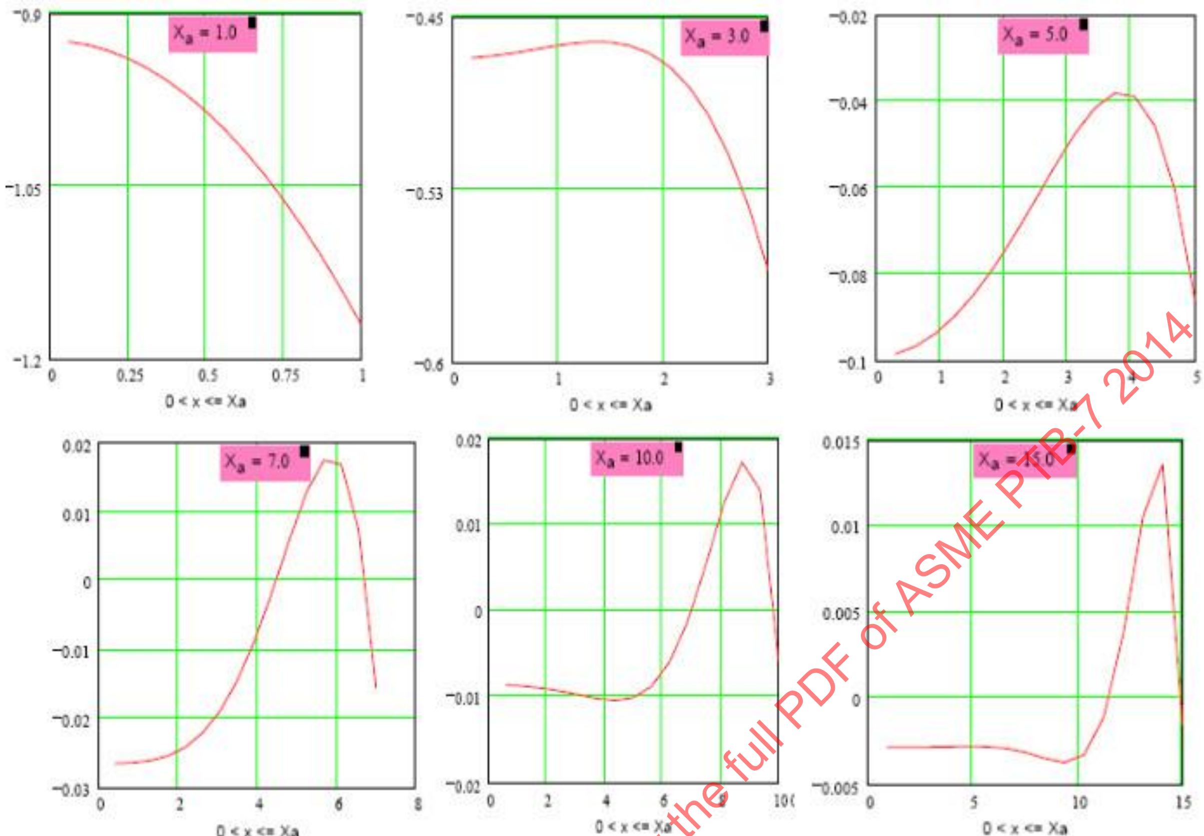


Figure 25 — Bending Stress Distribution Throughout the TS for Q3=0.0 and Xa=1, 3, 5, 7, 10 and 15

(b) **Shear stress**

[V.6]

$$Q_r(x) = \frac{1}{a_o}\left[M_a\, Q_\alpha(x) + (a_o\, V_a)\, Q_\beta(x)\right] = \frac{(a_o\, V_a)}{a_o}\left[Q_3\, Q_\alpha(x) + Q_\beta(x)\right] = \frac{a_o}{2}\underbrace{\left[Q_3\, Q_\alpha(x) + Q_\beta(x)\right]}_{F_Q(x)} P_e$$

$$Q_r(x) = \frac{a_o}{2} F_Q(x)\, P_e \qquad F_Q(x) = Q_3\, Q_\alpha(x) + Q_\beta(x) \quad \text{with:} \quad \begin{cases} Q_\alpha(x) = \dfrac{ber'\, bei'x - bei'\, ber'x}{Z_a} X_a \\ Q_\beta(x) = \dfrac{\psi_2\, bei'x - \psi_1\, ber'x}{Z_a} \end{cases}$$

Note: At TS periphery (x=Xa) $\left.\begin{matrix} Q_\alpha(X_a) = 0 \\ Q_\beta(X_a) = 1 \end{matrix}\right\} F_Q(X_a) = 1 \;\Rightarrow\; Q_r(X_a) = V_a$:

the boundary condition is satisfied.

The shear stress in the TS, averaged throughout TS thickness, is given by:

$$\tau(x) = \frac{Q_r(x)}{\mu\, h} = \frac{a_o}{2\,\mu\, h} F_Q(x)\, P_e$$

$$\boxed{\tau(x)=\frac{1}{2\mu}\frac{a_o}{h}F_Q(x)P_e} \qquad \boxed{F_Q(x)=Q_3\,Q_\alpha(x)+Q_\beta(x)}$$

The maximum value of τ(x) when x varies from 0 to X_a is obtained from the maximum value $F_{Q,max}$ of $F_Q(x)$. $F_Q(x)$ depends on parameters X_a and Q_3 which are known for a given HE. A parametric study has been performed, which enables to determine, for each couple [X_a , Q_3]: $F_{Q,max}$ = MAX [F_Q (x)]

Annex Q provides for $1 \le X_a \le 20$ and $-0.8 \le Q_3 \le +0.8$:

- values and graphs of $F_Q(x)$ for $0 \le x \le X_a$
- values and graphs of $F_{Q,max}$
- location of the maximum of $F_Q(x)$: x_{max}

The maximum stress is obtained for: $F_Q = |F_{Q,max}| = MAX\left[|F_Q(x)|\right]$

Maximum shear stress in the TS is written: $\tau_{max}=\frac{F_Q}{2\mu}\cdot\frac{a_o}{h}\cdot P_e \qquad F_Q = MAX\left[|F_Q(x)|\right]$

Shear stress at periphery is written: $\boxed{\tau=\tau(X_a)=\frac{1}{4\mu}\frac{D_o}{h}P_e}$ as $F_Q(X_a)=1$

This formula does not provide necessarily the maximum shear stress as $F_Q(x)$ is not always maximum at periphery. However, a parametric study has shown that the maximum of τ(x) appears for values of Q_3 and X_a for which the TS bending stress σ(x) controls the design.

TEMA rules provide the same formula, but use the equivalent diameter D_L corresponding to the perimeter of the outermost ligaments, instead of the equivalent diameter D_o of outer tube limit circle. The equivalent diameter D_L is calculated from the perimeter of the tube layout, $C_P = \pi D_L$ and the area $A_P = \pi D_L^2 / 4$ enclosed by this perimeter. This leads to $D_L = 4A_p / C_P$

D_L is always lower than D_o and leads to a lower TS shear stress: $\tau=\frac{1}{4\mu}\frac{D_L}{h}P_e$

ASME 2013 edition has adopted that formula under the form $\boxed{\tau=\left(\frac{1}{4\mu}\right)\left(\frac{1}{h}\right)\left(\frac{4A_p}{C_p}\right)P_e}$

If τ≤0.8S : $P_e \le 3.2\mu\frac{h}{D_L}S$ the shear stress does not control and τ does not need to be calculated

8.5 Axial Membrane Stress in Tubes

Axial force $F_t^*(r)$ in tube row at radius r is obtained from [V.1a]:

$$q(r)=-\frac{V_t(r)}{\pi\, a_o^2 / N_t}+\Delta p^* \qquad \Delta p^* = x_s P_s - x_t P_t$$

$$V_t(x)=\frac{\pi\,a_o^2}{N_t}\left[\Delta p^* - q(x)\right] \quad \text{with [VIII.1]} \quad q(x)=\underbrace{\frac{X_a^4}{2}\left[Q_3\,Z_w(x)+Z_d(x)\right]}_{F_t(x)}P_e = F_t(x)\,P_e$$

$$\sigma_t(x)=\frac{V_t(x)}{s_t}=\frac{\pi\,a_o^2}{N_t\,s_t}\left[\Delta p^* - F_t(x)\,P_e\right] \quad \text{with [III.2.b1]} \quad \frac{\pi\,a_o^2}{N_t\,s_t}=\frac{1}{x_t-x_s}$$

$$\boxed{\sigma_t(x)=\frac{1}{x_t-x_s}\left[\Delta p^* - F_t(x)\,P_e\right]} \qquad \boxed{F_t(x)=\frac{X_a^4}{2}\left[Q_3\,Z_w(x)+Z_d(x)\right]}$$

$\sigma_t(x)$ is either positive (tubes in tension) or negative (tubes in compression). The extreme values (maximum and minimum) of $\sigma_t(x)$ must be determined to obtain the maximum stresses in the tubes. These extreme values of $\sigma_t(x)$ are obtained for the minimum $F_{t,min}$ and maximum $F_{t,max}$ values of $F_t(x)$:

$$\boxed{F_{t,min}=MIN\left[F_t(x)\right] \qquad F_{t,max}=MAX\left[F_t(x)\right]}$$

Annex O provides for $1 \le X_a \le 20$ and $-0.8 \le Q_3 \le +0.8$:

- values and graphs of $F_t(x)$ for $0 \le x \le X_a$
- values and graphs of $F_{t,min}$ and $F_{t,max}$
- locations of the minimum and maximum of $F_t(x)$: x_{min} and x_{max}
- values of $F_t(X_a)$ and F_q

The minimum and maximum of $\sigma_t(x)$ are given by:

$$\boxed{\sigma_{t,1}=\frac{1}{x_t-x_s}\left[\left(x_s P_s - x_t P_t\right)-P_e F_{t,min}\right]} \qquad \boxed{\sigma_{t,2}=\frac{1}{x_t-x_s}\left[\left(x_s P_s - x_t P_t\right)-P_e F_{t,max}\right]}$$

If tubes are in compression ($\sigma_{t,1}$ or $\sigma_{t,1}$ negative), special consideration must be given to their load carrying ability, which could lead to failure by buckling if a substantial number of tubes were above their buckling limit.

See Annex R which provides the allowable buckling stress limit for tubes.

Note: The general of $\sigma_t(x)$ can be written, using the general formula of q(x) given by [V.4]:

$$q(x)=\frac{X_a^4}{a_o^2}\left[M_a\,Z_w(x)+\left(a_o V_a\right)Z_d(x)\right]$$

$$\sigma_t(x)=\frac{1}{x_t-x_s}\left\{\Delta p^* - \frac{X_a^4}{a_o^2}\left[\underbrace{M_a\,Z_w(x)+\left(a_o\,V_a\right)Z_d(x)}_{A}\right]\right\} \qquad M_a=\left(a_o\,V_a\right)Q_1+Q_2$$

$$[A]=\left(a_o V_a\right)Q_1\,Z_w(x)+Q_2\,Z_w(x)+\left(a_o\,V_a\right)Z_d(x)=\frac{a_o^2\,P_e}{2}\left[Z_d(x)+Q_1\,Z_w(x)\right]+Q_2\,Z_w(x)$$

$$\sigma_t(x)=\frac{1}{x_t-x_s}\left\{\Delta p^* - X_a^4\left[\frac{P_e}{2}\left(Z_d(x)+Q_1\,Z_w(x)\right)+\frac{Q_2}{a_o^2}\,Z_w(x)\right]\right\}$$

This equation can be used whether $P_e \neq 0$ or $P_e = 0$. However, it does not permit to determine the minimum $F_{t,min}$ and maximum $F_{t,max}$ of $F_t(x)$ using the parametric study of X_a and Q_3 to obtain the curves $F_{t,min}$ and $F_{t,max}$ as a function of X_a and Q_3.

8.6 Stresses in the Shell

(a) **Axial membrane stress**

[VI.2a-3] $a_s' V_s = a_o V_a + \frac{a_o^2}{2} P_t + \frac{a_s^2 - a_o^2}{2} P_s = a_o V_a + \frac{a_s^2 - a_o^2}{2}(P_s - P_t) + \frac{a_s^2}{2} P_t \qquad a_s' = \frac{D_s + t_s}{2}$

- axial force in the shell:

$$V_s = \frac{a_o^2 P_e}{2 a_s'} + \frac{a_o^2}{2 a_s'}\left(\rho_s^2 - 1\right)\left(P_s - P_t\right) + \frac{a_s^2}{2 a_s'} P_t \qquad \text{[VIII.6a]}$$

- axial membrane stress in the shell:

$$\sigma_{s,m} = \frac{V_s}{t_s} = \frac{a_o^2}{(D_s + t_s) t_s}\left[P_e + \left(\rho_s^2 - 1\right)\left(P_s - P_t\right)\right] + \frac{a_s^2}{(D_s + t_s) t_s} P_t$$

1st term is due to the equivalent pressure P_e acting on the tubesheet
2nd term is due to the pressures P_s and P_t acting on the unperforated ring
3rd term is due to the end load pressure P_t acting on the channel head

Note for the other types of HEs:
U tubes HE: $P_e = P_s - P_t$ *(see PART 5)*

$$\sigma_{s,m} = \frac{a_o^2}{(D_s + t_s) t_s} \rho_s^2 (P_s - P_t) + \frac{a_s^2 P_t}{(D_s + t_s) t_s} = \frac{a_s^2}{(D_s + t_s) t_s} P_s$$

$$\sigma_{s,m} = \frac{D_s^2}{4(D_s + t_s) t_s} P_s$$

Immersed floating head: $P_e = P_s - P_t$ *(see PART 4)*

$$\sigma_{s,m} = \frac{D_s^2}{4(D_s + t_s) t_s} P_s$$

In both cases the classical cylinder formula is obtained.
Externally sealed floating head: $P_e = (1 - \rho_s^2) P_s - P_t$ *(see PART 4)*

$$\sigma_{s,m} = \frac{a_o^2}{(D_s + t_s) t_s}\left[(1 - \rho_s^2) P_s - P_t + \left(\rho_s^2 - 1\right) P_s - \left(\rho_s^2 - 1\right) P_t\right] + \frac{a_s^2}{(D_s + t_s) t_s} P_t = 0$$

There is no axial force acting in the shell of an externally sealed floating head

Internally sealed floating head: $P_e = \left(1 - \rho_s^2\right)(P_s - P_t)$ *(see PART 4)*

$$\sigma_{s,m} = \frac{a_o^2}{(D_s + t_s) t_s}\left[\left(1 - \rho_s^2\right)(P_s - P_t) + \left(\rho_s^2 - 1\right)\left(P_s - P_t\right)\right] + \frac{a_s^2}{(D_s + t_s) t_s} P_t = 0$$

$$\sigma_{s,m} = \frac{D_s^2}{4(D_s + t_s) t_s} P_t$$ *The classical cylinder formula is obtained.*

(b) **Axial displacement of the shell**

For the half-shell of length l [IV.2e]: $\delta_{s,Total} = \frac{l V_s}{J E_s t_s} + \delta_s(\theta_s) + \delta_s(v_s) + \delta_J(P_s)$

$$\delta_{\mathrm{s,Total}} = \frac{l\, V_{\mathrm{s}}}{J\, E_{\mathrm{s}}\, t_{\mathrm{s}}} + l\, \alpha_{\mathrm{s,m}} \left(T_{\mathrm{s,m}} - T_{\mathrm{a}}\right) - \frac{l\, D_{\mathrm{s}}^2\, P_{\mathrm{s}}}{2\, E_{\mathrm{s}} \left(D_{\mathrm{s}} + t_{\mathrm{s}}\right) t_{\mathrm{s}}}\, \nu_{\mathrm{s}} + \frac{\pi}{16} \frac{D_{\mathrm{J}}^2 - D_{\mathrm{s}}^2}{K_{\mathrm{J}}}\, P_{\mathrm{s}}$$

For the shell of length L=2l: $\Delta_{\mathrm{s}} = 2\,\delta_{\mathrm{sT}}$

$$\boxed{\Delta_{\mathrm{s}} = \frac{L}{J\, E_{\mathrm{s}}\, t_{\mathrm{s}}} V_{\mathrm{s}} + L\, \alpha_{\mathrm{s,m}} \left(T_{\mathrm{s,m}} - T_{\mathrm{a}}\right) - \frac{L D_{\mathrm{s}}^2\, P_{\mathrm{s}}}{2\, E_{\mathrm{s}} \left(D_{\mathrm{s}} + t_{\mathrm{s}}\right) t_{\mathrm{s}}}\, \nu_{\mathrm{s}} + \frac{\pi}{8} \frac{D_{\mathrm{J}}^2 - D_{\mathrm{s}}^2}{K_{\mathrm{J}}}\, P_{\mathrm{s}}}$$

1st term is due to the axial force V_s acting in the in the shell
2nd term is due to the displacement of the shell subjected to the mean temperature $T_{s,m}$
3rd term is due to the effect of the Poisson's ratio ν_s of the shell
4th term is due to the pressure P_s acting on the sidewalls of the expansion joint

(c) **Bending stress**

The bending moment M_s in the shell at its connection with the TS exists only when the shell is integral with the TS (configurations a, b, c). As explained in Annex J, the shell must have a minimum length $l_{s,\min} = 1.8\sqrt{D_s\, t_s}$ adjacent to the TS.

[VI.1a]: $M_{\mathrm{s}} = k_{\mathrm{s}} \left(1 + \frac{t_{\mathrm{s}}'}{2}\right) \theta_{\mathrm{s}} + \left(k_{\mathrm{s}}\, \beta_{\mathrm{s}}\, \delta_{\mathrm{s}}\right) P_{\mathrm{s}}$ with $\theta_s = \theta_a$ given by [V.5]:

$$\theta_a = \frac{a_o}{D^*}\left[M_{\mathrm{a}}\, Z_{\mathrm{m}} + \left(a_o\, V_{\mathrm{a}}\right) Z_{\mathrm{v}}\right] = \frac{a_o}{D^*}\left[\left(a_o\, V_{\mathrm{a}}\right)\left(Z_{\mathrm{v}} + Q_1\, Z_{\mathrm{m}}\right) + \left(Q_2\, Z_{\mathrm{m}}\right)\right] = \frac{12\left(1-\nu^{*2}\right)}{E^*\, h^3}\, a_o \left[\frac{a_o^2}{2} P_{\mathrm{e}} \left(Z_{\mathrm{v}} + Q_1\, Z_{\mathrm{m}}\right) + \left(Q_2\, Z_{\mathrm{m}}\right)\right]$$

$$\boxed{\theta_{\mathrm{a}} = \frac{6\left(1-\nu^{*2}\right)}{E^*} \left(\frac{a_o}{h}\right)^3 \left[P_{\mathrm{e}} \left(Z_{\mathrm{v}} + Q_1\, Z_{\mathrm{m}}\right) + \frac{2}{a_o^2}\left(Q_2\, Z_{\mathrm{m}}\right)\right]} \qquad \text{[VIII.6b]}$$

$$M_{\mathrm{s}} = k_{\mathrm{s}} \left\{ \left(1 + \frac{t_{\mathrm{s}}'}{2}\right) \frac{6\left(1-\nu^{*2}\right)}{E^*} \left(\frac{a_o}{h}\right)^3 \left[P_{\mathrm{e}} \left(Z_{\mathrm{v}} + Q_1\, Z_{\mathrm{m}}\right) + \frac{2}{a_o^2}\left(Q_2\, Z_{\mathrm{m}}\right)\right] + \left(P_{\mathrm{s}}\, \beta_{\mathrm{s}}\, \delta_{\mathrm{s}}\right) \right\} \qquad t_s' = h\, \beta_s$$

Bending stress $\sigma_{\mathrm{s,b}}$ is written: $\sigma_{\mathrm{s,b}} = 6\dfrac{M_{\mathrm{s}}}{t_s^{\,2}}$

$$\boxed{\sigma_{\mathrm{s,b}} = \frac{6}{t_{\mathrm{s}}^2} k_{\mathrm{s}} \left\{ \beta_s\, \delta_s\, P_s + \frac{6\left(1-\nu^{*2}\right)}{E^*} \left(\frac{a_o}{h}\right)^3 \left(1 + \frac{h\, \beta_s}{2}\right) \left[P_{\mathrm{e}} \left(Z_{\mathrm{v}} + Q_1\, Z_{\mathrm{m}}\right) + \frac{2}{a_o^2}\left(Q_2\, Z_{\mathrm{m}}\right)\right] \right\}} \qquad \text{[VIII.6c']}$$

Note 1: If $P_{\mathrm{e}} \neq 0$ the equation can be written as a function of Q_3:

$$P_{\mathrm{e}} \left(Z_{\mathrm{v}} + Q_1\, Z_{\mathrm{m}}\right) + \frac{2}{a_o^2}\left(Q_2\, Z_{\mathrm{m}}\right) = P_{\mathrm{e}} \left\{ Z_{\mathrm{v}} + Z_{\mathrm{m}} \left[Q_1 + \frac{2}{a_o^2\, P_{\mathrm{e}}} Z_{\mathrm{m}}\, Q_2 \right] \right\} = P_{\mathrm{e}} \left(Z_{\mathrm{v}} + Q_3\, Z_{\mathrm{m}}\right)$$

Note 2: In Section 6.1(a): $\delta_{\mathrm{s}} = \dfrac{a_{\mathrm{s}}^2}{E_{\mathrm{s}}\, t_{\mathrm{s}}}\left(1 - \dfrac{\nu_{\mathrm{s}}}{2}\right) = \dfrac{a_{\mathrm{s}}^2}{E_{\mathrm{s}}\, t_{\mathrm{s}}} - \dfrac{\nu_{\mathrm{s}}}{2} \dfrac{a_{\mathrm{s}}^2}{E_{\mathrm{s}}\, t_{\mathrm{s}}}$

The equation used by SOLER [9] in his book, page 429, looks more general:

$$\delta_{\mathrm{s}}^{AS} = \frac{a_{\mathrm{s}}^2}{E_{\mathrm{s}}\, t_{\mathrm{s}}} - \nu_s \frac{a_{\mathrm{s}}^2}{E_{\mathrm{s}}\, t_{\mathrm{s}}} \frac{V_{\mathrm{s}}}{P_{\mathrm{s}}} = \frac{a_{\mathrm{s}}^2}{E_{\mathrm{s}}\, t_{\mathrm{s}}} - \nu_s \frac{a_{\mathrm{s}}^2}{E_{\mathrm{s}}\, t_{\mathrm{s}}} \frac{t_{\mathrm{s}}}{P_{\mathrm{s}}}\, \sigma_{\mathrm{s,m}}$$

where V_s *is the axial force acting in the shell, which leads to:* $\sigma_{s,m} = \dfrac{V_s}{t_s}$

In page 447 the term $\nu_s \dfrac{a_s}{t_s}\dfrac{\sigma_{s,m}}{P_s}$ *is neglected as* $\sigma_{s,m}$ *is not yet determined. However that term is reintroduced in the final equation giving* $\sigma_{s,b}$*, which is not correct.*

If the classical shell formula is used for $\sigma_{s,m}$*:* $\sigma_{s,m} = \dfrac{a_s}{2t_s} P_s$

δ_s^{AS} *is written:*

$$\delta_s^{AS} = \frac{a_s^2}{E_s\, t_s} - \nu_s \frac{a_s^2}{2\, E_s\, t_s} = \frac{a_s^2}{E_s\, t_s}\left(1 - \frac{\nu_s}{2}\right) = \delta_s$$

8.7 Stresses in the Channel

(a) **axial membrane stress**

Integral channel (configuration a):

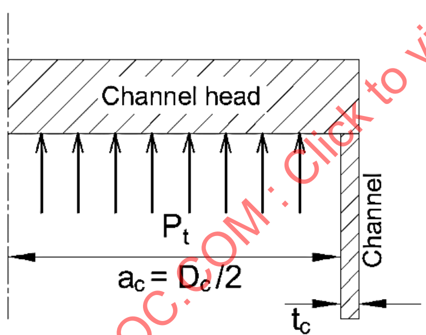
$$a_c = \frac{D_c}{2}$$

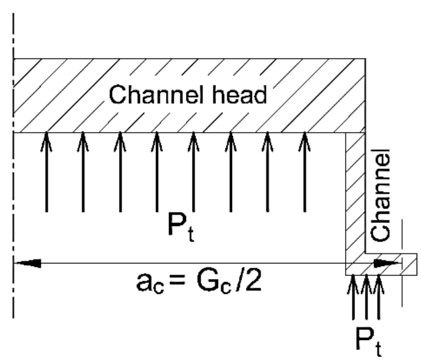
$$\sigma_{c,m} = \frac{\pi\left(\dfrac{D_c}{2}\right)^2 P_t}{\pi\left(D_c + t_c\right)t_c} = \frac{a_c^2}{\left(D_c + t_c\right)t_c} P_t$$

Gasketed channel (configurations b, c, d):

$$a_c = \frac{G_c}{2}$$

$$\sigma_{c,m} = \frac{\pi\left(\dfrac{G_c}{2}\right)^2 P_t}{\pi\left(D_c + t_c\right)t_c} = \frac{a_c^2}{\left(D_c + t_c\right)t_c} P_t$$





General formula: $\sigma_{c,m} = \dfrac{a_c^2}{\left(D_c + t_c\right)t_c} P_t$

(b) **Bending stress**

The bending moment M_c in the channel at its connection with the TS exists only when the shell is integral with the TS (configuration a). As explained in Annex J, the channel must have a minimum length $l_{c,\min} = 1.8\sqrt{D_c\, t_c}$ adjacent to the TS.

[VI.1b]: $M_c = k_c\left(1 + \dfrac{t_c'}{2}\right)\theta_c + \left(k_c\, \beta_c\, \delta_c\right) P_c$ with: $\theta_c = -\theta_a$ and θ_a given by [VIII.7b]

$$M_c = k_c\left\{-\left(1 + \frac{t_c'}{2}\right)\frac{6\left(1-\nu^{*2}\right)}{E^*}\left(\frac{a_o}{h}\right)^3\left[P_e\left(Z_v + Q_1\, Z_m\right) + \frac{2}{a_o^2}\left(Q_2\, Z_m\right)\right] + \left(P_c\, \beta_c\, \delta_c\right)\right\} \quad t_c' = h\,\beta_c$$

$$\sigma_{c,b} = \frac{6}{t_c^2} k_c \left\{ \beta_c \, \delta_c \, P_c - \frac{6\left(1-\nu^{*2}\right)}{E^*} \left(\frac{a_o}{h}\right)^3 \left(1+\frac{h\,\beta_c}{2}\right) \left[P_e \left(Z_v + Q_1 Z_m\right) + \frac{2}{a_o^2}\left(Q_2 Z_m\right) \right] \right\}$$

9 DETERMINATION OF THE ALLOWABLE STRESS LIMITS

9.1 General Considerations

The tubesheet rules of Part UHX are intended to generally follow the stress classification of Appendix 4.1 of Section VIII, Division 2 [11] for primary and primary plus secondary stresses. The scope of Part UHX does not include any consideration of peak stresses or any requirements for fatigue.

(a) **Design (pressure) loading cases**

The bending stress resulting from a pressure loading in a flat plate is a primary bending stress. Any yielding of the plate material results in a permanent deformation, and the deformation may continue to occur until the plate fails (or the deformation is so large that the plate cannot perform its intended function). When the tubesheet is extended as a flange, the loading generated by the bolting moment is considered primary. Thus, primary bending stress limits are appropriate when considering the tubesheet bending stress resulting from pressure loading acting alone or in combination with the bolt loading when applicable, i.e. the so-called "design pressure loading cases".

The stresses in the shell and channel are somewhat more complex to categorize. The axial membrane stresses in the shell and channel remote from the tubesheet resulting from pressure loadings are primary.

The bending stresses at the shell-to-tubesheet junction and the channel-to-tubesheet junction result from restrained differential motion at these junctions. As such, these bending stresses have the basic characteristic of a secondary stress. However, a very important distinction has to be made regarding the status of these stresses, based on footnote 2 of Table 4-120.1 of Section VIII Division 2 [11] . If the discontinuity bending moment at the edge of a flat plate is required to maintain the bending stress elsewhere in the plate to within its allowable stress the shell/channel bending stress is classified as primary bending and should be limited to the primary bending stress limit.
Accordingly, when an elastic stress analysis includes the rotational stiffness of the shell and channel in determining the tubesheet stress under primary loading, the discontinuity bending stress should be categorized as primary bending stress and be limited accordingly.

However, there may be instances where the design needs not consider the full strengthening effect of the shell/channel. For example, if one choses to not include the stiffening effect of the shell and channel for the tubesheet analysis, the shell and channel bending stresses could be correctly categorized as secondary and be limited according to the secondary stress considerations.

Or, if the shell/channel bending stresses resulting from pressure/bolting loads do not satisfy the primary bending stress limits, then full credit cannot be taken for the stiffness of that component. In such a case, it is deemed appropriate to apply a "knockdown" factor to the stiffness of the shell or channel component by reducing its modulus of elasticity. The "knockdown" factor used in the UHX "Elastic-Plastic analysis" is based on evaluations of the extent of strengthening offered by the fully plastic moment at the shell and channel junction when the primary bending stress limits are not satisfied.

(b) **Operating (pressure + thermal) loading cases**

The stresses resulting from the temperature difference between shell and tubes (operating pressure + thermal loading cases) are secondary in that they are self-limiting. The code limits on secondary stress are derived to accomplish "shakedown" to elastic action". UHX rules consider the tubesheet, shell, channel, and tube stresses to be secondary stresses under the action of thermal loads.

Note: It has been a long, standing practice of TEMA to divide the loads resulting from thermal expansion by a factor of two, including the tube loads. Thus, the TEMA allowable tube tensile stress *for thermal loading cases is effectively increased by a factor of two. This practice has not led to any noted problems or deficiencies in allowed tube loads, and this practice is continued for the Part UHX rules for allowable tensile stress when considering thermal load conditions.*

9.2 Allowable Stress Limit in the Tubesheet

- The bending stress σ due to pressure loading (design loading cases) is a primary bending stress (P_b) to be limited to 1.5S.
- The bending stress due to pressure + thermal loading (operating loading cases) is a secondary bending stress (Q) to be limited to S_{PS}.
- The shear stress is limited to 0.8S, as this is the practice in most codes (TEMA, CODAP, EN)

9.3 Allowable Stress Limit in the Tubes

- The axial membrane stress σ_t due to pressure loading (design loading cases) is a primary membrane stress (P_m) to be limited to S_t.
- The axial membrane stress due to pressure + thermal loading (operating loading cases) is strain induced and therefore a secondary stress (Q) to be limited to $S_{PS,t}$
  However, this limit has been lowered to $2S_t$ for two reasons:
  1) If the stress level in the tubes is too high, it may happen that a substantial number of tubes fail and that the tube bundle could not sustain the required loading, especially for tubes in compression.
  2) Consistency must be ensured with Appendix A, which limits the maximum axial load in the tube-to-tubesheet joint to $2S_t$.

If the tubes are under compression, tube buckling may restrict the tubes load carrying ability. This is true for either pressure or thermal load conditions. Accordingly, no distinction is made between primary and secondary allowable compressive loads in the tubes.
The tube axial stresses are limited to the maximum buckling stress limit $S_{t,b}$, determined in Annex R.

9.4 Allowable Membrane Stress Limit in the Shell

- The axial membrane stress $\sigma_{s,m}$ due to pressure loading (design loading cases) is a primary membrane stress (P_m) to be limited to S_s.
- The axial membrane stress due to pressure + thermal loading (operating loading cases) is strain induced and therefore is a secondary stress (Q) to be limited to $S_{PS,s}$.

If the shell is under compression, the axial stress must be limited to the maximum buckling stress limit $S_{s,b}$ determined by applying UG-23(b).

9.5 Allowable Membrane + Bending Stress Limit in the Shell

- The membrane + bending stress σ_s of the shell at its junction to the tubesheet due to pressure loading (design loading cases) is a primary bending stress (P_b) to be limited to $1.5S_s$. If this limit is exceeded, an elastic-plastic analysis may be performed as mentioned in Section 9.1a.
- The membrane + bending stress due to pressure + thermal loading (operating loading cases) is a secondary stress (Q) to be limited to $S_{PS,s}$.

9.6 Allowable Membrane + Bending Stress Limit in the Channel

The same rules as for the shell above apply.

9.7 Conclusions

The analytical method is based on the concept of the TS replaced by an equivalent solid plate resting on an elastic foundation to which the classical discontinuity analysis is applied.
The analytical development enables to determine:

- at any radius r of the TS, the deflection w(x), rotation θ(x), net effective pressure q(x), bending stress σ(x), shear stress τ(x)and axial stress in the tubes $\sigma_t(x)$, where $x = r\sqrt[4]{k_w / D^*}$. Their maximum has been obtained thanks to a parametric study performed on the two dimensionless coefficients X_a and Q_3 of the HE.
- in the shell, the axial force and resulting axial displacement and membrane stress, and the bending stress at its connection with TS and channel.
- in the channel, the membrane stress, and the bending stress at its connection with TS and shell.

The results obtained confirm the correctness of the design rules given in UHX-13.5.

Specific rules complete the UHX-13.5 design rules to cover:

- The effect of different shell material or thickness adjacent to the tubesheet (UHX-13.6)
- The effect of plasticity at tubesheet-shell-channel joint (UHX-13.7)
- The effect of radial thermal expansion adjacent to the tubesheet (UHX-13.8)
- The calculation procedure for simply supported tubesheet (UHX-13.9)
- The design of tubesheet flange extension (UHX-9)

10 ADDITIONAL RULES

10.1 Effect of Different Shell Thickness and Material Adjacent to the TS (UHX-13.6)

(a) **General:** When the shell is integral with the TS (configurations a, b or c), if the stresses are above their allowable stress limits (i.e. $\sigma_s > 1.5S_s$ for pressure loadings or $\sigma_s > S_{PS,s}$ for thermal loadings), the calculated stresses can be reduced by thickening the shell adjacent to the TSs. As explained in Annex J, the shell must have a minimum length adjacent to the TS of $l_{s,\min} = 1.8\sqrt{D_s\, t_s}$.

Accordingly, the shell must be thickened over a minimum length $l_{s,\min,1} = 1.8\sqrt{D_{s,1}\, t_{s,1}}$ adjacent to the TSs. The thickened TS sections should have the same thickness $t_{s,1}$ and the same material to comply with the assumption that the HE is a symmetrical unit (see Section 3.4a). However the thickened lengths l_1 and l'_1 adjacent to each TS can be different. The material can be different over these lengths with modulus $E_{s,1}$ and thermal coefficient $\alpha_{s,m,1}$.

Additional notations are as follows, using subscript 1 for the quantities linked to the shell thickening. See Figure 26

$E_{s,1}$ = modulus of elasticity for shell material adjacent to tubesheets at T_s
l_1, l_1'= lengths of shell of thickness $t_{s,1}$ adjacent to tubesheets ($l_1 \geq l_{s,\min,1}$ $l_1' \geq l_{s,\min,1}$)
$D_{s,1}$= internal shell diameter adjacent to tubesheets
$t_{s,1}$ = shell thickness adjacent to tubesheets
$S_{s,1}$ = allowable stress for shell material adjacent to tubesheets at T_s
$S_{y,s,1}$ = yield strength for shell material adjacent to tubesheets at T_s.
$S_{PS,s,1}$ = allowable primary plus secondary stress for shell material at T_s
$\alpha_{s,m,1}$ = mean coefficient of thermal expansion of shell material adjacent to tubesheets at $T_{s,m}$

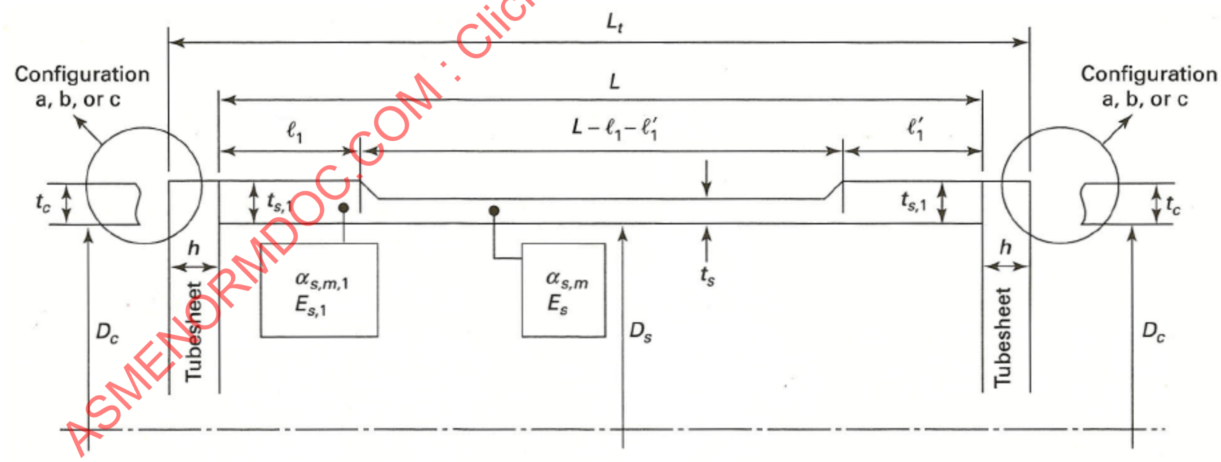


Figure 26 — Shell with Increased Thickness Adjacent to TSs

(b) **Shell axial stiffness** K_s^* is determined from:

$$\frac{1}{K_s^*} = \frac{1}{K_{s,0}} + \frac{1}{K_{s,1}} + \frac{1}{K'_{s,1}}$$ where:

$K_{s,o} = \dfrac{\pi\, t_s \left(D_s + t_s\right) E_s}{L_o}$ is the axial stiffness of the basic shell section of length $L_o = L - l_1 - l_1'$

$K_{s,1} = \dfrac{\pi\, t_{s,1} \left(D_{s,1} + t_{s,1}\right) E_{s,1}}{l_1}$ is the axial stiffness of the shell section of length l_1

$K_{s,1}' = \dfrac{\pi\, t_{s,1} \left(D_{s,1} + t_{s,1}\right) E_{s,1}}{l_1'}$ is the axial stiffness of the shell section of length l_1'

$$\frac{1}{K_s^*} = \frac{L_o}{\pi\, t_s \left(D_s + t_s\right) E_s} + \frac{l_1 + l_1'}{\pi\, t_{s,1} \left(D_{s,1} + t_{s,1}\right) E_{s,1}}$$

$$K_s^* = \frac{\pi}{\dfrac{L_o}{t_s \left(D_s + t_s\right) E_s} + \dfrac{l_1 + l_1'}{t_{s,1} \left(D_{s,1} + t_{s,1}\right) E_{s,1}}}$$

$D_{s,1} + t_{s,1}$ can be assimilated to $D_s + t_s$ for the following reasons:

- $D_{s,1}=D_s$ in most cases
- $t_{s,1}$ is small compared to $D_{s,1}$ and can be replaced by t_s
- calculation performed on Example E4.18.7 (see Annex V), using $t_{s,1}$=1.0 (instead of 0.625) and $l_1=l_1$'=15.0 (instead of 0.0), shows that K_s* decreases from 8.866 to 8.860 (0.1 % difference) when $D_{s,1}+ t_{s,1}$, is replaced by $D_s + t_s$. There is no effect on TS stress σ and shell bending stress $\sigma_{s,b}$.

  Accordingly the formula is written: $$K_s^* = \frac{\pi (D_s + t_s)}{\dfrac{L - (l_1 + l_1')}{t_s\, E_s} + \dfrac{l_1 + l_1'}{t_{s,1}\, E_{s,1}}}$$

(c) **Axial differential thermal expansion between tubes and shell** is written:

$$\gamma^* = \alpha_{t,m} (T_{t,m} - T_a) L - \left[\alpha_{s,m} (T_{s,m} - T_a) L_o + \alpha_{s,m,1} (T_{s,m} - T_a)(l_1 + l_1')\right]$$

$$\gamma^* = (T_{t,m} - T_a)\alpha_{t,m} L - (T_{s,m} - T_a)\left[\alpha_{s,m} (L - l_1 - l_1') + \alpha_{s,m,1} (l_1 + l_1')\right]$$

(d) **Design procedure is affected as follows:**
Quantities concerned by the shell thickening are those which are involved by the TS-shell connection and which involve E_s, t_s, K_s, or γ. Accordingly:

- K_s must be replaced by K_s*, which affects $K_{s,t}$ and J.
- β_s, k_s, δ_s and $\sigma_{s,m}$, $\sigma_{s,b}$ are calculated replacing t_s with $t_{s,1}$ and E_s with $E_{s,1}$.
- must be replaced by γ*, which affects P_γ

If the material of the thickened shell section is different from the material of the current shell section, use the allowable shell stress limits $S_{s,1}$ and $S_{PS,s,1}$.

10.2 Effect of Plasticity at Tubesheet-Shell-Channel Joint (UHX-13.7)

(1) **General:** If the bending stress in the shell at its junction to the tubesheet exceeds $1.5S_s$ (i.e. yield) for the pressure loadings only, then some type of plastic hinge is developed. The elastic analysis accounts for the edge moment, based only on discontinuity considerations. It does not recognize that the fully plastic moment can never be physically exceeded. Thus the amount of edge restraint that reduces the stress in the center of the tubesheet will be overestimated as the plastic hinge increases.

Taking that into consideration through an elastic-plastic analysis, Note (2) of Table 4-120.1 of ASME Section VIII Div.2 classifies the shell bending stress as a secondary stress (Q) to be limited to $S_{PS,s}$. Thus the membrane + bending stress σ_s of the shell at its junction to the tubesheet due to pressure loading (design loading cases) can go up to $S_{PS,s}$, provided that an elastic-plastic analysis is performed.

When the shell/channel bending stresses resulting from pressure loads do not satisfy the primary stress limit, then full credit cannot be taken for the stiffness of that component. In such a case, it is deemed appropriate to apply a "knockdown" factor to the stiffness of the shell or channel component by reducing its modulus of elasticity. This will increase the rotation of the joint up to a value obtained when there is no more support from the shell and channel loading, to the extreme case of a simply supported TS which is covered in Section 10.4. The "knockdown" factor is based on evaluations of the extent of strengthening offered by the fully plastic moment at the shell and channel junction.

In 1985, Soler [13] proposed to use a reduced modulus of elasticity E_s* for the shell (and E_c* for the channel) based on the degree of overstress in the shell: $E_s^* = E_s\,(S_s^* / \sigma_{s,b})$ where S_s* is the allowable stress limit. This formula has been modified for use in U-tube TS HEs (UHX-12); $E_s^* = E_s\,\sqrt{S_s^* / \sigma_{s,b}}$. The same formula applies for the channel, using subscript c.

Series of elastic-plastic finite element calculations have been performed in 1990 by Soler [14] on a short cylindrical shell to improve this straightforward formula for fixed TS HEs. The results enable to determine a reduced modulus of elasticity E_s* which gives the same rotation obtained by the elastic solution at the TS-shell-channel joint:

$E_s^* = E_s\left[1 - f(\sigma_{s,b} / S_s^*)\right]$ where function $f(\sigma_{s,b} / S_s^*)$ =0 when $\sigma_{s,b} = S_s^*$ and increases when $\sigma_{s,b} \geq S_s^*$.

(2) **UHX formula:** The above formula has been refined later, using additional elastic-plastic calculations as follows.

For the shell: $E_s^* = E_s\left(1.4 - 0.4\dfrac{|\sigma_{s,b}|}{S_s^*}\right)$ For the channel: $E_c^* = E_c\left(1.4 - 0.4\dfrac{|\sigma_{c,b}|}{S_c^*}\right)$

The reduced effective modulus has the effect of reducing the shell and/or channel stresses obtained from the elastic calculation. However, due to load shifting, this usually leads to an increase of the tubesheet stress. Accordingly, this simplified elastic-plastic procedure can only be performed when the TS stress obtained in the elastic calculation is below the allowable stress limit 1.5S for pressure loading.

The maximum allowable bending stress limit in the shell and channel for design loading cases 1, 2, 3 and 4 is defined as:

$$S_s^* = \text{MIN}\left[\left(S_{y,s}\right),\ \left(\frac{S_{PS,s}}{2}\right)\right] \qquad S_c^* = \text{MIN}\left[\left(S_{y,c}\right),\ \left(\frac{S_{PS,c}}{2}\right)\right]$$

Above this value, which is more or less the yield stress, the shell and channel start to yield.

(3) **UHX procedure** is given hereafter for the shell. It applies to the channel, using subscript c instead of s.

(1) The procedure applies when $\left|\sigma_{s,b}\right| > S_s^* \Rightarrow E_s^* < E_s$

Cases where $\left|\sigma_{s,b}\right| = S_s^* \Rightarrow E_s^* = E_s$ or when $\left|\sigma_{s,b}\right| < S_s^* \Rightarrow E_s^* > E_s$ are covered by the normal elastic procedure and are out of the scope of the elastic-plastic procedure. Therefore E_s* formula can be written:

$\boxed{E_s^* = E_s \ fact_s}$ where $\boxed{fact_s = MIN\left[\left(1.4 - 0.4\frac{\left|\sigma_{s,b}\right|}{S_s^*}\right), (1)\right]}$

Fact$_s$ varies from 1 when $\left|\sigma_{s,b}\right| = S_s^*$ to 0.6 when $\left|\sigma_{s,b}\right| = S_{PS,s} = 2S_s^*$.

(2) Quantities affected by the elastic-plastic procedure are those which are involved in the TS-shell-channel joint and which involve Es. Accordingly:

- k_s affects λ_s, which leads to new values for F, Φ, Q_1, Q_{Z1}, Q_{Z2}, U ; P_W, P_{rim}, P_e ; Q_2, Q_3, F_m and finally the tubesheet bending stress σ.
- δ_s is not affected because it is used only in $\omega_s = \rho_s \beta_s k_s \delta_s \left(1 + h\beta_s\right)$ in which E_s is cancelled out by the product $k_s\delta_s$.

If |σ| ≤ 1.5*S*, the design is acceptable. Otherwise, the HE geometry must be reconsidered.
For Example 3 given in Annex V, the results for the elastic calculation are as follows for controlling loading case 2:
Stiffening coefficient Φ=9.0, σ=23084<1.5S=23700, σ_s=30035>1.5Ss=23700, but lower than $S_{PS,s}$=47400
Similar results for the channel.
For the elastic-plastic calculation: the stiffening coefficient Φ decreases to 7.6, and σ =22205<1.5S=23700.

In this example the elastic-plastic calculation leads to a decrease of the TS stress. In most cases it is the opposite, where the elastic-plastic calculation increases the TS stress.

10.3 Effect of Radial Thermal Expansion Adjacent to the Tubesheet (UHX-13.8)

When there is a significant temperature gradient at the TS-shell-channel joint, it may be necessary to account for the resulting radial differential thermal expansion at the joint. This occurs when the TS is integral with the shell (configurations a, b, c) or the channel (configuration a).
Additional notations are as follows (see Figure 27):

T' = tubesheet metal temperature at the rim

T_c' = channel metal temperature at the tubesheet

T_s' = shell metal temperature at the tubesheet

α' = mean coefficient of thermal expansion of tubesheet material at T'

α_c' = mean coefficient of thermal expansion of channel material at T_c'

α_s' = mean coefficient of thermal expansion of shell material at T_s'

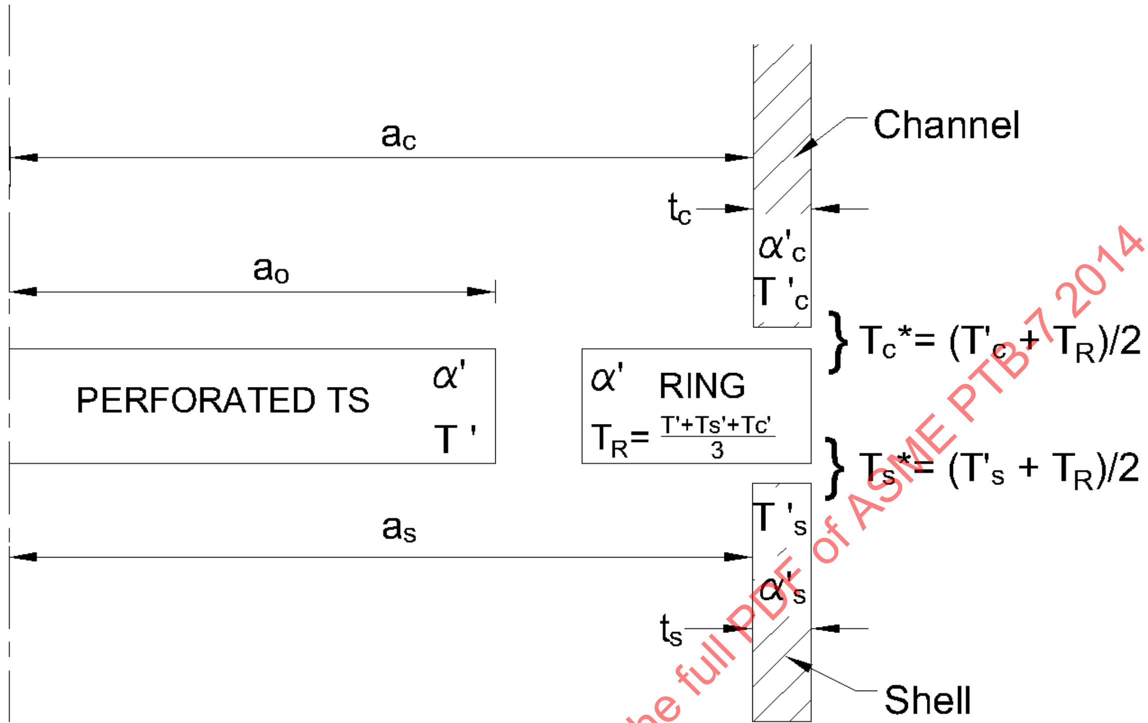


Figure 27 — Temperature Gradient at TS-Shell-Channel Joint

Average temperature of the unperforated rim T_r: $T_R = \dfrac{T' + T_s' + T_c'}{3}$

Average temperature of the shell T_s^* and channel T_c^* at their junction to the tubesheet:

$$T_s^* = \frac{T_s' + T_R}{2} \qquad T_c^* = \frac{T_c' + T_R}{2}$$

Radial strain of the shell due to T_s* and T_R: $\varepsilon_s = \alpha_s' \left(T_s^* - T_a \right) - \alpha' \left(T_r - T_a \right)$

The radial displacement of the shell is given by: $w_s(P_s^*) = a_s \, \varepsilon_s$ which can be written in the same way as $w_s(P_s)$ given by [A-VI.1a-1]:

$$w_s(P_s) = \underbrace{\frac{a_s^2}{E_s \, t_s}\left(1 - \frac{\nu_s}{2}\right)}_{\delta_s} P_s = \delta_s P_s \quad \text{with} \quad \delta_s = \frac{a_s^2}{E_s \, t_s}\left(1 - \frac{\nu_s}{2}\right)$$

$$w_s(P_s^*) = \underbrace{\frac{a_s^2}{E_s \, t_s}}_{\delta_{s,o}} \underbrace{\frac{E_s \, t_s}{a_s} \varepsilon_s}_{P_s^*} = \delta_{s,o} \, P_s^* \quad \text{with} \quad \boxed{\delta_{s,o} = \frac{a_s^2}{E_s \, t_s}} \quad \text{and} \quad \boxed{P_s^* = \frac{E_s \, t_s}{a_s} \varepsilon_s}$$

The total radial displacement of the shell w_s given in VI.1a accounting for $w_s(P_s^*)$ becomes:

$$w_s = \frac{Q_s}{\beta_s^2\, k_s} + \frac{M_s}{k_s\, \beta_s} + w_s(P_s) + w_s\,(P_s^*) = \frac{Q_s}{\beta_s^2\, k_s} + \frac{M_s}{k_s\, \beta_s} + \delta_s P_s + \delta_{s,o}\, P_s^*$$

with: $$P_s^* = \frac{E_s\, t_s}{a_s}\left[a_s'\,\left(T_s^* - T_a\right) - \alpha'\,(\,T_r - T_a)\right]$$

Thus $\delta_{s,o} P_s^*$ must be added to $\delta_s P_s$ in each equation where $\delta_s P_s$ appears.

The total radial displacement of the channel w_c is written in the same way:

$$w_c = \frac{Q_c}{\beta_c^2\, k_c} + \frac{M_c}{k_c\, \beta_c} + \delta_c P_c + \delta_{c,o} P_c^*$$

with: $$P_c^* = \frac{E_c\, t_c}{a_c}\left[a_c'\,\left(T_c^* - T_a\right) - \alpha'\,(\,T_r - T_a)\right]$$

Thus $\delta_{c,o} P_c^*$ must be added to $\delta_c P_c$ in each equation where $\delta_c P_c$ appears.

Equation [VI.2b'] giving the moment of the ring becomes:

$$R\, M_R = -\,a_o\, M_a + a_o^2\, V_a\,(\rho_s - 1) + P_s\,\frac{a_o^3}{4}\left[\left(\rho_s^2 - 1\right)\left(\rho_s - 1\right)\right] - P_c\,\frac{a_o^3}{4}\left[\left(\rho_c - 1\right)\left(\rho_c^2 + 1\right) - 2\,(\rho_s - 1)\right]$$
$$-\left[a_s'\, k_s\left(1 + t_s' + \frac{t_s'^2}{2}\right) + a_c'\, k_c\left(1 + t_c' + \frac{t_c'^2}{2}\right)\right]\theta_a + a_o\left[\omega_c\, P_c - \omega_s\, P_s\right] + a_o\left[\omega_{c,o}\, P_c^* - \omega_{s,o}\, P_s^*\right]$$

where: $\omega_{s,o} = \rho_s\, \beta_s\, k_s\, \delta_{s,o}\,(1 + h\, \beta_s)$ $\quad \omega_{c,o} = \rho_c\, \beta_c\, k_c\, \delta_{c,o}\,(1 + h\, \beta_c)$

Equation giving P_{rim} in VII.2 is written:

$$P_{rim} = -\frac{U}{a_o^2}\left[\frac{a_o^2}{4}\left[\left(\rho_s^2 - 1\right)(\rho_s - 1)\, P_s - \left(\left(\rho_c^2 + 1\right)(\rho_c - 1) - (\rho_s - 1)\right) P_c\right]\right] + \frac{U}{a_o^2}\left[\omega_s\, P_s - \omega_c\, P_c\right]$$

A term P_ω must be added to P_{rim}: $$P_\omega = \frac{U}{a_o^2}\left(\omega_{s,o}\, P_s^* - \omega_{c,o}\, P_c^*\right)$$

Accordingly the equivalent pressure P_e becomes: $$P_e = \frac{J\, K_{s,t}}{1 + J\, K_{s,t}\, F_q'}\left[P_s' - P_t' + P_\gamma + P_w + P_{rim} + P_\omega\right]$$

Coefficient Q_2 becomes: $$Q_2 = \frac{\left(\omega_s^*\, P_s - \omega_c^*\, P_c\right) - \left(\omega_{s,o}\, P_s^* - \omega_{c,o}\, P_c^*\right) + \left(W^*\, \dfrac{\gamma_b}{2\pi}\right)}{1 + \Phi\, Z_m}$$

$\sigma_{s,b}$ and $\sigma_{c,b}$ become:

$$\sigma_{s,b} = \frac{6}{t_s^2} k_s\left\{\beta_s(\delta_s P_s + \delta_{s,o} P_s^*) + \frac{6\left(1 - \nu^{*2}\right)}{E^*}\left(\frac{a_o}{h}\right)^3\left(1 + \frac{h\beta_s}{2}\right)\left[P_e\,\left(Z_v + Z_m\, Q_1\right) + \frac{2}{a_o^2}\, Q_2 Z_m\right]\right\}$$

$$\sigma_{c,b} = \frac{6}{t_c^2} k_c \left\{ \beta_c(\delta_c P_c + \delta_{s,o} P_c^*) - \frac{6(1-\nu^{*2})}{E^*}\left(\frac{a_o}{h}\right)^3 \left(1+\frac{h\beta_c}{2}\right)\left[P_e \ (Z_v + Z_m \ Q_1) + \frac{2}{a_o^2} \ Q_2 Z_m\right]\right\}$$

10.4 Calculation Procedure for Simply Supported Tubesheets (UHX-13.9)

(a) **General:** There are instances where the design does not need to consider the full strengthening effect of the shell/channel when they are integral with the TS. A Code Case was issued in 2005 to answer the concern of a HE manufacturer for externally sealed floating head HEs designed by TEMA. Use of UHX rules would need to increase significantly both the thicknesses of the floating TS and the attached channel, which is not always possible due to diameter and layout constrains. The Code Case proposed to consider the TS as simply supported.

If one chooses to not include the stiffening effect of the shell and channel for the tubesheet analysis when they are overstressed, this assumes that a plastic hinge has formed at the TS-shell-channel junction. Accordingly, the shell and channel bending stresses can be categorized as secondary and be limited according to the secondary stress limitations.
These considerations are a limit case of the elastic-plastic considerations of X-2, using for the shell and channel moduli of elasticity reduced to zero. Accordingly, the design procedure is similar and must be conducted in two phases.

(b) **Phase 1:** Perform the normal calculation of the TS stress σ and its limitations. Calculate the shell and channel stresses which are considered as secondary and must be limited respectively to $S_{PS,s}$ and $S_{PS,c}$. The shell and channel minimum length limitations $l_{s,min}$ and $l_{s,min}$ no more apply as the stiffening effect due to the bending rigidity of these components will not be considered in Phase 2.
For Example E4.18.8 given in Annex V, the results are as follows for controlling loading case 2: Stiffening coefficient Φ=9.0, σ =23084<1.5S=23700, σs =30035>1.5S_s=23700, but lower than $S_{PS,s}$=47400.
Similar results for the channel.

(c) **Phase 2:** Ignore the bending rigidity of the shell i.e., from [VI.1a]: k_s=0 and δ_s=0, which leads to M_s=0. Same for the channel; k_c=0 and δ_c=0. Then calculate the TS stress σ for pressure loading (design loading cases) and check that σ≤1.5S.

For Example 3, the stiffening coefficient Φ decreases to 0.14, and σ =15170<1.5S=23700 σ_s =1561<1.5S_s=23700

As already mentioned above, the same result would be obtained by imposing E_s*=0 and E_c*=0 in the elastic-plastic method.

In this example the simply supported calculation leads to a decrease of the TS stress. In most cases it is the opposite, where the simply supported calculation increases the TS stress.

10.5 Tubesheet Effective Bolt Load (UHX-8)

Tubesheets with flanged extensions are affected by the bolt loads in two ways:
- externally (at the flanged end). The design of the tubesheet extension is covered in 10.6
- internally (in the perforated region). The application of the bolt load for a fixed tubesheet exchanger is more complicated than for a regular flange.

The technical basis for the effect of that load on the perforated region is investigated in detail in Parts 3, 4 and 5.

For some loading cases, the bolt load is a secondary, self-limiting load. As an example, when considering Loading Case 2 (shell side pressure only) for a configuration b geometry, the bolt load only produces a secondary effect in that it does not directly resist pressure, and if the tubesheet-flange deflects sufficiently, then the bolt load may be totally relieved.

Likewise, for those loading cases where the bolts directly resist the pressure load, the bolt loads used for the design should be consistent with Appendix 2, which requires that the gasket seating bolt load be used to check the flange at ambient temperature for protection against damage due to over bolting. The "primary" bolt load, W_{m1} is used in Appendix 2 for designing the flange under operating conditions. In order to be consistent with Appendix 2, W_{m1} must be used as the bolt load for loading cases where primary stresses are determined i.e. Pressure Design Loading Cases.

Pressure and Thermal Operating Loading Cases require consideration of secondary stresses for which it is appropriate to include the design bolt load from Appendix 2. A new term "tubesheet effective bolt load W* ", is introduced in 2013 Edition and selected from a new table UHX-8.1 where the designer can select the appropriate bolt load to be used for the respective Configuration and Loading Case combinations that accounts for the above considerations. So, part UHX is consistent with the philosophy of Appendix 2 bolt loads.

Table UHX-8.1

TUBESHEET EFFECTIVE BOLT LOAD, W*

	Loading Case			
	1	**2**	**3**	**4-7**
Configuration				
a	**0**	**0**	**0**	**0**
b	W_{m1c}	**0**	W_{m1c}	W_c
c	W_{m1c}	**0**	W_{m1c}	W_c
d	W_{m1c}	W_{m1s}	W_{m1max}	W_{max}
e	**0**	W_{m1s}	W_{m1s}	W_s
f	**0**	W_{m1s}	W_{m1s}	W_s
A	**0**	**0**	**0**	**0**
B	W_{m1c}	**0**	W_{m1c}	W_c
C	W_{m1c}	**0**	W_{m1c}	W_c
D	**0**	**0**	**0**	**0**

Where:
W_c = channel flange design bolt load for the gasket seating condition
W_s = shell flange design bolt load for the gasket seating condition
$W_{max} = MAX[(W_c), (W_s)]$
W_{m1c} = channel flange design bolt load for the operating condition
W_{m1s} = shell flange design bolt load for the operating condition
$W_{m1max} = MAX[(W_{m1c}), (W_{m1s})]$
W^* = tubesheet effective bolt load selected from Table UHX-8.1 for the respective Configuration and Loading Case

10.6 Tubesheet Flange Extension (UHX-9)

(a) **General:** UHX-9 provides the rules to determine the required thickness of flanged TS extensions when bolt loads are transmitted to them (configurations b, d, e and B) and to flanged and unflanged

TS extensions when no bolt loads are applied to them (configurations c, d, f and C). The rules in UHX-9 are not applicable to Configurations a, A and D.

The required thickness of the flanged TS extension calculated in accordance with UHX-9 may differ from that required for the interior of the TS calculated in accordance with UHX-12, UHX-13, or UHX-14.

Figure 28 depicts thickness h_r for some representative configurations.

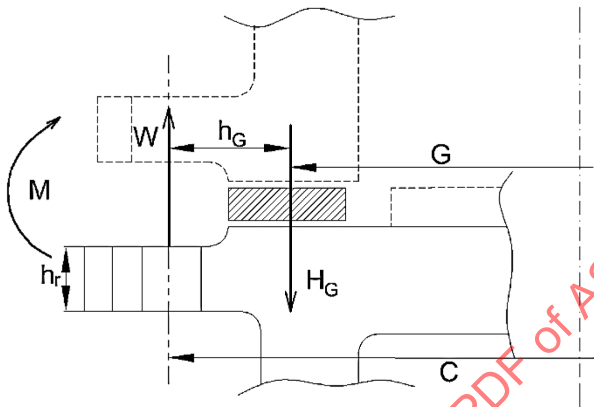


Figure 28 — Tubesheet Flanged Extension

(b) **Flanged TS extension with bolt loads applied:**
The moment acting on the TS flanged extension is written:

$$M = W\, h_G = W\,\frac{C-G}{2}$$

The bending stress in the TS flanged extension is written:

$$\sigma = \frac{6\,M}{\pi\, G\, h_r^2} = \frac{6\,W\,h_G}{\pi\, G\, h_r^2} = \frac{1.91\,W\,h_G}{G\,h_r^2}$$ and is limited to the allowable TS stress.

For the gasket seating condition, the minimum required thickness of the tubesheet flanged extension, h_r, is obtained for $\sigma = S_a$:

$$h_r = \sqrt{\frac{1.9W\,h_G}{S_a G}}$$

A similar formula must be applied for the operating condition (W=Wm1, S). Accordingly:

$$h_r = \text{MAX}\left[\sqrt{\frac{1.9Wh_G}{S_a G}},\ \sqrt{\frac{1.9W_{m1}h_G}{SG}}\right]$$

For design conditions, $W = W_{m1}$ from equation (1) of Appendix 2, Section 2-5(c)(1) [10] .
For gasket seating condition, $W = W$ from equation (5) of Appendix 2, Section 2-5(e) [10] .
S = allowable stress for the material of the tubesheet extension at design temperature
S_a = allowable stress for the material of the tubesheet extension at ambient temperature

(c) **Flanged and Unflanged TS extension with no bolt load applied:**
For flanged Configuration d and for unflanged Configurations D and C having no bolt loads applied to the extension, the shear load due to effect of pressure at the gasket ID must be considered. The derivation of the shear stress equation is shown in Section 8.4(b). The minimum required thickness of the extension, h_r, shall be the maximum of the values determined for each design loading case as follows:

$$\boxed{h_r = \left(\frac{D_E}{3.2S}\right)\left|P_s - P_t\right|}$$

D_E = maximum of the shell and channel gasket inside diameters, but not less than the maximum of the shell and channel flange inside diameters

(d) **Unflanged TS extension with bolt loads applied:**
The calculation procedure for unflanged Configurations c and f are not provided in UHX-9, but the minimum required thickness of the extension, h_r, shall be calculated in accordance with ASME Section VIII, Division 1,Mandatory Appendix 2, 2-8(c) for loose type flanges with laps.

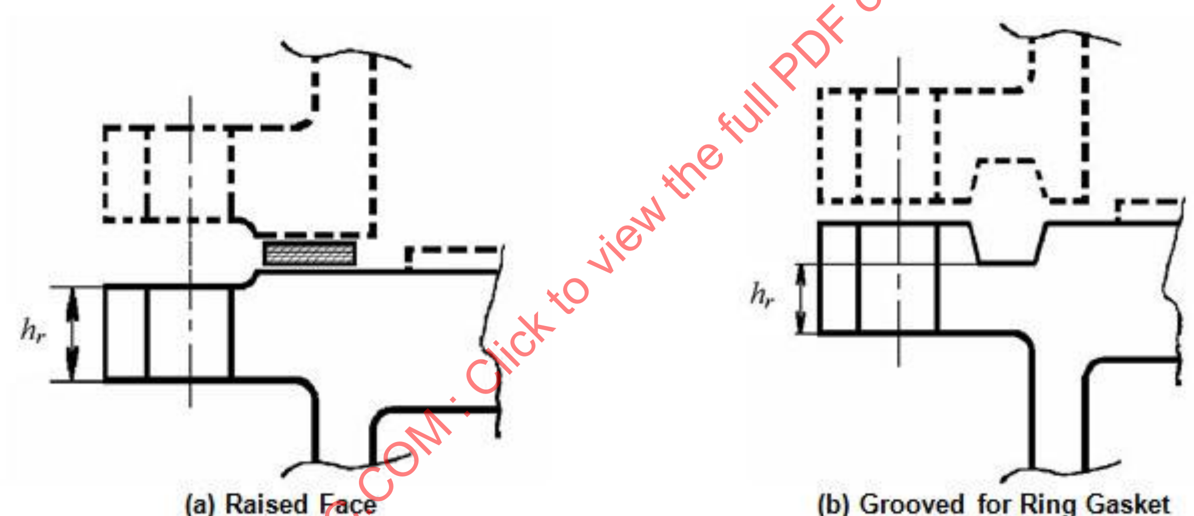


Figure 29 — Minimum Required Thickness of the Tubesheet Flanged Extension

10.7 HE Set-up with a Thin-Walled Expansion Joint (UHX-13.16)

The joint must comply with the rules of Appendix 26.

10.8 HE Set-up with a Thick-Walled Expansion Joint (UHX-13.17)

The expansion joint must comply with the rules of Appendix 5 which provides allowable stress limits. The design of the joint may be performed using higher allowable stress limits, defined in Table UHX-17, which allow the expansion joint to yield and decrease its stiffness. Accordingly, the HE must be re-designed for design (pressure) loading cases using a zero expansion joint stiffness (K_J=0).
The equivalent pressure P_e must be recalculated using K_J=0. Using Note 1 of Section 7.2:

$$P_{s1}^{'} = P_s\left[\left(JK_{s,t}\right)x_s + (JK_{s,t})2\left(1-x_s\right)\nu_t + (J)2\left(\frac{D_s}{D_o}\right)^2 \nu_s - (\rho_s^2 - 1) - \frac{\left(1-J\right)}{2}\frac{\left[D_J^2 - D_s^2\right]}{D_o^2}\right]$$

$$P_{t1}^{'} = P_t\left[\left(JK_{s,t}\right)x_t + \left(JK_{s,t}\right)2\left(1-x_t\right)\nu_t + 1\right]$$

Using J=0:

$$P_s^{'} = P_s\left[1-\rho_s^2 - \frac{1}{2}\left[\frac{D_J^2}{D_o^2} - \rho_s^2\right]\right] = P_s\left[1-\frac{1}{2}\left[\frac{D_J^2}{D_o^2} + \rho_s^2\right]\right]$$

$$P_{t1}^{'} = P_t\left[\left(JK_{s,t}\right)x_t + \left(JK_{s,t}\right)2\left(1-x_t\right)\nu_t + 1\right] = P_t$$

$$P_e = \frac{1}{1+J\ K_{s,t}\ F_q^{'}}\left[P_{s1}^{'} - P_{t1}^{'}\right] = \boxed{P_s\left[1-\frac{1}{2}\left[\rho_s^2 - \frac{D_J^2}{D_o^2}\right]\right] - P_t}$$

11 HOW TO USE THE RULES

The calculation procedure can be summarized as follows:

- Set the data listed in Section 3.2a
- Calculate the design coefficients listed in Section 3.2b (x_s,x_t ; ρ_s,ρ_c ; $K_{s,t}$; J)
- Calculate first characteristic parameter X_a and coefficients Q_1,Q_2 ; ω_s^*,ω_c^*
- Calculate coefficients Q_{Z1}, Q_{Z2}, U, the equivalent pressure P_e, and second characteristic parameter Q_3
- Calculate the maximum stresses in TS, tubes, shell and channel and limit their values to the maximum allowable stress limits.

Because of the complexity of the procedure, it is likely that users will computerize the solution. Annex V provides a Mathcad calculation sheet for a fixed TS HE defined in PTB-4 Example E4.18.7. The calculation sheet is divided in 2 parts:

Part 1 follows strictly the various steps (Steps 1 to 11) of UHX-13.5 calculation procedure.
Part 2 provides the equations developed in VIII, which enable to calculate:

- at any radius of the perforated tubesheet: net pressure, deflection, rotation, bending stress, shear stress
- at any radius of the tube bundle: axial membrane stress.
- at tubesheet-shell-channel connection: loads and displacements acting on the shell.

12 CHECKING OF THE RESULTS

12.1 Comparison with FEA

(a) **General**

SG-HTE proceeded to Finite Element Analysis to check UHX rules. Example E4.18.7 (configuration a) given inAnnex V was selected for the comparison.

FEA was performed by Joel Gordon, member of SG-HTE, using ANSYS with plate elements for the TS and shell elements for shell and channel. The results are in full agreement with results obtained by Tony Norton, member of SG-THE, using a KSHELL analysis with maximum discrepancies less than 5%.

So as to reflect the UHX assumptions as closely as possible, FEA calculation was performed as follows:

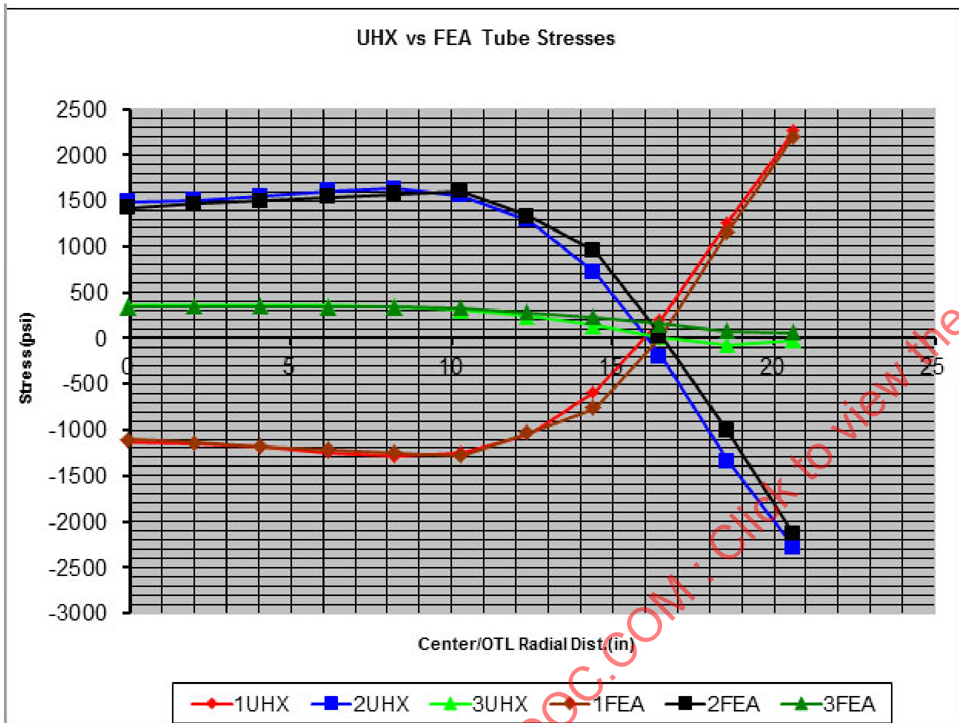
- The TS is replaced by an equivalent unperforated plate with an effective ligament efficiency μ* and equivalent effective constants E* and ν*,
- The tubes are replaced by an elastic foundation of modulus $k_w = N_t k_t / \left(\pi a_o^2\right)$,
- The unperforated rim is narrow enough such that it tends toward a rigid behavior.

(b) **Results**

(1) **for tube axial stress σ_t(r),** calculated from the TS displacements, FEA and UHX show very good agreement, with discrepancies less than 5% at the center (r=0) and at theperiphery of the TS (r=a_o). The stress distributions throughout the TS track each other almost perfectly as shown by the shape of the curves given in

(2) Figure 30 for design (pressure) loading cases 1, 2 and 3. It must be noted that the curves meet at the same point (x =5.5; σ_t=0).This surprising result is demonstrated analytically in Annex S.

(3) **For TS bending stress** σ(r) the comparison is not so good: the TS stress obtained by UHX is significantly higher (by about 50%) than the stress obtained by FEA. SG-HTE is still looking for an explanation for this discrepancy. This might be due to the unperforated rim, considered as a rigid ring in the analytical treatment.



	dist. (in)	$\sigma_t(1)$ psi	$\sigma_t(2)$ psi	$\sigma_t(3)$ psi	FEA 1	FEA 2	FEA 3
TS center (r=0)	0,0	-1129	1486	357	-1106	1425	342
	2,06	-1145	1503	358	-1141	1462	344
	4,12	-1191	1551	360	-1176	1499	348
	6,19	-1250	1608	358	-1211	1536	343
	8,25	-1289	1634	345	-1246	1573	338
	10,32	-1249	1558	309	-1281	1610	333
	12,38	-1046	1286	241	-1031	1330	277
	14,44	-581	718	137	-758	955	233
	16,50	209	-195	14	23	18	153
	18,57	1265	-1338	-73	1161	-1011	81
TS periph(r=20.63)	20,63	2259	-2277	-17	2204	-2135	59

Series: 1 (UHX) 2 (UHX) 3 (UHX) 1 (FEA) 2 (FEA) 3 (FEA)

Figure 30 — Comparison of Tube Stresses Calculated Per UHX and FEA (Example E4.18.7)

12.2 Comparison with CODAP French Rules

(a) **General**

ASME and CODAP (French Code for Pressure Vessels – Part Design, Chapter C7.3 "Fixed TS HEs) rules are based on the same analytical approach (TS replaced by an equivalent solid plate of effective elastic constants E* and ν*, tube bundle replaced by an equivalent elastic foundation, shell and channel disconnected from the TS with relevant edge loads). In 1992 ASME and CODAP decided to reconcile their rules so as to have unified rules: scope, notations, figures, TS configurations, loading cases, ligament efficiencies, effective elastic constants, stress design formulas.

However additional assumptions have been maintained in the basic rules of CODAP (sections C7.3 devoted to Fixed TS HEs and C7.4 devoted to FL TS HEs), which simplify the analytical treatment:

- TS is assumed to be perforated up to the internal shell diameter D_s. Accordingly the unperforated TS rim is ignored
- Treatment of the TS-shell-channel connection is simplified
- Effect of the bolting load (configurations b, c, d) on the TS is ignored. In return, the maximum allowable stress is lowered down.
- TS radial displacement at shell-channel connection is ignored
- Effect of P_s and P_t on shell and channel at their connection with TS is ignored

Note: In June 2012, new chapters C7.5 and C7.6, based on UHX-13 and UHX-14, have been introducted in CODAP code as alternative rules to sections C7.3 and C7.4.

These simplifications, applied to the UHX method, lead to the CODAP formulas, as shown hereafter.

(b) **Unperforated rim ignored:** The TS is assumed to be perforated till the internal diameter D_s of the shell. Therefore:

$$\boxed{D_o = D_s = D_c \quad \Rightarrow \quad a_o = a_s = a_c \quad \Rightarrow \quad \rho_s = 1 \quad \rho_c = 1} \qquad A = D_o \ \Rightarrow K = \frac{A}{D_0} = 1$$

X_a is written: $$\boxed{X_a = X = \sqrt[4]{24\left(1-\nu^{*2}\right) N_t \frac{E_t\, t_t \left(d_t - t_t\right)}{E^*\, L\, h^3} \left(\frac{D_s}{2}\right)^2}}$$

X_a, named X in CODAP, is the first characteristic parameter of the HE.

(c) **effect of P_s and P_t on shell and channel at their connection with TS ignored**

$$\delta_s\, P_s = 0 \quad \Rightarrow \quad \delta_s = 0 \quad \omega_s = 0 \quad \omega_s^* = 0$$
$$\delta_c P_c = 0 \quad \Rightarrow \quad \delta_c = 0 \quad \omega_c = 0 \quad \omega_c^* = 0$$

(d) **TS radial displacement at shell-channel connection ignored**

$$\left.\begin{aligned} w_s = 0 \quad &\Rightarrow \quad M_s = k_s \theta_s \\ w_c = 0 \quad &\Rightarrow \quad M_c = k_c \theta_c \end{aligned}\right\} M_s + M_c = k_s\theta_s + k_c\theta_c = (k_s + k_c)\theta_E \qquad as \ \ \theta_s = \theta_c = \theta_E$$

Moment M_E acting at TS periphery is such that:

$$M_E = -\left(M_s + M_c\right) = -\underbrace{\left(k_s + k_c\right)}_{K_\theta} \theta_E = -K_\theta\, \theta_E$$

The second characteristic parameter of the exchanger is defined as: $$\boxed{Z = \frac{K_\theta}{k\, D^*} = \frac{D_s}{2\, X} \frac{k_s + k_c}{D^*}}$$

CODAP considers the liaison of the TS with shell and channel as an elastic restraint:

- If $Z = 0$: the TS is simply supported

- If $Z=\infty$: the TS is clamped

(e) **Coefficient Φ**

$$\left.\begin{array}{l} w_s=-\frac{h}{2}\theta_s=0\Rightarrow t_s^{'}=0\Rightarrow\lambda_s=\frac{6D_s}{h^3}k_s \\ w_c=-\frac{h}{2}\theta_c=0\Rightarrow t_c^{'}=0\Rightarrow\lambda_c=\frac{6D_c}{h^3}k_c \end{array}\right\}\Phi=\frac{1-\nu^{*2}}{E^*}(\lambda_s+\lambda_c)=\frac{1-\nu^{*2}}{E^*}\frac{6}{h^3}(D_sk_s+D_ck_c)=\underbrace{\frac{1}{2D^*}(D_sk_s+D_ck_c)}_{XZ}$$

Accordingly: $\Phi=X\ Z$ plays the role of Z in CODAP:

- If the plate is simply supported (no support from shell and channel): Φ=0 in ASME , Z=0 in CODAP
- If the plate is clamped (full support from shell and channel): Φ=∞ in ASME , Z=∞ in CODAP

(f) **Second coefficient Q_3 of the HE**

In ASME: $Q_1=\frac{(\rho_s-1)-\Phi\ Z_v}{1+\Phi\ Z_m}$ $\quad Q_2=\frac{(w_s^*P_s-w_c^*P_c)+\left(\frac{\gamma_b}{2\pi}W^*\right)}{1+\Phi Z_m}$ $\quad Q_3=Q_1+\frac{2\ Q_2}{P_e\ a_0^2}$

In CODAP, these formulas become, with ρ_s=1, ω^*_s=0, ω^*_c=0, W^*=0:

$$[Q_1]_{CODAP}=\frac{-\Phi\ Z_v}{1+\Phi\ Z_m}\qquad [Q_2]_{CODAP}=0\qquad [Q_3]_{CODAP}=Q_1=\frac{-\Phi\ Z_v}{1+\Phi\ Z_m}=-\frac{(X\ Z)\ Z_v}{1+(X\ Z)\ Z_m}$$

$$\boxed{Z=\frac{-Q_3}{Q_3\ Z_m+Z_v}\frac{1}{X}}$$

The couple [X,Z] in CODAP plays the role of couple [X_a,Q_3] in ASME to determine coefficients F_q, F_m, F_Q and F_t.

(g) **Equivalent design pressure**

In ASME: $P_e=\frac{J\ K_{s,t}}{1+J\ K_{s,t}\left[Q_{Z1}+(\rho_s-1)\ Q_{Z2}\right]}\left[P_s^{'}-P_t^{'}+P_\gamma+P_W+P_{rim}+P_\omega\right]$

- $Q_{Z1}=(Z_w\ Q_1+Z_d)\frac{X_a^4}{2}$ Substituting $[Q_1]_{CODAP}$, Z_w and Z_d, it can be shown that:
- $Q_{Z1}=\left[F_q\right]_{CODAP}$ Thus, with ρ_s=1: $1+J\ K_{s,t}\left[Q_{Z1}+(\rho_s-1)\ Q_{Z2}\right]=1+J\ K_{s,t}\left[F_q\right]_{CODAP}$
- $P_s^{'}=\left[x_s+2(1-x_s)\ \nu_t+\frac{2}{K_{s,t}}\left(\frac{D_s}{D_o}\right)^2\nu_s-\frac{\rho_s^2-1}{J\ K_{s,t}}-\frac{1-J}{2\ J\ K_{s,t}}\frac{D_J^2-D_s^2}{D_o^2}\right]P_s$

  with D_s=D_o and ρ_s=0:

  $$\boxed{P_s^{'}=\left[x_s+2(1-x_s)\ \nu_t+\frac{2}{K_{s,t}}\nu_s-\frac{1-J}{2JK_{s,t}}\frac{D_J^2-D_s^2}{D_o^2}\right]P_s=\left[P_s^{'}\right]_{CODAP}}$$

- $$\boxed{P_t^{'}=\left[x_t+2(1-x_t)\ \nu_t+\frac{1}{J\ K_{s,t}}\right]P_t=\left[P_t^{'}\right]_{CODAP}}$$

- $$P_\gamma = \frac{N_t\, K_t}{\pi\, a_o^2}\,\gamma = \frac{N_t\, K_t}{\pi\, R_e^2}\,\gamma = K_w\,\gamma = \left[P_\gamma\right]_{CODAP}$$
- $$P_w = \frac{U}{a_o^2}\,\frac{\gamma_b}{2\,\pi}\,W^* = 0$$ due to W*=0 in CODAP
- $$P_{rim} = -\frac{U}{a_o^2}\left[\omega_s^*\, P_s - \omega_c^*\, P_t\right] = 0$$ due to ω^*_s=0 and ω^*_c=0 in CODAP.
- $$P_\omega = -\frac{U}{a_o^2}\left[\omega_s\, P_s^* - \omega_c\, P_t^*\right] = 0$$ due to ω_s=0 and ω_c=0 in CODAP.

Finally: $$P_e = \frac{J\, K_{s,t}}{1 + J\, K_{s,t} F_q}\left[P_s^{'} - P_t^{'} + P_\gamma\right] = \left[P_e\right]_{CODAP}$$

(h) **TS bending stress**

In ASME: $F_m(x) = \dfrac{Q_v(x) + Q_3\, Q_m(x)}{2}$

Substituting $Q_v(x)$ and using: $\left[Q_3\right]_{CODAP} = \dfrac{\phi\, Z_v}{1 + \phi\, Z_m}$

it can be shown that: $F_m(x) = \left[F_m(x)\right]_{CODAP}$ and:

$$\sigma_r(x) = \frac{1{,}5 F_m(x)}{\mu^*}\left(\frac{2a_o}{h}\right)^2 P_e = \left[\sigma_r(x)\right]_{CODAP}$$

(i) **TS shear stress**

In ASME: $F_Q(x) = Q_3\, Q_\alpha(x) + Q_\beta(x)$ $\qquad \tau(x) = \dfrac{1}{2\mu}\,\dfrac{a_o}{h}\,F_Q(x)\, P_e$

The maximum shear stress is at periphery ($x=X_a$), for which:
$Q_\alpha(X_a) = 0\ ,\ Q_\beta(X_a) = 1 \Rightarrow F_Q(X_a) = 1$

Thus: $$\tau = \frac{1}{2\mu}\,\frac{a_o}{h}\,P_e = \left[\tau\right]_{CODAP}$$

(j) **Tube axial stress**

In ASME: $F_t(x) = \dfrac{X_a^4}{2}\left[Q_3\, Z_w(x) + Z_d(x)\right]$

Substituting $Z_w(x)$ and $Z_d(x)$ and using $\left[Q_3\right]_{CODAP} = \dfrac{\phi\, Z_v}{1 + \phi\, Z_m}$

it can be shown that: $F_t(x) = \left[F_t(x)\right]_{CODAP}$ and:

$$\sigma_t(x) = \frac{1}{x_t - x_s}\left[\Delta p^* - F_t(x)\, P_e\right] = \left[\sigma_t(x)\right]_{CODAP}$$

(k) **Shell stresses**

(1) **Axial membrane stress**

In ASME: $\sigma_{s,m} = \dfrac{a_o^2}{(D_s + t_s)\, t_s}\left[P_e + \left(\rho_s^2 - 1\right)(P_s - P_t)\right] + \dfrac{a_s^2}{(D_s + t_s)\, t_s} P_t$

In CODAP, that formula becomes, with $a_0 = \dfrac{D_s}{2}$ *and* $\rho_s = 1$:

$$\boxed{\sigma_{s,m} = \frac{D_s^2}{4\, t_s\,(D_s + t_s)}(P_e + P_t) = \left[\sigma_{s,m}\right]_{CODAP}} \quad [\text{ XI.2.11a}]$$

(2) **Bending stress**

In ASME: $M_{s,b} = k_s \left\{\beta_s\, \delta_s\, P_s + \dfrac{6\left(1-\nu^{*2}\right)}{E^*}\left(\dfrac{a_o}{h}\right)^3\left(1+\dfrac{h\,\beta_s}{2}\right)\left[P_e\,(Z_v + Q_1\, Z_m) + \dfrac{2}{a_o^2}(Q_2\, Z_m)\right]\right\}$

In CODAP, that formula becomes with, $\beta_s = 0 \quad \delta_s = 0 \quad a_o = \dfrac{D_s}{2}$

$$[Q_1]_{CODAP} = \frac{-\Phi Z_v}{1+\Phi Z_m} \qquad [Q_2]_{CODAP} = 0:$$

$$M_{s,b} = \frac{k_s}{2}\frac{1}{D^*}\left(\frac{D_s}{2}\right)^3 \frac{Z_v}{1+\Phi\, Z_m} P_e = \left[M_{s,b}\right]_{CODAP} \qquad \boxed{\sigma_{s,b} = \frac{6}{t_s^2} M_{s,b} = \left[\sigma_{s,b}\right]_{CODAP}}$$

(l) **Channel stresses**

(1) **Axial membrane stress:** $\boxed{\sigma_{c,m} = \dfrac{a_c^2}{(D_c + t_c)\, t_c} P_t = \dfrac{D_s^2}{4(D_c + t_c)\, t_c} P_t = \left[\sigma_{c,m}\right]_{CODAP}}$

(2) **Bending stress** is obtained the same way as for the shell: $\boxed{\sigma_{c,b} = \dfrac{6}{t_c^2} M_{c,b} = \left[\sigma_{c,b}\right]_{CODAP}}$

(m) **In conclusion**, all CODAP formulas have been retrieved, although they were obtained at a time where there was no connection between ASME and CODAP committees. This tends to prove that both methods are correct and coherent. This coherency extends to TEMA as shown in Section 12.3. In CODAP, the ignorance of unperforated rim and bolting loads are the most important simplifications and have a significant impact on the results. Other simplifications have a less important impact.

Calculation of a HE by CODAP method can be obtained from a UHX-13.3 software, such as Mathcad software shown in Annex V for Example UHX.20.2.3, by imposing the simplifications listed in Sections 12.2(b) to (e) above.

Like ASME, CODAP results for tube axial stress $\sigma_t(r)$ are in good agreement with FEA results as shown by Figure 31 which provides the tube stress distributions throughout the TS.

dist.(in)	CODAP1	CODAP2	CODAP3
0	-1168	1549	381
2,063	-1186	1570	384
4,126	-1241	1632	391
6,189	-1318	1719	401
8,252	-1388	1797	409
10,315	-1392	1799	407
12,378	-1234	1615	381
14,441	-792	1107	315
16,504	41	155	195
18,567	1257	-1229	29
20,626	2562	-2700	-137

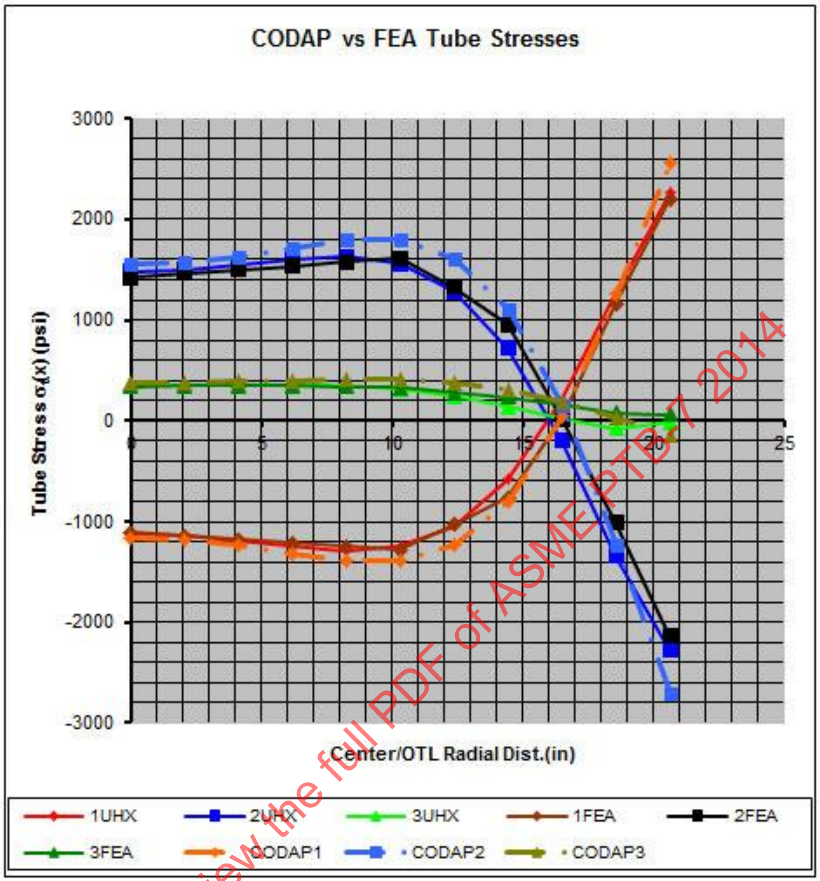


Figure 31 — Tube Stress Distribution Obtained by UHX, CODAP and FEA throughout the TS from r = 0 to r = ao

12.3 Comparison with TEMA Rules

(a) **General:** TEMA rules for fixed TSs have been developed by Gardner, based on its 1952 paper [2] . The analytical approach is the same as for ASME and CODAP with several assumptions to provide more simple rules. CODAP assumptions are used, plus additional simplifications. The TS is assumed to be either Simply Supported (SS) (i.e. no restraint from the shell and channel) or Clamped (CL) (i.e. full restraint from the shell and channel).

The original TEMA formula for the determination of the TS thickness in bending:

$T = F\dfrac{G}{2}\sqrt{\dfrac{P}{S}}$ is based on flat circular plates formula, which can be either SS:

$$\left.\begin{aligned} h = F\frac{G}{2}\sqrt{\frac{P}{\sigma_{alw}}} &= 1.11\frac{G}{2}\sqrt{\frac{P}{S}} \qquad if\ \sigma_{alw} = S \\ &= 0.91\frac{G}{2}\sqrt{\frac{P}{S}} \qquad if\ \sigma_{alw} = 1.5S \end{aligned}\right\} \quad TEMA\ uses\ the\ mean\ value\ F = 1.0$$

or CL:

$$\left.\begin{aligned} h = F\frac{G}{2}\sqrt{\frac{P}{\sigma_{alw}}} &= 0.87\frac{G}{2}\sqrt{\frac{P}{S}} \qquad \text{if } \sigma_{alw} = S \\ &= 0.71\frac{G}{2}\sqrt{\frac{P}{S}} \qquad \text{if } \sigma_{alw} = 1.5S \end{aligned}\right\} \quad \textit{TEMA uses the mean value } F = 0.8$$

(b) **Ligament efficiency**

That formula was modified later to introduce a ligament efficiency η based on the mean length of the ligament $\boxed{T = F\frac{G}{3}\sqrt{\frac{P}{\eta S}}}$, as proposed by Miller [3] (see Section 4.1 of PART 2):

For Triangular pitch: $\eta = 1 - \frac{0.907}{(p/d_t)^2}$ For Square pitch: $\eta = 1 - \frac{0.785}{(p/d_t)^2}$

Accordingly TEMA ligament efficiency η is lower than ASME ligament efficiency μ*.
The new formula was set-up so that the value of T remains approximately the same for the triangular pitch value η=0.42, obtained from the minimum pitch imposed by TEMA ($p=1.25d_t$).
TEMA simplifications applied to ASME-CODAP methods lead to the TEMA formulas, as shown hereafter.

(c) **TEMA bending formula**

ASME-CODAP formula for TS bending can be written using the TEMA format:

$$h = \sqrt{\frac{1.5F_m(X)}{\mu^*}}D_o\sqrt{\frac{P_e}{\sigma_{alw}}} = \sqrt{\frac{9\times 1.5F_m(X)}{\mu^*/\eta}}\frac{D_o}{3}\sqrt{\frac{P_e}{\eta\sigma_{alw}}}$$

In TEMA, the TS is assumed uniformly perforated up to the shell: $D_o=D_s$ (noted G in TEMA for fixed TS).
σ_{alw} =S for design (pressure) loadings (loading cases 1, 2, 3 and 4), σ =2S for operating (thermal + pressure) loadings (loading cases 1, 2, 3 and 4)

Using η/μ*=1.33, which is quite usual:

$$h = \sqrt{12F_m(X)}\frac{D_s}{3}\sqrt{\frac{P}{\eta S}} = F_{CODAP}\frac{D_s}{3}\sqrt{\frac{P}{\eta S}} \qquad F_{CODAP} = \sqrt{12F_m(X)}$$

Thus, in CODAP-ASME the TS thickness depends strongly on coefficient $F_m(X)$, whereas TEMA coefficient F has fixed values. Figure 32 gives the variation of F_{CODAP} as a function of X:

- when the TS is SS (obtained for Z=0 in CODAP).
- when the TS is CL (obtained for Z=∞ in CODAP).

TEMA considers the TS as SS when $t_s/D_s<0.02$ with F=1, and CL when $t_s/D_s>.05$ with F=0.8. In between TEMA uses a linear interpolation (see Figure 33).

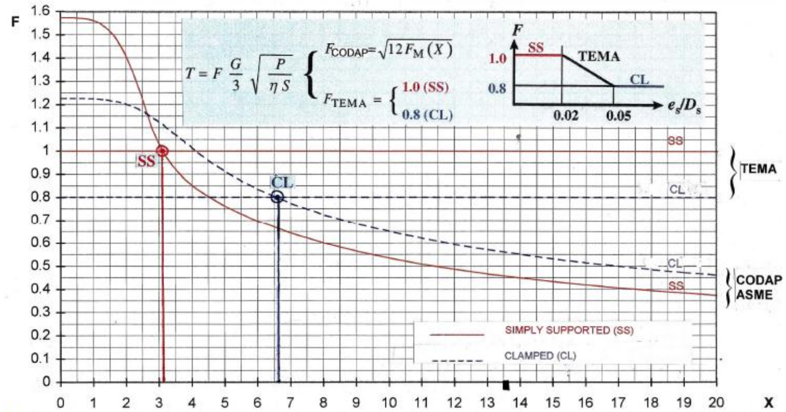


Figure 32 — TEMA and ASME-CODAP Coefficient F for X Varying from X=0 to X=20

Figure 32 shows that:

- for low values of X_a $(X_a < 3)$ TEMA rules generally lead to an unconservative thickness;
- for high values of X_a $(X_a > 6)$ TEMA rules generally lead to conservative thickness
- values of X_a between 3 and 6, TEMA rules lead to tubesheet thickness that is close to ASME.

Due to simplifications mentioned above, TEMA does not ensure an overall and consistent design margin for all heat-exchangers. Accordingly, TEMA has imposed limitations on shell diameter D_s and design pressure P: $D_s \le 60$ inch $P \le 3{,}000$ Psi $PD_s \le 60{,}000$ Psi inch (see Figure 34).

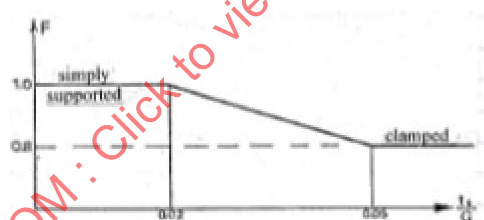


Figure 33 — TEMA Coefficient F

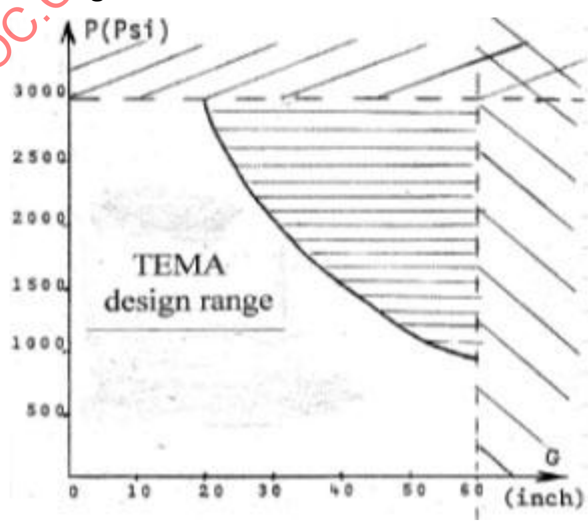


Figure 34 — TEMA Design Range

However, it must be pointed out that the value of coefficient *F* has been remarkably well chosen as it represents approximately the mean value of coefficient F(X). Finally TEMA formula is written:

$$\boxed{h = F\,\frac{D_s}{3}\sqrt{\frac{P_e}{\eta S}} = F\,\frac{G}{3}\sqrt{\frac{P_{TEMA}}{\eta S}}} \quad \text{where:}\begin{cases} P_{TEMA}\ \textit{is the TEMA effective pressure} \\ G\ \textit{is the shell internal diameter}\ D_s \end{cases}$$

(d) **TEMA effective pressure** is given by: $\boxed{P_{TEMA} = P^{'}_{t,TEMA} - P^{'}_{s,TEMA} + P_{d,TEMA}}$ with:

$$P^{'}_{s,TEMA} = \frac{0.4J\left[1.5 + K_{s,t}\left(1.5 + x_s\right)\right] - \frac{1-J}{2}\frac{D_J^2 - D_s^2}{D_s^2}}{1 + JK_{s,t}F_q}P_s \qquad P^{'}_{t,TEMA} = \frac{1 + 0.4JK_{s,t}\left(1.5 + x_t\right)}{1 + JK_{s,t}F_q}P_t$$

$$P_{d,TEMA} = \frac{4JE_s t_s / L}{\left(D_{s,e} - 3t_s\right)\left(1 + JK_{s,t}F_q\right)} \qquad with: \begin{cases} \Delta L = \left[\alpha_{s,m}(T_{s,m} - T_a) - \alpha_{t,m}(T_{t,m} - T_a)\right]L = -\gamma \\ D_{s,e} = external\ shell\ diameter = D_s + 2t_s \end{cases}$$

ASME-CODAP formula for equivalent pressure is written:

$$P_e = \frac{\left(JK_{s,t}\right)P^{'}_s - \left(JK_{s,t}\right)P^{'}_t + \left(JK_{s,t}\right)P_\gamma}{1 + JK_{s,t}F_q} = P^{'}_{s,1} - P^{'}_{t,1} + P_{\gamma,1} \quad \text{with:}$$

- $$P^{'}_{s,1} = \frac{\left(JK_{s,t}\right)P^{'}_s}{1 + JK_{s,t}F_q} = \frac{\left(JK_{s,t}\right)\left[x_s + 2\left(1 - x_s\right)\nu_t + \frac{2\nu_s}{K_{s,t}} - \frac{1-J}{2JK_{s,t}}\frac{D_J^2 - D_s^2}{D_o^2}\right]}{1 + JK_{s,t}F_q}P_s$$

$$= \frac{J\left[K_{s,t}x_s + 2K_{s,t}\left(1 - x_s\right)\nu_t + 2\nu_s\right] - \frac{1-J}{2}\frac{D_J^2 - D_s^2}{D_o^2}}{1 + JK_{s,t}F_q}P_s \qquad Using\ \upsilon_s = 0.3\ \ \upsilon_t = 0.3\ and\ D_o = D_s:$$

$$\boxed{P^{'}_{s,1} = \frac{0.4J\left[1.5 + K_{s,t}\left(1.5 + x_s\right)\right] - \frac{1-J}{2}\frac{D_J^2 - D_s^2}{D_s^2}}{1 + JK_{s,t}F_q}P = P^{'}_{s,TEMA}} \qquad \boxed{P^{'}_{s,TEMA} = \frac{JK_{s,t}}{1 + JK_{s,t}F_q}P^{'}_s}$$

- $$P^{'}_{t,1} = \frac{\left(JK_{s,t}\right)P^{'}_t}{1 + JK_{s,t}F_q} = \frac{\left(JK_{s,t}\right)\left[x_t + 2\left(1 - x_t\right)\nu_t + \frac{1}{JK_{s,t}}\right]}{1 + JK_{s,t}F_q}P_t = \frac{J\left[K_{s,t}x_t + 2K_{s,t}\left(1 - x_t\right)\nu_t + 1\right]}{1 + JK_{s,t}F_q}P_t \qquad Using\ \upsilon_t = 0.3:$$

$$\boxed{P^{'}_{t,1} = \frac{1 + 0.4JK_{s,t}\left(1.5 + x_t\right)}{1 + JK_{s,t}F_q}P_t = P^{'}_{t,TEMA}} \qquad \boxed{P^{'}_{t,TEMA} = \frac{JK_{s,t}}{1 + JK_{s,t}F_q}P^{'}_t}$$

- $$P_{\gamma,1} = \frac{\left(JK_{s,t}\right)F_q N_t K_t}{\left(1 + JK_{s,t}F_q\right)\pi a_O^2}\gamma = \frac{4JK_s}{\left(1 + JK_{s,t}F_q\right)\pi D_o^2}\gamma = \frac{4JE_s t_s\left(D_s + t_s\right)}{\left(1 + JK_{s,t}F_q\right)D_s^2}\frac{\gamma}{L}$$

$$\frac{D_s+t_s}{D_s^2}=\frac{D_{s,e}-t_s}{\left(D_{s,e}-2t_s\right)^2}=\frac{1}{D_{s,e}}\frac{1-\dfrac{t_s}{D_{s,e}}}{\left(1-\dfrac{2t_s}{D_{s,e}}\right)^2}\quad\frac{1}{D_{s,e}}\frac{1}{\left(1-\dfrac{4t_s}{D_{s,e}}\right)\left(1+\dfrac{t_s}{D_{s,e}}\right)}\quad\frac{1}{D_{s,e}}\frac{1}{\left(1-\dfrac{3t_s}{D_{s,e}}\right)}=\frac{1}{\left(D_{s,e}-3t_s\right)}$$

$$\boxed{P_{\gamma,1}=\frac{4JE_st_s}{\left(1+JK_{s,t}F_q\right)\left(D_{s,e}-3t_s\right)}\frac{\gamma}{L}=\frac{4JE_st_s}{\left(1+JK_{s,t}F_q\right)\left(D_{s,e}-3t_s\right)}\frac{-\Delta L}{L}=-P_d}$$

$$\boxed{P_{d,TEMA}=-\frac{JK_{s,t}}{1+JK_{s,t}F_q}P_\gamma}$$

Finally:

$$\boxed{P_e=\frac{\left(JK_{s,t}\right)P_s^{'}-\left(JK_{s,t}\right)P_t^{'}+\left(JK_{s,t}\right)P_\gamma}{1+JK_{s,t}F_q}=P_{s,1}^{'}-P_{t,1}^{'}+P_{\gamma,1}=P_{s,TEMA}^{'}-P_{t,TEMA}^{'}-P_{d,TEMA}=-P_{TEMA}}$$

(e) **Coefficient X**

$$X_a=X=\sqrt[4]{24\left(1-\nu^{*2}\right)N_t\frac{E_t\,t_t\left(d_t-t_t\right)}{E^*\,L\,h^3}\left(\frac{D_s}{2}\right)^2}=\sqrt[4]{\frac{6\left(1-\nu^{*2}\right)}{E^*}\frac{1}{L}\underbrace{N_tE_tt_t\left(d_t-t_t\right)}_{E_st_s\left(D_s+t_s\right)/K_{s,t}}\left(\frac{D_s^2}{h^3}\right)}$$

Introducing the deflection efficiency:

$$\eta^*=\frac{D^*}{D}=\frac{E^*}{1-\nu^{*2}}\frac{1-\nu^2}{E}\quad \textit{which leads to}:\frac{1-\nu^{*2}}{E^*}=\frac{1-\nu^2}{E\eta^*}\quad \text{, it follows:}$$

$$X_a=X=\sqrt[4]{\frac{6\left(1-\nu^2\right)}{\eta*E}\frac{E_st_s\left(D_s+t_s\right)}{LK_{s,t}}\frac{D_s^2}{h^3}}\quad \textit{Neglecting } t_s \textit{ compared to } D_s:\ \boxed{X_{TEMA}=\sqrt[4]{\frac{6\left(1-\nu^2\right)}{\eta*}\frac{E_st_s}{ELK_{s,t}}\left(\frac{D_s}{h}\right)^3}}$$

(f) **Coefficient F_q** is given on Figure 35 as a function of X for SS (Z=0) and CL (Z=∞) TS.

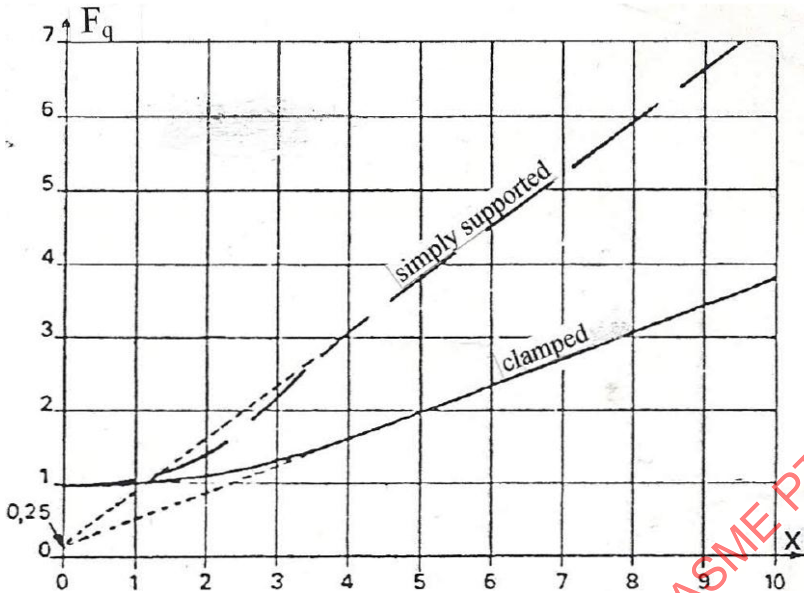


Figure 35 — Coefficient Fq as a Function of X for SS and CL TS

F_q can be approximated as: $F_q = 0.25 + \alpha X \quad with: \begin{cases} \alpha = \dfrac{\sqrt{2}}{2} \text{ when the TS is SS} \\ \alpha = \dfrac{\sqrt{2}}{4} \text{ when the TS is CL} \end{cases}$

TEMA coefficient F equals: $\begin{cases} F = 1.0 \text{ when the TS is SS} \\ F = 0.8 \text{ when the TS is CL} \end{cases}$

Thus, α can be written: $\alpha = (F - 0.6)\dfrac{\sqrt{2}}{0.8}$ and F_q is written:

$$F_q = 0.25 + (F - 0.6)\frac{\sqrt{2}}{0.8}\sqrt[4]{\frac{6(1-\nu^2)}{\eta *}\frac{E_s t_s}{ELK_{s,t}}\left(\frac{D_s}{h}\right)^3} = 0.25 + (F - 0.6)\sqrt[4]{\underbrace{\frac{24(1-\nu^2)}{0.8^4 \eta *}}_{A}\frac{E_s t_s}{ELK_{s,t}}\left(\frac{D_s}{h}\right)^3}$$

For the determination of coefficient A, TEMA uses:

$\upsilon = 0.3 \text{ and } \eta^* = 0.178 \text{ which leads to } A = 300 \text{ and:}$

$F_q = 0.25 + (F - 0.6)\sqrt[4]{\dfrac{300 E_s t_s}{ELK_{s,t}}\left(\dfrac{D_s}{T}\right)^3}$ Figure 35 shows that lowest value of F_q is 1. Thus:

$$\boxed{F_q = MAX\left[(1), \left(0.25 + (F - 0.6)\sqrt[4]{\frac{300 E_s t_s}{ELK_{s,t}}\left(\frac{D_s}{T}\right)^3}\right)\right]}$$

(g) **TEMA shear formula**

CODAP formula for shear stress is written: $\tau = \dfrac{1}{2\mu}\cdot\dfrac{D_o}{2h}\cdot P_e$ with $\tau \le 0.8S$

Thus, the minimum TS shear thickness is written: $h = \frac{1}{3.2\mu} \cdot \frac{D_o}{S} \cdot P_e$

ASME 2013 uses for D_o the equivalent diameter D_L of the tube center limit parameter, which leads to:

$$\boxed{T = \frac{0.31 D_L}{\mu} \frac{P}{S}}$$

(h) **Tube longitudinal stress** is calculated by TEMA at periphery of bundle:

$$\sigma_{t,o} = \frac{1}{x_t - x_s}\left[\left(P_s x_s - P_t x_t\right) - F_q P_e\right]$$

with:

[III.2.b1] $x_t - x_s = \frac{N_t s_t}{\pi a_o^2} = \frac{N_t t_t (d_t - t_t)}{a_o^2}$ $\quad a_o = D_s/2 \quad -P_e = P_{TEMA} = P'_{t,TEMA} - P'_{s,TEMA} + P_{d,TEMA}$

$$\boxed{\sigma_{t,o} = \frac{F_q D_s^2}{4 N_t t_t (d_t - t_t)}\left[\frac{x_s P_s - x_t P_t}{F_q} + P'_{t,TEMA} - P'_{s,TEMA} + P_{d,TEMA}\right] = \frac{F_q D_s^2}{4 N_t t_t (d_t - t_t)}\left[\underbrace{\left(P'_{t,TEMA} - \frac{x_t P_t}{F_q}\right)}_{P_2} - \underbrace{\left(P'_{s,TEMA} - \frac{x_s P_s}{F_q}\right)}_{P_3} + P_{d,TEMA}\right]}$$

- If $\sigma_{t,o} > 0$ (tubes in traction) $\sigma_{t,o}$ is limited to $2S_t$
- If $\sigma_{t,o} \leq 0$ (tubes in compression) $\sigma_{t,o}$ is limited to S_t and to the maximum permissible buckling stress limit S_{tb} named S_c in TEMA.

(i) **Shell longitudinal stress** is obtained from [XI.2.11a]: $\sigma_{s,m} = \frac{D_s^2}{4 t_s (D_s + t_s)}\left(P_e + P_t\right)$

with: $D_s \cong D_s + t_s$ *and* $P_e = -P_{TEMA} = P'_{s,TEMA} - P'_{t,TEMA} - P_{d,TEMA}$

$$\boxed{\sigma_{s,m} = \frac{D_s^2}{4 t_s (D_s + t_s)}\left(\underbrace{P_t - P'_{t,TEMA}}_{P_1} + P'_{s,TEMA} - P_{d,TEMA}\right)}$$

Shell and channel bending stresses are not calculated.

12.4 Comparison with Circular Plates Subject to Pressure

The classical formulas of circular plates subjected to pressure for SS plates are:

$$\sigma_r(x) = 1.5 \underbrace{\frac{(3+\upsilon)\left(1 - \frac{r^2}{R^2}\right)}{16}}_{F^{SS}(r)} \left(\frac{2R}{h}\right)^2 \left(P_s - P_t\right) \qquad with: \begin{cases} F^{SS}(0) = \frac{3+\upsilon}{16} & at\ TS\ center\ (r=0) \\ F^{SS}(R) = 0 & at\ TS\ periphery\ (r=R) \end{cases}$$

for CL plates:

$$\sigma_r(x) = 1.5 \underbrace{\frac{(1+\upsilon)-(3+\upsilon)\frac{r^2}{R^2}}{16}}_{F^{CL}(r)} \left(\frac{2R}{h}\right)^2 (P_s - P_t) \quad with: \begin{cases} F^{CL}(0) = \frac{1+\upsilon}{16} & at\ TS\ center\ (r=0) \\ F^{CL}(R) = -\frac{1}{8} & at\ TS\ periphery\ (r=R) \end{cases}$$

The following modifications must be applied to UHX-13.3 rules.

(a) **No unperforated rim**: $\begin{cases} a_o = a_s = R \quad D_o = D_s \quad \Rightarrow \quad \rho_s = 1 \quad \omega_s^* = \omega_s \\ a_o = a_c = R \quad D_o = D_c \quad \Rightarrow \quad \rho_c = 1 \quad \omega_c^* = \omega_c \end{cases}$

(b) **No holes and no tubes**: $\mu^* = 1 \quad E^* = E \quad \upsilon^* = \upsilon$

$$N_t = 0 \quad \Rightarrow \quad x_s = 1 \quad x_t = 1 \Rightarrow X_a = \sqrt[4]{24\left(1-\nu^{*2}\right) N_t \frac{E_t\, t_t\,(d_t - t_t)}{E^*\, L\, h^3} \left(\frac{D_s}{2}\right)^2} = 0$$

When there no more tubes, there is no more elastic foundation and X_a=0. For X_a=0 several coefficients are to infinity
(Z_d, Z_w, Q_m, Q_v,…). Accordingly, these coefficients will be calculated for $X_a \rightarrow 0$, as shown in Annex F.

(c) **No bellows**: $K_J = 0 \quad \Rightarrow \quad J = \frac{K_J}{K_J + K_s} = 1$

(d) **No bolted flange: W*=0** **No differential thermal expansion: γ =0**

(e) **No effect of P_s and P_t on shell and channel at their connection with TS**:

$$\begin{cases} \delta_s P_s = 0 \quad \Rightarrow \quad \delta_s = 0 \quad \omega_s = 0 \quad \omega_s^* = 0 \\ \delta_c P_c = 0 \quad \Rightarrow \quad \delta_c = 0 \quad \omega_c = 0 \quad \omega_c^* = 0 \end{cases}$$

(f) **Calculation of A and Φ**: $A = D_o \Rightarrow K = \frac{A}{D_0} = 1 \Rightarrow \Phi = \frac{1-\nu^{*2}}{E^*}(\lambda_s + \lambda_c)$

From [VI.3]:

$$Q_1 = -\frac{1}{4}\frac{\Phi}{(1+\upsilon^*)+\Phi} \qquad Q_2 = \frac{\left(\omega_s^* P_s - \omega_c^* P_c\right) + \left(W \frac{\gamma_b}{2\pi}\right)}{1+\Phi Z_m} = 0$$

$$Q_3 = Q_1 + \frac{2}{a_0^2\, P_e} Q_2 = Q_1 = -\frac{1}{4}\frac{\Phi}{(1+\upsilon^*)+\Phi}$$

Using Z_w and Z_d from Annex G:

$$Q_{Z1} = (Z_w Q_1 + Z_d)\frac{X_a^4}{2} = \left[\frac{1}{4(1+\upsilon^*)}\frac{-\Phi}{4(1+\upsilon^*)+\Phi} + \frac{2}{X_a^4}\right]\frac{X_a^4}{2} = 1$$

Using Z_v and Z_m from Annex G:

$$Q_{Z2} = \left(Z_v Q_1 + Z_m\right)\frac{X_a^4}{2} = \left[\frac{1}{4\left(1+\upsilon^*\right)}\frac{-\Phi}{4\left(1+\upsilon^*\right)+\Phi} + \frac{1}{1+\upsilon^*}\right]\frac{X_a^4}{2} = 0$$

$$U = \frac{Z_w + \left(\rho_s - 1\right)Z_m}{1+\Phi Z_m} X_a^4 = \frac{Z_w}{1+\Phi Z_m} X_a^4 = \frac{1}{4\left(1+\upsilon^*\right)}\frac{1}{1+\Phi Z_m} X_a^4 = 0$$

(g) **Equivalent pressure** is written:

$$P_e = \frac{1}{\dfrac{1}{JK_{s,t}} + \left[Q_{Z1} + \left(\rho_s - 1\right)Q_{Z2}\right]}\left[P_s^{'} - P_t^{'} + P_\gamma + P_w + P_{rim}\right]$$

Thus: $\boxed{P_e = P_s - P_t}$

- If the plate is simply supported (no support from shell and channel): Φ=0 in ASME , Z=0 in CODAP
- If the plate is clamped (full support from shell and channel): Φ=∞ in ASME , Z=∞ in CODAP

(h) **Coefficient F_m** is written: $F_m\left(x\right) = \dfrac{Q_v(x) + Q_3 Q_m(x)}{2}$

where $Q_v(x)$ and $Q_m(x)$ are given by Annex G when $X_a \rightarrow 0$:

$$Q_m(x \rightarrow 0) = \frac{\left(1+\upsilon^*\right)}{2Z_a} bei' = \frac{\left(1+\upsilon^*\right)}{\dfrac{1+\upsilon^*}{2} X_a}\frac{X_a}{2} = 1$$

$$Q_v(x \rightarrow 0) = \frac{\left(1+\upsilon^*\right)}{2Z_a X_a}\Psi_1 = \frac{\left(1+\upsilon^*\right)}{\dfrac{1+\upsilon^*}{2} X_a^2}\frac{3+\upsilon^*}{16} X_a^2 = \frac{3+\upsilon^*}{8}$$

Coefficient F_m at TS center is given by formula:

$$\boxed{F_m\left(x \rightarrow 0\right) = \frac{Q_3}{2} + \frac{3+\upsilon^*}{16} = -\frac{1}{8}\frac{\Phi}{\left(1+\upsilon^*\right)+\Phi} + \frac{3+\upsilon^*}{16}}$$

Coefficient F_m at TS periphery is given by formula:

$$\boxed{F_m\left(X_a\right) = \frac{Q_3}{2} = -\frac{1}{8}\frac{\Phi}{\left(1+\upsilon^*\right)+\Phi}}$$

(i) **TS bending stress** is written: $\sigma_r\left(x\right) = 1.5\, F_m(x)\left(\dfrac{2R}{h}\right)^2\left(P_s - P_t\right)$

If the TS is SS: $\Phi = 0 \;\Rightarrow\; Q_3 = 0 \;\Rightarrow\; \begin{cases} F_m^{SS}(0) = \dfrac{3+\upsilon}{16} & \text{at TS center} \\ F_m^{SS}(R) = 0 & \text{at TS periphery} \end{cases}$

If the TS is CL: $\Phi = \infty \quad \Rightarrow \quad Q_3 = -\frac{1}{4} \quad \Rightarrow \begin{cases} F_m^{CL}(0) = -\frac{1}{8} + \frac{3+\upsilon}{16} = \frac{1+\upsilon}{16} & at\ TS\ center \\ F_m^{CL}(R) = -\frac{1}{8} & at\ TS\ periphery \end{cases}$

Accordingly classical formulas for circular plates subjected to pressure given in this section have been retrieved. Similar calculations can also be performed for the TS deflection w(x) and rotation θ(x).

These results can be obtained from a UHX-13.3 software, such as Mathcad software shown in Annex V, by imposing the simplifications listed in (a) to (e) above.

12.5 Conclusions

Applying the relevant simplifications, it has been analytically demonstrated that UHX-13.3 method leads to CODAP, TEMA and circular plates' formulas. This intends to prove the correctness of ASME method, which is confirmed by FEA comparisons for the TS deflection.

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[11] ASME Section VIII Division 1, 2007 Edition, Appendix 4.

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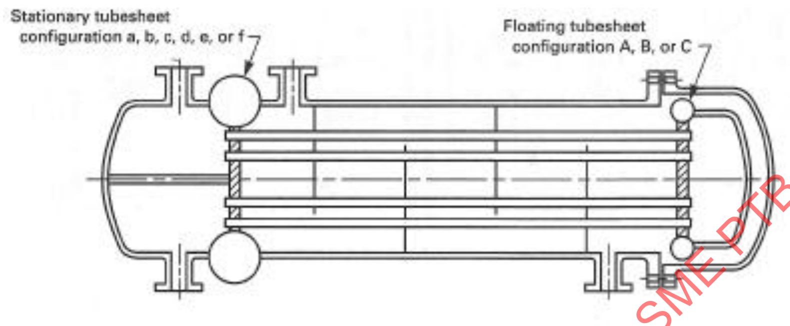
PART 4
FLOATING TUBESHEETS

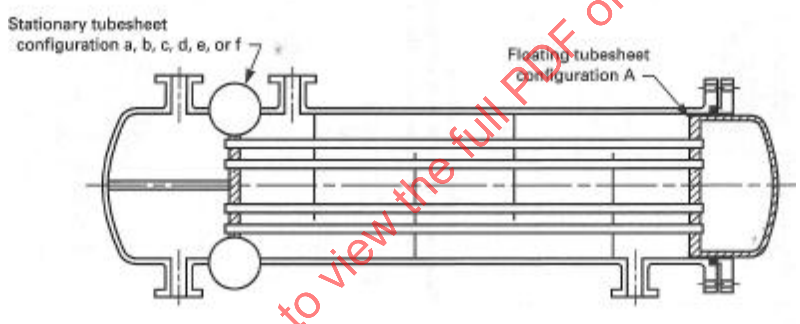
1 SCOPE

PART 4 applies to floating tubesheet (TS) Heat Exchangers (HE)s that have one stationary tubesheet and one floating tubesheet. Three types of HEs are covered as shown in Figure 36:

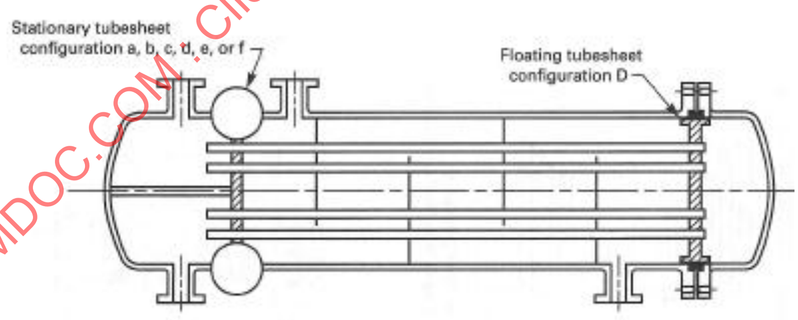
- immersed floating head (sketch a)
- externally sealed floating head (sketch b)
- internally sealed floating tubesheet (sketch c)



(a) Typical Floating Tubesheet Exchanger With an Immersed Floating Head



(b) Typical Floating Tubesheet Exchanger With an Externally Sealed Floating Head



(c) Typical Floating Tubesheet Exchanger With an Internally Sealed Floating Tubesheet

Figure 36 — Floating Tubesheet Heat Exchangers

(see TS configurations in Figure 37 and Figure 38)

2 HISTORICAL BACKGROUND

In the past decades many authors have proposed theoretical methods for the design of floating TS HEs. The most important contributions are provided below.

Gardner [1] in 1948 was the very first to develop an analytical approach by taking into consideration the support afforded by the tubes and the weakening effect of the TS holes. The TS is considered as either simply supported or clamped at its periphery to simulate the rotational restraint afforded by the shell and the channel, which compels the designer to make a more or less arbitrary choice between these two extreme cases. The method was adopted by Dutch code STOOWEZEN in 1975

K.A.G. Miller [2] at the same time, proposed a similar approach that was published in the British Code BS 1515 in 1965.

Galletly [3] in 1959 improved these design methods by accounting for the degree of rotational restraint of the TS at its periphery by the shell and the channel. This method was adopted by the French Pressure Vessel Code CODAP in 1982 and by the European Pressure Vessel Standard EN13445 in 2002.

Gardner [4] in 1969 improved his 1942 method by proposing a direct determination of the TS thickness which accounts for the unperforated rim and the TS-shell-channel connection. The method was adopted by ISO/DIS-2694 in 1973, by BS5500 in 1976 and by CODAP in 1995.

Soler [5] in 1984 developed a similar method accounting for the unperforated rim and the TS-shell-channel connection. The method is derived from fixed TS method and published for the first time in Non mandatory Appendix AA of Section VIII Division 1. In 2003 it was published in a new Part UHX of Section VIII Division 1 “Rules for Shell and Tubes Heat Exchangers” which became mandatory in 2004.

3 GENERAL

3.1 TS Configurations (UHX-14.1)

(a) **The stationary TS** is attached to the shell and the channel by welding (integral TS) or by bolting (gasketed TS) in accordance with the following 6 configurations (see Figure 37):

- Configuration a: tubesheet integral with shell and channel;
- Configuration b: tubesheet integral with shell and gasketed with channel, extended as a flange;
- Configuration c: tubesheet integral with shell and gasketed with channel not extended as a flange;
- Configuration d: tubesheet gasketed with shell and channel extended as a flange or not;
- Configuration e: tubesheet gasketed with shell and integral with channel, extended as a flange;
- Configuration f: tubesheet gasketed with shell and integral with channel not extended as a flange.

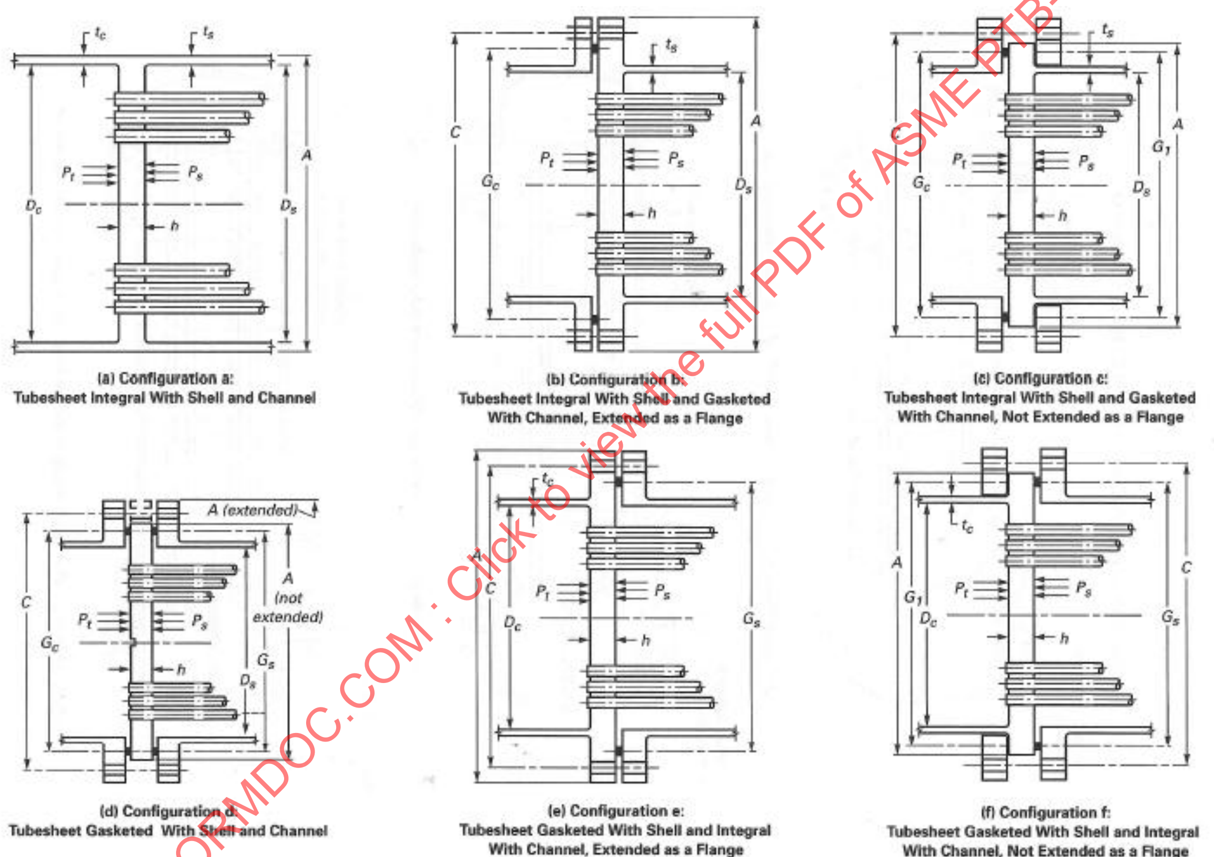


Figure 37 — Stationary Tubesheet Configurations

(b) **The floating TS** is free to move along the HE longitudinal axis in accordance with the following 4 configurations (see Figure 38):

- Configuration A: integral tubesheet;
- Configuration B: gasketed, tubesheet extended as a flange;
- Configuration C: gasketed, tubesheet not extended as a flange;
- Configuration D: internally sealed tubesheet.

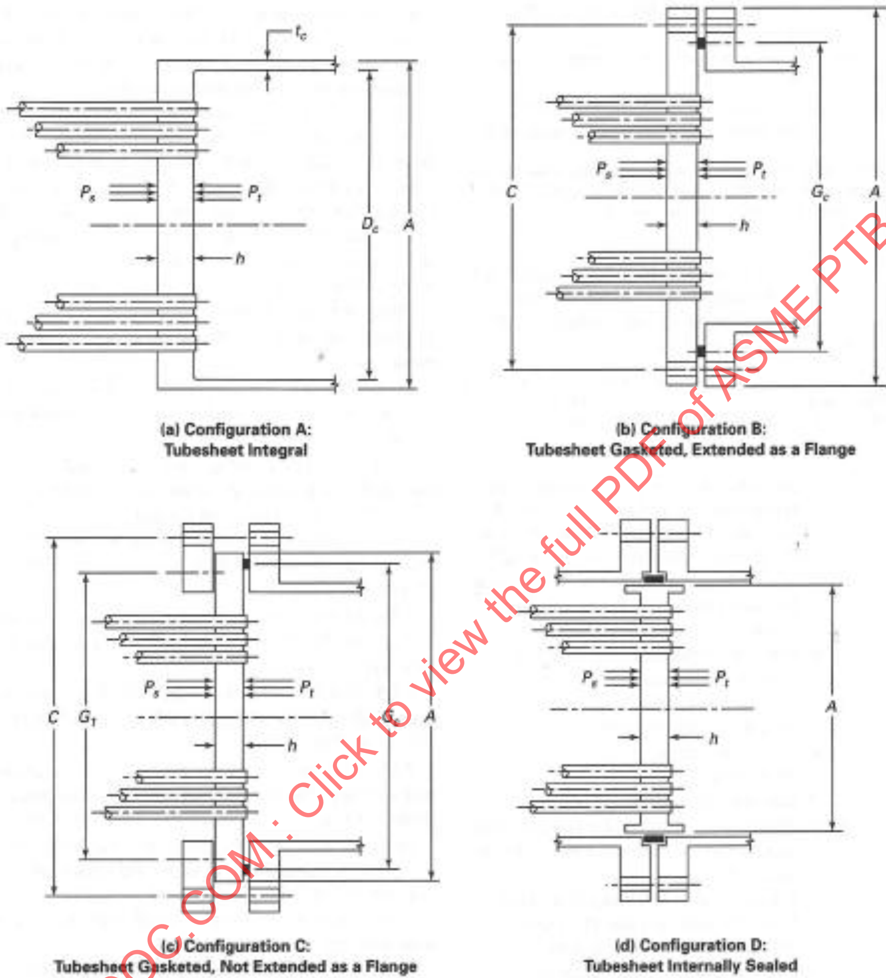


Figure 38 — Floating Tubesheet Configurations

3.2 Notations

(a) **Data for the design of the HE are as follows** (UHX-14.3)

Symbols D_o, E^*, h_g', μ, μ^* and ν^* are defined in Section 2 of PART 2.

A	=	outside diameter of tubesheet
a_c	=	radial channel dimension
a_o	=	equivalent radius of outer tube limit circle
a_s	=	radial shell dimension
C	=	bolt circle diameter
D_c	=	inside channel diameter
D_s	=	Inside shell diameter
d_t	=	nominal outside diameter of tubes
E	=	modulus of elasticity for tubesheet material at T

E_c = modulus of elasticity for channel material at T_c
E_s = modulus of elasticity for shell material at T_s
E_t = modulus of elasticity for tube material at T_t
G_c = diameter of channel gasket load reaction
G_s = diameter of shell gasket load reaction
G_1 = midpoint of contact between flange and tubesheet
h = tubesheet thickness
L_t = tube length between outer tubesheet faces
N_t = number of tubes
P_e = effective pressure acting on tubesheet
P_s = shell side design or operating pressure. For shell side vacuum use a negative value for P_s.
P_t = tube side design or operating pressure. For tube side vacuum use a negative value for P_t.

Notation P_c, instead of P_t, is used throughout the analytical development so as to maintain the symmetry of the equations involving the shell (subscript s) and the channel (subscript c).

S = allowable stress for tubesheet material at T
S_c = allowable stress for channel material at T_c
S_s = allowable stress for shell material at T_s
S_t = allowable stress for tube material at T_t
S_y = yield strength for tubesheet material at T
$S_{y,c}$ = yield strength for channel material at T_c
$S_{y,s}$ = yield strength for shell material at T_s
$S_{y,t}$ = yield strength for tube material at T_t
S_{PS} = allowable primary plus secondary stress for tubesheet material at T
$S_{PS,c}$ = allowable primary plus secondary stress for channel material at T_c
$S_{PS,s}$ = allowable primary plus secondary stress for shell material at T_s
T = tubesheet design temperature
T_a = ambient temperature, 70ºF (20ºC)
T_c = channel design temperature
T_s = shell design temperature
T_t = tube design temperature
t_c = channel thickness
t_s = shell thickness
t_t = nominal tube wall thickness
W^* = tubesheet effective bolt load determined in accordance with UHX-8
ν = Poisson's ratio of tubesheet material
ν_c = Poisson's ratio of channel material
ν_s = Poisson's ratio of shell material
ν_t = Poisson's ratio of tube material

(b) **Design coefficients** (UHX-14.5.1 to 4)

The following coefficients, specific to each component of the HE, will be used in the analytical treatment. They complete the data given above.

(1) **Perforated TS**

Equivalent diameter of outer tube limit circle (see Section 4.3(a) of PART 2): $D_o = 2r_o + d_t$

Equivalent radius of outer tube limit circle: $a_o = \dfrac{D_o}{2}$

TS coefficients relating to the tubes:

- Shell side: $x_s = 1 - N_t \left(\frac{d_t}{2a_o}\right)^2$; $1 - x_s = N_t \left(\frac{d_t}{2\,a_o}\right)^2$
- Tube side: $x_t = 1 - N_t \left(\frac{d_t - 2t_t}{2a_o}\right)^2$; $1 - x_t = N_t \left(\frac{d_t - 2t_t}{2\,a_o}\right)^2$
- $x_t - x_s = N_t \left(\frac{d_t^2 - (d_t - 2t_t)^2}{4\,a_o^2}\right) = \frac{N_t \cdot s_t}{\pi\,a_o^2} = \frac{N_t \cdot k_t}{E_t} \cdot \frac{l}{\pi\,a_o^2} = \frac{N_t \cdot K_t}{E_t} \cdot \frac{L}{\pi\,a_o^2}$
- Ligament efficiency: $\mu^* = \frac{p^* - d^*}{p^*}$
- Effective tube hole diameter d* and effective pitch p* are defined in Section 4.3(d) and (c) of PART 2
- Effective elastic constants E* and ν* are given in Section 5.6 of PART 2 as a function of μ^* and h/*p* (triangular or square pitch).
- Bending stiffness: $D^* = \frac{E^* \cdot h^3}{12\left(1 - \nu^{*2}\right)}$
- Effective tube side pass partition groove depth given in Section 4.3(f) of PART 2: h'_g
- Effective pressure acting on tubesheet: P_e

(2) **Tube bundle**

Tube cross-sectional area: $s_t = \frac{\pi}{4}\left[d_t^2 - (d_t - 2t_t)^2\right] = \pi\, t_t\,(d_t - t_t) = \frac{\pi\, a_o^2}{N_t}(x_t - x_s)$

Axial stiffness K_t of one tube: $K_t = \frac{E_t\, s_t}{L} = \frac{\pi\, t_t\,(d_t - t_t)\,E_t}{L}$

Axial stiffness k_t of one half tube of length l=L/2: $k_t = \frac{E_t\, s_t}{l} = \frac{2\pi\, t_t\,(d_t - t_t)\,E_t}{L} = 2\,K_t$

Effective elastic foundation modulus equivalent to the half tube bundle:

$$k_w = \frac{N_t \cdot k_t}{\pi\, a_o^2} = \frac{2\,N_t \cdot K_t}{\pi\, a_o^2} = \frac{2\,N_t \cdot E_t \cdot t_t\,(d_t - t_t)}{L\,a_o^2} = \frac{2\,E_t}{L}(x_t - x_s) = \frac{E_t}{l}(x_t - x_s)$$

$$k = \sqrt[4]{\frac{k_w}{D^*}} \quad ; \quad x = k\,r \quad ; \quad 0 \le r \le a_o \quad \Rightarrow \quad 0 \le x \le k\,a_o \qquad k a_o = X_a$$

Axial stiffness ratio tubes/TS:

$$X_a = k\,a_o = \sqrt[4]{\frac{k_w}{D^*}}\,a_o = \left[24\left(1 - \nu^{*2}\right) N_t \frac{E_t\, t_t\,(d_t - t_t)\,a_o^2}{E^*\,L\,h^3}\right]^{1/4}$$

(3) **Shell**

Radial shell dimension: a_s $\quad \rho_s = \frac{a_s}{a_o}$

ST TS Integral configurations (a, b and c): $a_s = D_s / 2$
Gasketed configurations (d): $a_s = G_s / 2$

FL TS configurations (A, B, C and D): $a_s = a_c$

Mean shell radius: $a_s^{'} = \dfrac{D_s + t_s}{2}$

Shell cross-sectional area: $s_s = \pi\, t_s \left(D_s + t_s\right)$

Axial stiffness k'ₛ of the half shell of length l=L/2: $k_s^{'} = \dfrac{E_s\, s_s}{l} = \dfrac{2\pi\, t_s \left(D_s + t_s\right) E_s}{L} = 2\, K_s$

Shell coefficient: $\beta_s = \dfrac{\sqrt[4]{12\left(1 - \nu_s^2\right)}}{\sqrt{\left(D_s + t_s\right)\, t_s}}$

Bending stiffness: $k_s = \beta_s \dfrac{E_s \cdot t_s^3}{6\left(1 - \nu_s^2\right)}$

(4) **Channel**

Radial channel dimension: a_c $\rho_c = \dfrac{a_c}{a_o}$

Integral configurations (a, e, f, and A): $a_c = D_c / 2$
Gasketed configurations (b, c and d): $a_c = G_c / 2$
Gasketed configuration D: $a_c = A / 2$

Mean channel radius: $a_s^{'} = \dfrac{D_c + t_c}{2}$

Channel coefficient: $\beta_c = \dfrac{\sqrt[4]{12\left(1 - \nu_c^2\right)}}{\sqrt{\left(D_c + t_c\right) t_c}}$

Bending stiffness: $k_c = \beta_c \cdot \dfrac{E_c \cdot t_c^3}{6\left(1 - \nu_c^2\right)}$

(5) **Unperforated rim**
D_o = internal diameter
A = external diameter

Diameter ratio: $K = A / D_o$

3.3 Loading Cases (UHX-14.4)

The normal operating condition of the HE is achieved when the tube side pressure P_t and shell side pressure P_s act simultaneously. However, a loss of pressure is always possible. Accordingly, for safety reasons, the designer must always consider the cases where P_s=0 or P_t=0 for the normal operating condition(s).

The designer must also consider the startup condition(s), the shutdown condition(s) and the upset condition(s), if any, which may govern the design.

A floating TS HE is a statically indeterminate structure for which it is generally not possible to determine the most severe condition of coincident pressure and temperature. Thus, it is necessary to evaluate all the anticipated loading conditions mentioned above to ensure that the worst load combination has been considered in the design.
For each of these conditions, ASME, TEMA and CODAP used to consider the following pressure loading cases.

- Loading Case 1: Tube side pressure P_t acting only ($P_s = 0$).
- Loading Case 2: Shell side pressure P_s acting only ($P_t = 0$).
- Loading Case 3: Tube side pressure P_t and shell side pressure P_s acting simultaneously.

ASME 2013 Edition provides the detail of the pressure "design loading cases" to be considered for each operating condition specified by the user (normal operating conditions, startup conditions, the shutdown conditions,…). For the pressure loading cases, a table (table UHX-14.4-1) provides the values to be used for the design pressures P_s and P_t in the formulas, accounting for their maximum and minimum values. Additional operating (thermal + pressure) loading cases must be considered if the effect of the radial thermal expansion adjacent to the tubesheet is accounted for (see Section 10.1)

As the calculation procedure is iterative, a value *h* is assumed for the tubesheet thickness to calculate and check that the maximum stresses in tubesheet, tubes, shell, and channel are within the maximum permissible stress limits.

Because any increase of tubesheet thickness may lead to overstresses in the tubes, shell, or channel, a final check must be performed, using in the formulas the nominal thickness of tubesheet, tubes, shell, and channel, in both corroded and uncorroded conditions.

3.4 Design Assumptions (UHX-14.2)

A FL TS HE is a complex structure and several assumptions are necessary to derive a 'design by rules' method. Most of them could be eliminated, but the analytical treatment would lead to 'design by analysis' method requiring the use of a computer.

The design assumptions are as follows.

(a) HE
- The analytical treatment is based on the theory of elasticity applied to the thin shells of revolution.
- The HE is axi-symmetrical.
- The HE is a symmetrical unit with identical TSs so as to analyze a half-unit.

(b) TSs
- The two tubesheets are circular and identical (same diameter, uniform thickness, material, temperature and edge conditions). Deviations will be allowed for the FL TS to cover the 3 types of HE.
- The tubesheets are uniformly perforated over a nominally circular area, in either equilateral triangular or square patterns. This implies that each TS is fully tubed (no large untubed window)
- Radial displacement at the mid-surface of the TS is ignored
- Temperature gradient through TS thickness is ignored
- Shear deformation and transverse normal strain in the TS are ignored
- The unperforated rim of each TS is treated as a rigid ring without distortion of the cross section

(c) Tubes
- Tubes are assumed identical, straight and at same temperature
- Tubes are uniformly distributed in sufficient density to play the role of an elastic foundation for the TS
- The effect of the rotational stiffness of the tubes is ignored

(d) Shell and channel
- Shell and channel are cylindrical with uniform diameters and thicknesses
- If the channel head is hemispherical, it must be attached directly to the TS, without any cylindrical section between the head and the TS.

- shell and channel centerlines are the same.

(e) Weights and pressures drops

- Weights and pressures drops are ignored
- Pressures P_s and P_t are assumed uniform

3.5 Basis of Analytical Treatment

3.5.1 General

The design of a FL TS HE is complex as the two TSs are connected to each other through the tube bundle. Accordingly the structure is statically indeterminate. Many geometrical, mechanical and material properties are involved in the design as shown in Section 3.2(a) which lists the extensive input data. Although the FL TS differs from the ST TS (smaller channel diameter, edge conditions,…), the two tubesheets are assumed to be identical. This assumption may seem unrealistic, but if the real floating TS geometry is accounted for, the analytical treatment would lead to a 'design by analysis' method requiring the use of a computer.

As for a fixed TS HE, the analysis includes the effects of the shell and tube side pressures, the axial stiffening effect of the tubes, the stiffening effect of the unperforated ring at the tubesheet edge, and the stiffening effect of the integrally attached channel or shell to the tubesheet. For a tubesheet that is extended as a flange to which a channel or shell is to be bolted, the bolt load causes an additional moment in the tubesheet which is included in the total stress in the tubesheet in addition to the moments caused by pressure.

The analysis is based on classical discontinuity analysis methods to determine the moments and forces that the tubesheet, shell and channel must resist. These components are treated using the theory of elasticity applied to the thin shells of revolution.

Because the heat exchanger is assumed to be symmetric, only half of the heat exchanger is treated. The main steps of the ST TS design follow the analytical treatment of fixed TS HEs. The FL TS will be designed the same way in a second step, using its appropriate data (channel thickness and diameter, bolting data, edge conditions,…).

(a) The tubesheet is disconnected from the shell and channel. The shear load V_a and moment M_a are applied at the tubesheet edge as shown in Figure 39.

(b) The perforated tubesheet is replaced by an equivalent solid circular plate of diameter D_o and effective elastic constants E^* (effective modulus of elasticity) and ν^* (effective Poisson's Ratio) depending on the ligament efficiency μ^* of the tubesheet. This equivalent solid plate is treated by the theory of thin circular plates subjected to pressures P_s and P_t and relevant applied loads to determine the maximum stresses.

(c) The unperforated tubesheet rim is treated as a rigid ring whose cross section does not change under loading.

(d) The tubes are replaced by an equivalent elastic foundation of modulus k_w.

(e) The connection of the tubesheet with shell and channel accounts for the edge displacements and rotations of the 3 components.

(f) The shell and channel are subject to shell side and tube side pressures P_s and P_t and edge loads V_a and M_a to determine the maximum stresses.

(g) The maximum stresses in tubesheet, tubes, shell and channel are determined and limited to the appropriate allowable stress-based classifications of Section VIII Division 2 Part 4.

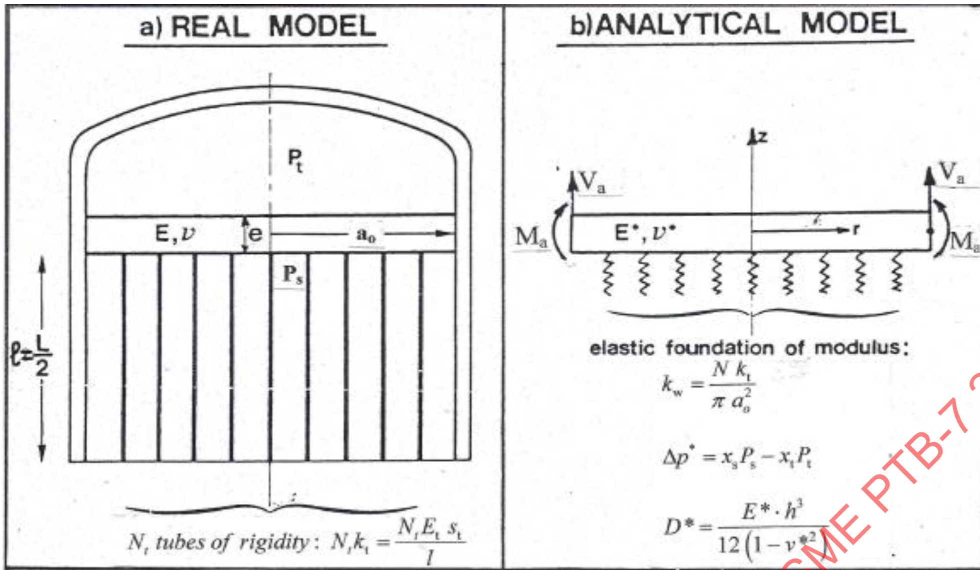


Figure 39 — Analytical Model Used in Design Method

3.5.2 Free Body Diagram for ST TS

Figure 40 shows, for a ST TS integral both sides (configuration a), the free body diagram of the component parts (perforated tubesheet, unperforated tubesheet rim, shell, channel). The figure details the relevant discontinuity forces (V_a, V_s, Q_s, V_c, Q_c) and moments (M_a, M_s, M_c, M_R) applied on each component, together with edge displacements.

In this figure:
Forces (V_a, V_s, Q_s, V_c, Q_c) and moments (M_a, M_s, M_c, M_R) are per unit of circumferential length
The following subscripts are used:

- s for shell,
- c for channel,
- R for unperforated rim

No subscript for the perforated TS
Notation P_c instead of P_t (tube side pressure) is used throughout the analytical development so as to maintain the symmetry of the equations involving the shell (subscript s) and the channel (subscript c).

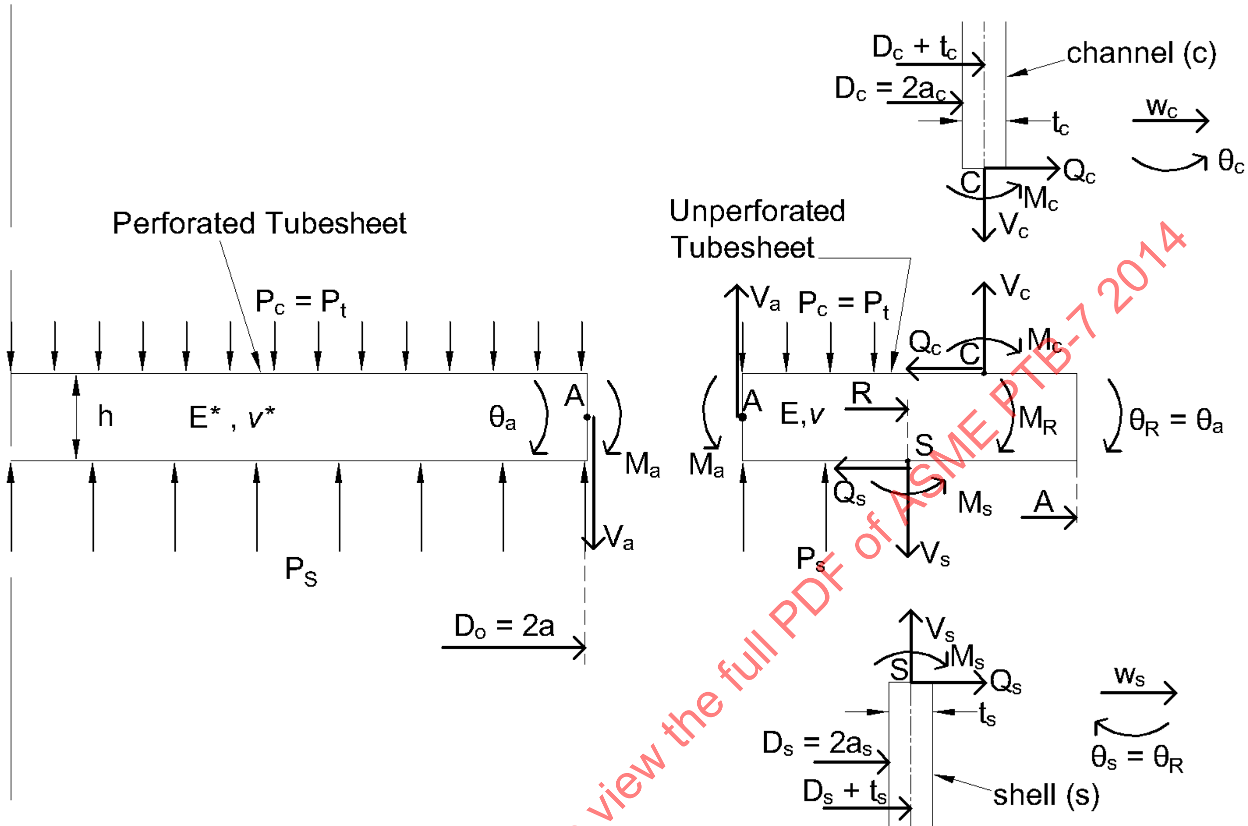


Figure 40 — Free Body Diagram of the Analytical Model for the ST TS

3.5.3 Free Body Diagram for FL TS

The FL TS is attached to the FL channel either by welding (integral configuration A) or by bolting (gasketed configurations B or C), but there is no shell, which implies V_s=0 and M_s=0.

Accordingly, the free body diagram is different from the free body diagram of the ST TS and depends on the HE type shown on Figure 36: immersed, externally sealed, internally sealed. Figure 41 provides the free body diagram of an immersed FL TS with configuration A.

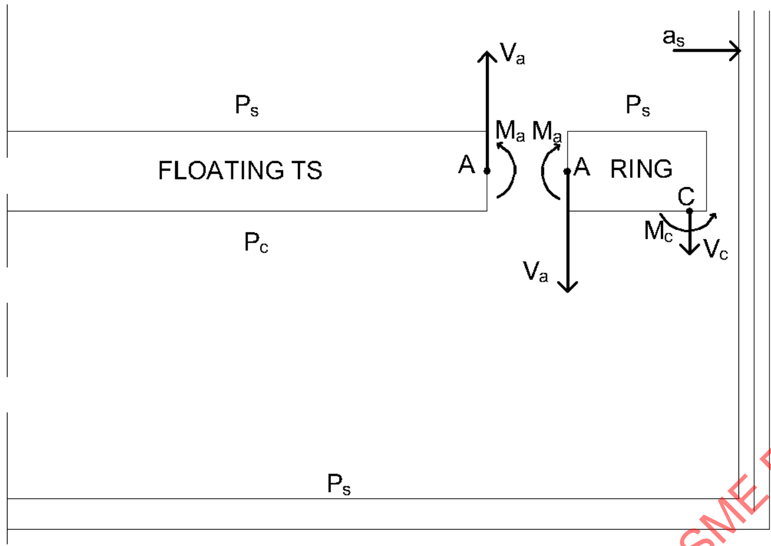


Figure 41 — Free Body Diagram of the Analytical Model for the FL TS

(floating channel is shown on Figure 48, Figure 49 and Figure 50 for the 3 types of FL TS HEs)

4 AXIAL DISPLACEMENTS AND FORCES ACTING ON THE TUBES AND ON THE SHELL

4.1 Axial Displacement and Force Acting on the Tubes (Figure 42)

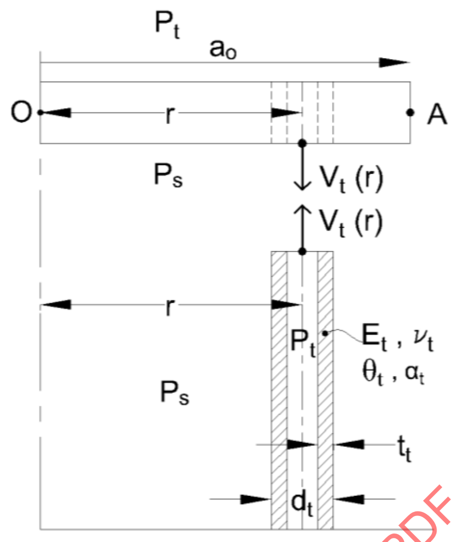


Figure 42 — Axial Displacement of Tubes

(a) **Axial Displacement of the tubes due to axial force $V_t(r)$ acting on the tube row at radius r:**

$$\delta_t\left(V_t\right)=\frac{V_t\left(r\right)}{k_t}=\frac{V_t\left(r\right)l}{E_t\ s_t}=\frac{l}{\pi\ E_t\ (d_t-t_t)\ t_t}V_t\left(r\right)$$

(b) **Axial force acting on each tube at radius r:** $V_t\left(r\right)=k_t\left[\delta_t(V_t)\right]$

(c) **Net effective pressure acting on the TS due to each tube at radius r of TS area** $\pi a_0^2 / N_t$:

$$q_t\left(r\right)=\frac{-V_t(r)}{\pi\ a_0^2\ /\ N_t}=-\frac{N_t k_t\ \delta_t\left(V_t\right)}{\pi\ a_0^2}=-\underbrace{\frac{N_t\ k_t}{\pi\ a_0^2}}_{k_w}\delta_t\left(V_t\right) \qquad \boxed{k_w=\frac{N_t\ k_t}{\pi\ a_0^2}} \qquad \boxed{q_t\left(r\right)=-k_w\ \delta_t\left(V_t\right)}$$

4.2 Axial Displacement and Force Acting on the Shell (Figure 43)

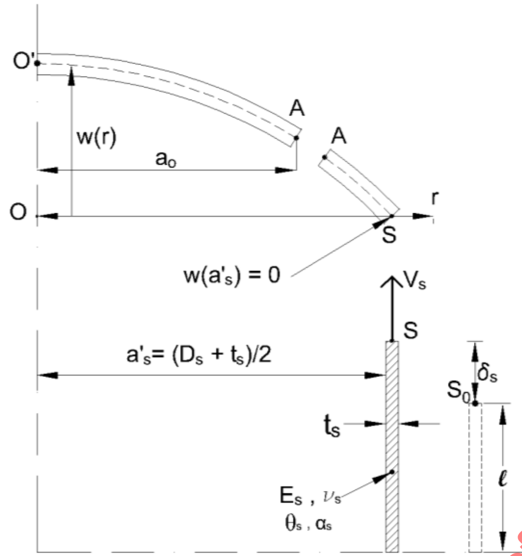


Figure 43 — Axial displacement of the Shell

(a) **Axial displacement of the shell due to axial force V_s acting on the shell:**

$$\delta_s(V_s) = \frac{V_s\ 2\pi\ a_s^{'}}{k_s^{'}} = \frac{V_s\ 2\pi\ a_s^{'}\ l}{E_s\ 2\pi\ a_s^{'} t_s} = \frac{l}{E_s\ t_s} V_s$$

V_s is per unit of length

$$a_s^{'} = mean\ shell\ radius = \frac{D_s + t_s}{2}$$

V_s is known and depends on the HE type (see VI.2).

(b) **Axial force acting on the shell:** $V_s = \frac{E_s t_s}{l}\,\delta_s(V_s)$

(c) **Axial displacement of tubes at radius r:**

$\delta_t(V_t) = \delta_s(V_s) + w(r)$ where w(r) is the TS deflection at radius r (see Figure 45)

(d) **TS deflection at radius r:** $\boxed{w(r) = \delta_t(V_t) - \delta_s(V_s)}$

5 DEFLECTION AND LOADS ACTING ON THE TUBESHEET

5.1 Equivalent Plate Resting on an Elastic Foundation (Figure 44)

(a) **Net effective pressure**

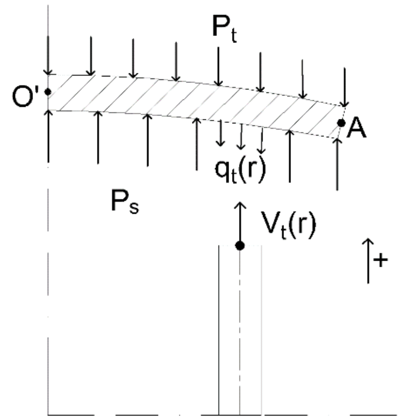


Figure 44 — Loads Acting on TS

due to tubes: $q_t(r) = -k_w\ \delta_t(V_t)$

Tubes act as an elastic foundation of equivalent modulus given by the axial rigidity of the half-bundle per unit of TS area:

$$k_w = \frac{N_t\ k_t}{\pi\ a_o^2} \qquad q_t(r) = -k_w\left[\delta_s(V_s) + w(r)\right]$$

due to pressures P_s and P_t acting on the equivalent plate (see Annex E);

$$q_P = x_s\ P_s - x_t\ P_t = \Delta p^*$$

net effective pressure:

$$q(r) = q_P + q_t(r) = \Delta p^* - \frac{V_t(r)}{\pi a_o^2 / N_t} \qquad \text{[V.1a]}$$

$$q(r) = \underbrace{\Delta p^* + k_w\left[-\delta_s(V_s)\right]}_{Q} - k_w\ w(r) \qquad Q = \Delta p^* - k_w\ \delta_s(V_s) \qquad \text{[V.1a']}$$

In this equation, the displacement δ_s of the shell subjected to axial force V_s is known and

$$q(r) = Q - k_w\ w(r)$$

(b) **Deflection of TS** (Figure 45)

The determination of the deflection given in Section 5.1(b) of PART 3 applies:

$$w(x) = A\,berx + B\,beix + \frac{Q}{k_w} \qquad \text{where: } x = k\ r = \sqrt[4]{\frac{k_w}{D^*}}\ r$$

Quantities $q(x)$, $\theta_r(x)$, $Q_r(x)$ and $M_r(x)$ are respectively given in Section 5.1(a), 5.1(c), 5.1(d) and 5.1(e) of PART 3

At TS periphery ($r = a_o$): $$X_a = k\ a_o = \sqrt[4]{\frac{k_w}{D^*}}\ a_o = \left[24\ \left(1-\nu^{*2}\right)\ N_t\ \frac{E_t\ t_t\ (d_t - t_t)\ a_o^2}{E^*\ L\ h^3}\right]^{1/4}$$

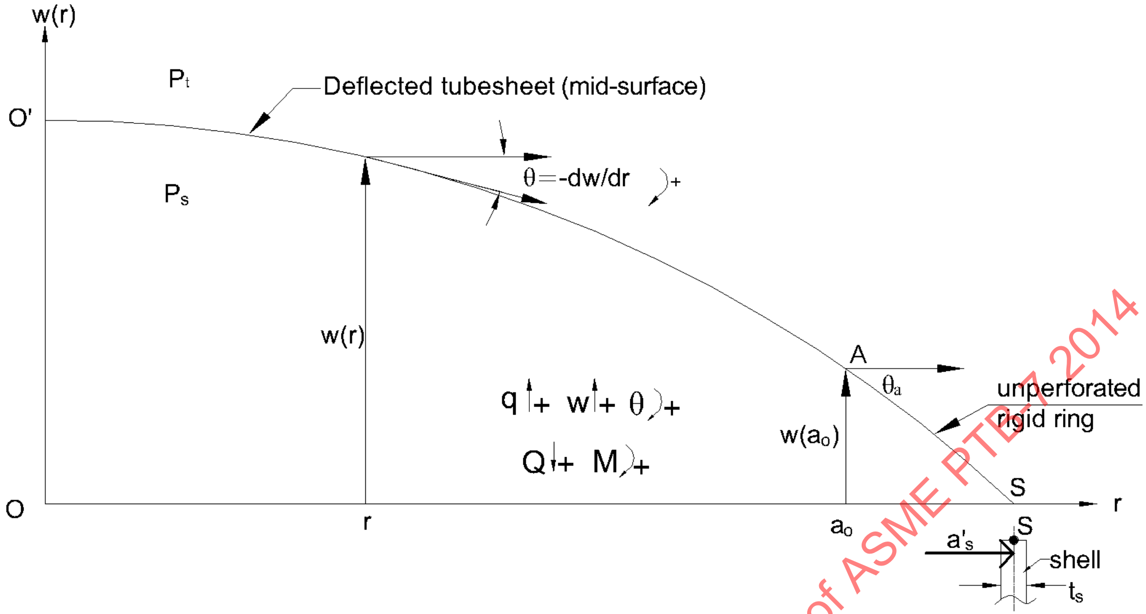


Figure 45 — TS Displacement

5.2 Determination of Integration Constants A and B

The determination of the integration constants A and B given in Section 5.2 of PART 3 applies. Substituting the expressions of A and B in V.1b enables to determine:

$$w(x), q(x), \theta(x), Q(x), M(x)$$

as functions of x, depending on V_a and M_a which are still unknown.

6 TREATMENT OF THE UNPERFORATED RIM

6.1 Edge Loads Applied on Shell and Channel at their Connection to the TS

ST TS: Section 6.1(a) and 6.1(b) of PART 3 apply.
FL TS: Section 6.1(a) of PART 3 does not apply as there is no shell.
Accordingly, the shell coefficients β_s, k_s, λ_s, δ_s: shall be taken equal to 0, and a_s shall be taken equal to a_c.
Section 6.1(b) of PART 3 applies.

6.2 Equilibrium of the Unperforated Rim

6.2.1 Due to Axial Loads

(a) The ring equilibrium for the ST TS is written (see Figure 46):

$$2\pi a_o V_a + \pi\left(a_s^2 - a_o^2\right)P_s + 2\pi a_c' V_c = \pi\left(a_c^2 - a_o^2\right)P_c + 2\pi a_s' V_s \qquad \text{[VI.2.1.1]}$$

where:

V_a = axial edge load acting at connection of ring with equivalent plate is still to be determined

V_s = axial force acting in the shell, which depends on the HE type (immersed, externally sealed, internally sealed).

V_c = axial force acting in the stationary channel: $2\,\pi\, a_c'\, V_c = \pi\, a_c^2\, P_c \quad \Rightarrow \quad a_c'\, V_c = \dfrac{a_c^2}{2}\, P_c$

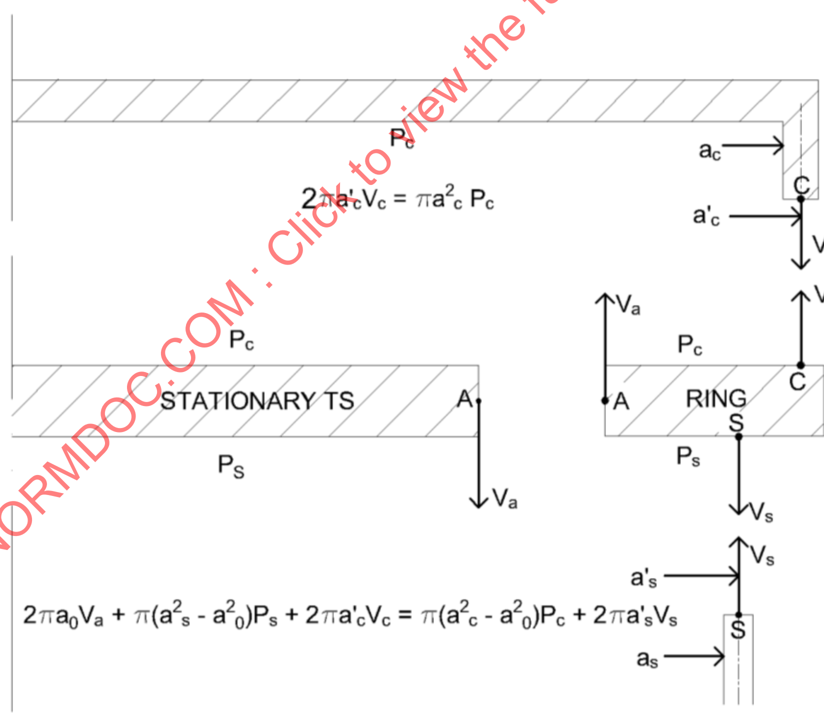


Figure 46 — Ring Equilibrium of the ST TS

(b) **The ring equilibrium for the FL TS is written** in the same way (see Figure 47), but there is no shell connected to the TS (V_s=0): $2\pi a_o V_a + \pi\left(a_s^2 - a_o^2\right)P_s + 2\pi a_c' V_c = \pi\left(a_c^2 - a_o^2\right)P_c$ [VI.2.1.2]

where:

V_a = axial edge load acting at connection of ring with equivalent plate is still to be determined

V_c = axial force acting in the floating channel, which depends on the HE type (immersed, externally sealed, internally sealed).

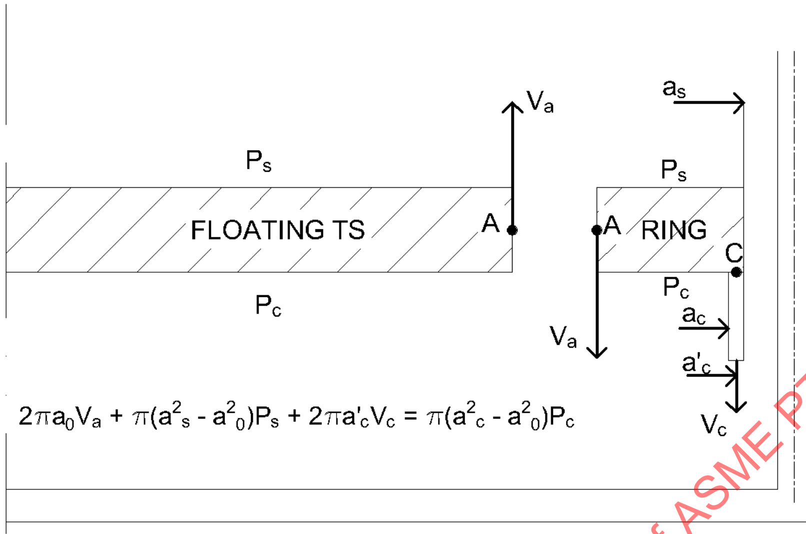


Figure 47 — Ring Equilibrium of the FL TS

(c) **Accordingly, the free body diagram of the FL TS is different from the free body diagram of the ST TS** and depends on the HE type shown on Figure 36: immersed, externally sealed, internally sealed, which must be considered separately to calculate the axial edge load V_a (point A) and the axial force V_E acting at the periphery of the ring (point E).

(1) Immersed FL HE (see Figure 48)

For ST TS: $\left.\begin{aligned} 2\pi a_s' V_s &= \pi a_s^2 P_s \\ 2\pi a_c' V_c &= \pi a_c^2 P_c \end{aligned}\right\}$ $\boxed{V_E = 2\pi a_c' V_c - 2\pi a_s' V_s = \pi a_c^2 P_c - \pi a_s^2 P_s}$

which, combined with [VI.2.1.1], leads to:

$$2\pi a_o V_a = \pi a_o^2 (P_s - P_c) \Rightarrow \boxed{V_a = \frac{a_o}{2}(P_s - P_c)}$$

For FL TS: $2\pi a_c' V_c = \pi a_c^2 P_c - \pi a_s^2 P_s$ $\boxed{V_E = 2\pi a_c' V_c = \pi a_c^2 P_c - \pi a_s^2 P_s}$

which, combined with [VI.2.1.2], leads to:

$$2\pi a_o V_a = \pi a_o^2 (P_s - P_c) \Rightarrow \boxed{V_a = \frac{a_o}{2}(P_s - P_c)}$$

(2) **externally sealed FL HE** (see Figure 49)

For ST TS: $\left.\begin{aligned} 2\pi a_s' V_s &= 0 \\ 2\pi a_c' V_c &= \pi a_c^2 P_c \end{aligned}\right\}$ $\boxed{V_E = 2\pi a_c' V_c - 2\pi a_s' V_s = \pi a_c^2 P_c}$

which, combined with [VI.2.1.1], leads to:

$$2\pi a_o V_a = \pi\left(a_o^2 - a_s^2\right)P_s - \pi a_o^2 P_c = \pi a_o^2\left[\left(1-\rho_s^2\right)P_s - P_c\right] \Rightarrow \boxed{V_a = \frac{a_o}{2}\left[\left(1-\rho_s^2\right)P_s - P_c\right]}$$

For FL TS: $2\pi a_c' V_c = \pi a_c^2 P_c$ $\boxed{V_E = 2\pi a_c' V_c = \pi a_c^2 P_c}$

which, combined with [VI.2.1.2], leads to:

$$2\pi a_o V_a = \pi\left(a_o^2 - a_s^2\right)P_s - \pi a_o^2 P_c = \pi a_o^2\left[\left(1-\rho_s^2\right)P_s - P_c\right] \Rightarrow \boxed{V_a = \frac{a_o}{2}\left[\left(1-\rho_s^2\right)P_s - P_c\right]}$$

(3) **internally sealed FL HE** (see Figure 50)

For ST TS: $\left.\begin{array}{l} 2\pi a_s' V_s = \pi a_s^2 P_c \\ 2\pi a_c' V_c = \pi a_c^2 P_c \end{array}\right\} \boxed{V_E = 2\pi a_c' V_c - 2\pi a_s' V_s = 0}$ (*assuming* $a_c' = a_s'$)

which, combined with [VI.2.1.1], leads to:

$$2\pi a_o V_a = \pi\left(a_o^2 - a_s^2\right)P_s - \pi\left(a_o^2 - a_s^2\right)P_c = \pi a_o^2\left(1-\rho_s^2\right)\left(P_s - P_c\right) \Rightarrow \boxed{V_a = \frac{a_o}{2}\left(1-\rho_s^2\right)\left(P_s - P_c\right)}$$

For FL TS: $V_c = 0 \quad \Rightarrow \quad \boxed{V_E = 2\pi a_c' V_c = 0}$

which, combined with [VI.2.1.2], leads to:

$$2\pi a_o V_a = \pi\left(a_o^2 - a_s^2\right)P_s - \pi\left(a_o^2 - a_s^2\right)P_c = \pi a_o^2\left[\left(1-\rho_s^2\right)\left(P_s - P_c\right)\right] \Rightarrow \boxed{V_a = \frac{a_o}{2}\left(1-\rho_s^2\right)\left(P_s - P_c\right)}$$

Thus, the axial edge load V_a (point A) and the axial force V_E acting at the periphery of the ring (point E) are the same for the ST TS and the FL TS, which enables the FL HE to be considered as a fixed TS HE for the analytical treatment.

However, the FL TS is different from the ST TS as its internal radius is smaller, it is not connected to the shell and the edge moments are different, except for the externally sealed HE which can be considered as a fixed TS HE with an expansion bellows of rigidity (see Note 2 of Section 7.2 of PART 3).

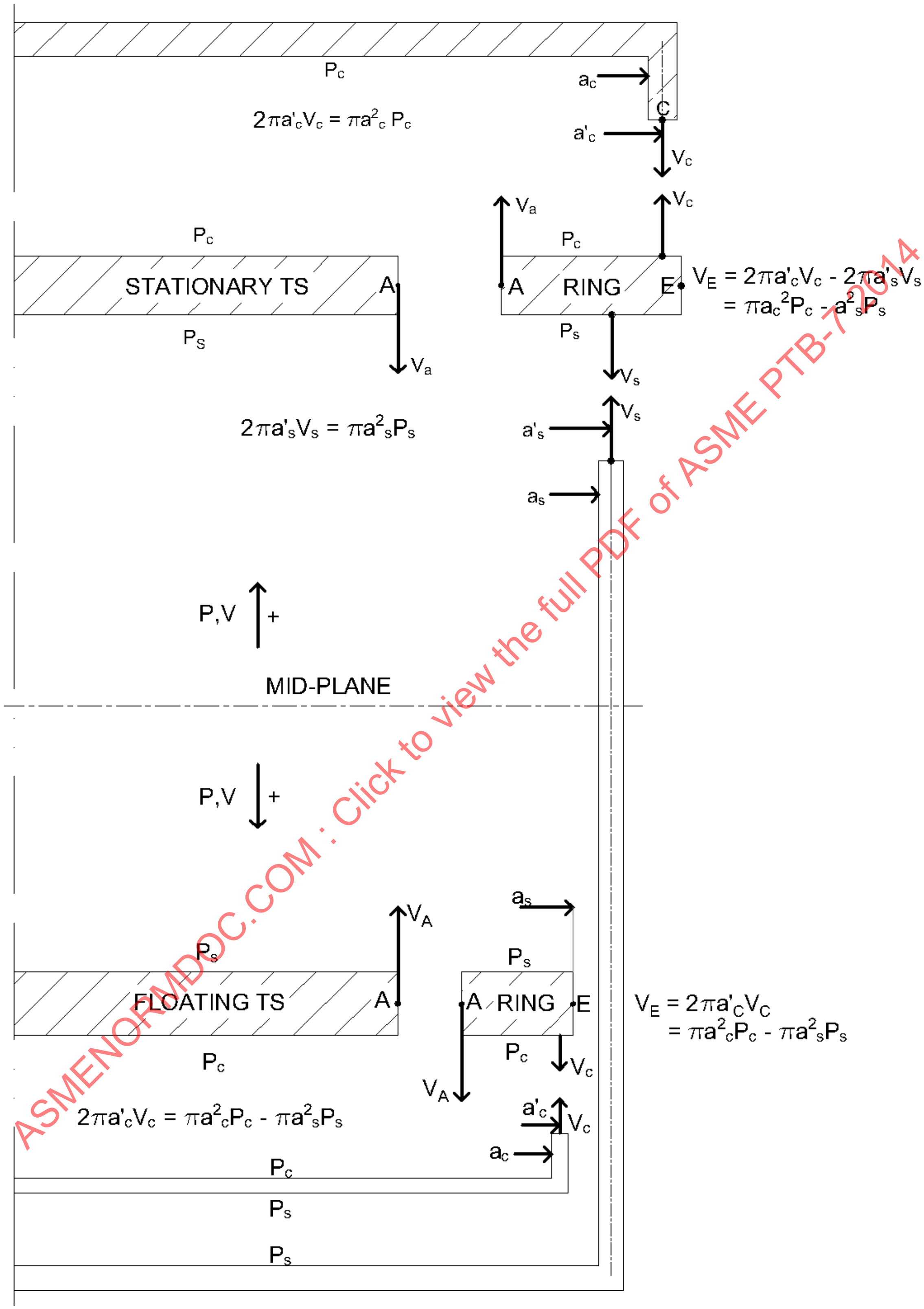


Figure 48 — Immersed Floating TS HE

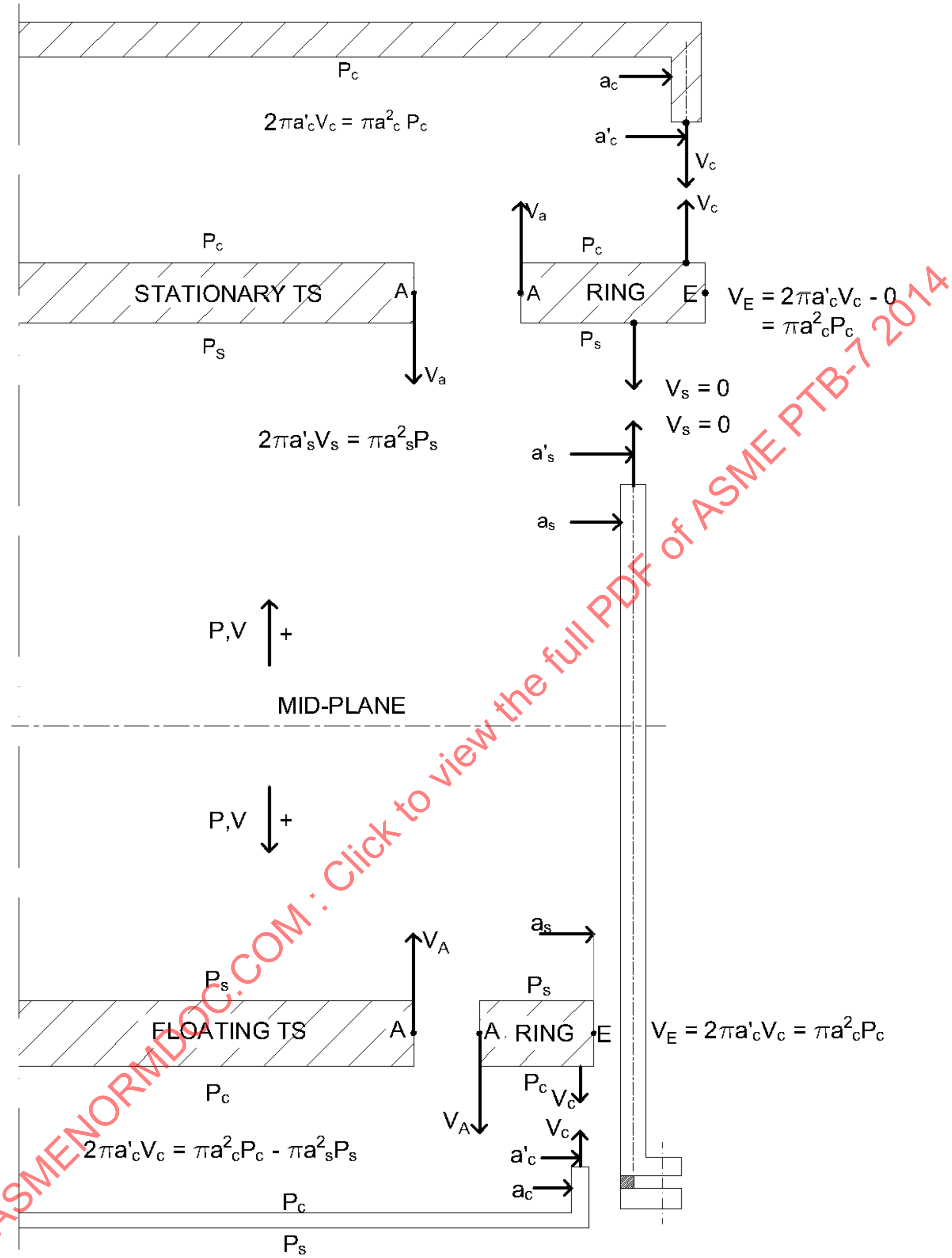


Figure 49 — Externally Sealed Floating TS HE

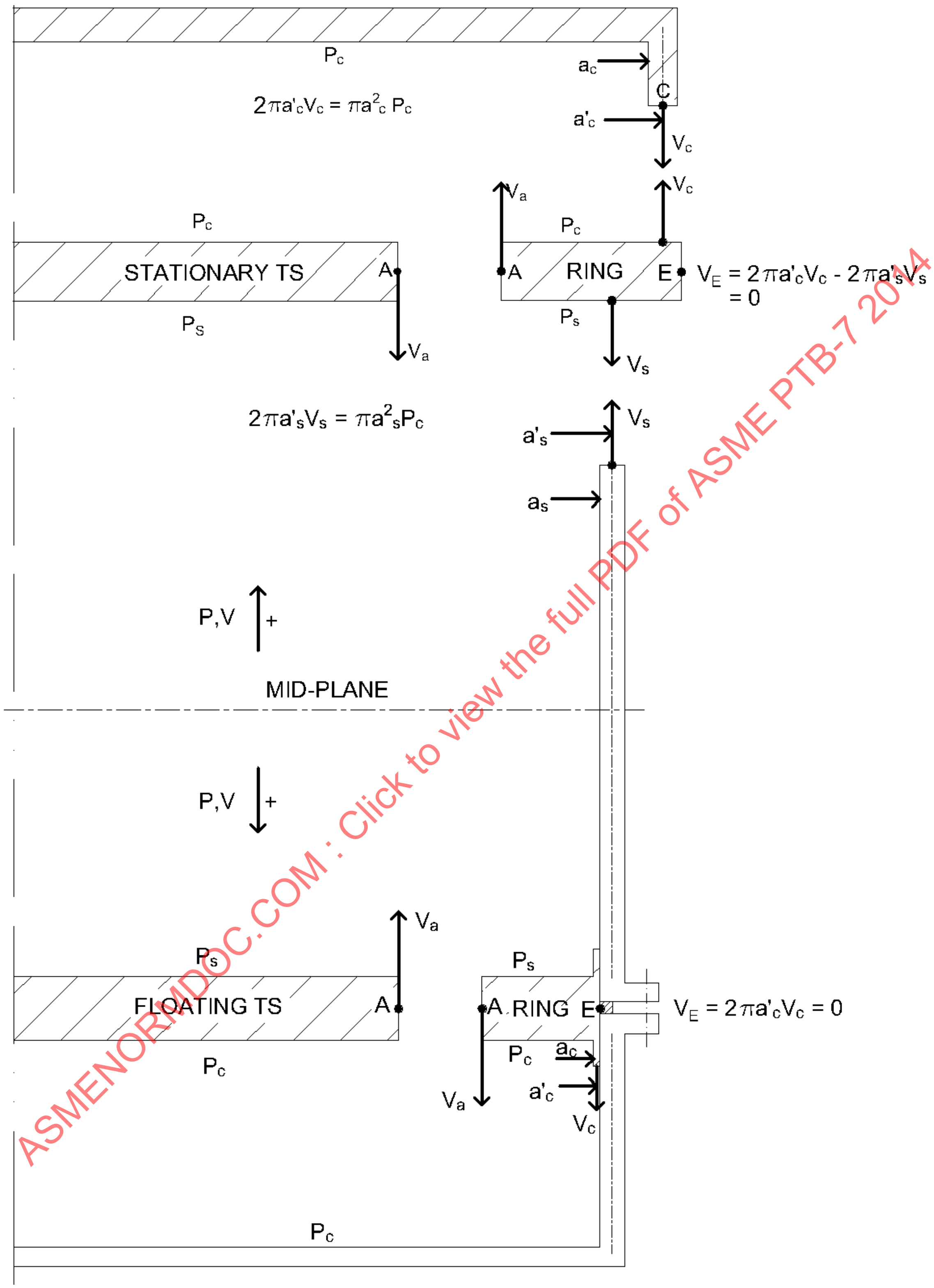


Figure 50 — Internally Sealed Floating TS HE

Note: The determination of the axial load V_s acting in the shell enables the calculation of the shell membrane stress:

$$\sigma_{s,m} = \frac{V_s}{t_s}$$

- *For immersed Floating TS HE:* $V_s = \frac{a_s^2 P_s}{2a_s^{'}}$ *leads to* $: \sigma_{s,m} = \frac{a_s^2}{(D_s + t_s) t_s} P_s$
- *For internally sealed Floating TS HE:* $V_s = 0$ *leads to* $: \sigma_{s,m} = 0$
- *For externally sealed Floating TS HE:* $V_s = \frac{a_s^2 P_c}{2a_s^{'}}$ *leads to* $: \sigma_{s,m} = \frac{a_s^2}{(D_s + t_s) t_s} P_c$

These formulas match the general shell membrane stress formula given in Section 8.6(a) of PART 3, using for P_e the relevant formula given in VII hereafter.

6.2.2 Due to Applied Moments

ST TS: Section 6.2(b) of PART 3 applies.
FL TS: Section 6.2(b) of PART 3 applies using β_s=0, k_s=0, λ_s=0 and δs=0, which leads to:

ω_s=0 and $\omega_s^* = a_o^2 \left[\frac{(\rho_s^2 - 1)(\rho_s - 1)}{4} \right]$

6.2.3 Edge Loads V_a and M_a Applied to the Tubesheet

(a) **Determination of M_a:** Section 6.3(a) of PART 3 applies, which leads to: $M_a = (a_o V_a) Q_1 + Q_2$

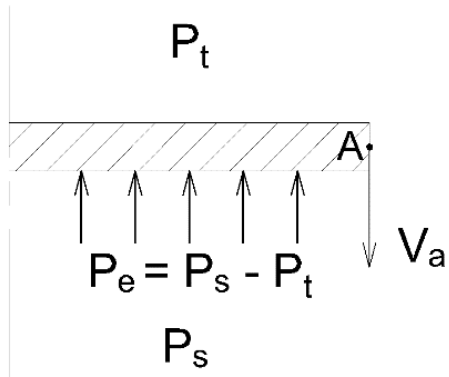
(b) **Determination of V_a:** V_a is known and depends on each HE type, as determined in VI.2.1.3. Accordingly, Section 6.3(b) and 6.3(c) of PART 3 are not relevant for FL TSs and quantities Q_{Z1}, Q_{Z2} and U do not apply.

7 EQUIVALENT PRESSURE ACTING ON THE TUBESHEET

A circular plate under uniform pressures P_s and P_t is subjected to a differential pressure $P_e = P_s - P_t$

The axial force V_a at periphery is determined from the plate equilibrium:

$$2\pi\, a_o\, V_a = P_e\, \pi\, a_o^2 \quad \Rightarrow \quad a_o\, V_a = \frac{a_o^2\, P_e}{2} \qquad P_e = \frac{2\, V_a}{a_o}$$



Where V_a is given in Section 6.2.1(c) for:

(a) **immersed FL HE:** $V_a = \frac{a_o}{2}(P_s - P_c) \Rightarrow \boxed{P_e = P_s - P_t}$

(b) **externally sealed FL HE:** $V_a = \frac{a_o}{2}\left[\left(1-\rho_s^2\right)P_s - P_c\right] \Rightarrow \boxed{P_e = \left(1-\rho_s^2\right)P_s - P_t}$

(c) **internally sealed FL HE:** $V_a = \frac{a_o}{2}\left[\left(1-\rho_s^2\right)\left(P_s - P_c\right)\right] \Rightarrow \boxed{P_e = \left(1-\rho_s^2\right)\left(P_s - P_t\right)}$

As shown in Section 6.2.1(c), these formulas apply both for the ST and FL TSs.

8 STRESSES IN THE HEAT-EXCHANGER COMPONENTS

Stress formulas calculated in Section 8 of PART 3 apply.

9 DETERMINATION OF ALLOWABLE STRESS LIMITS

The determination of the allowable stress limits developed in Section 9 of PART 3 apply.

10 ADDITIONAL RULES

The following additional rules apply in the same way as for the fixed TS HE covered in PART 3:

(1) Effect of radial thermal expansion adjacent to the tubesheet (UHX-14.6), covered in Section 10.3 of PART 3.

(2) Calculation procedure for simply supported TSs (UHX-14.7), covered in Section 10.4 of PART 3

(3) Effect of plasticity at tubesheet-shell-channel joint (UHX-14.8), covered in Section 10.2 of PART 3.

(4) Tubesheet flange extension (UHX-9), covered in Section 10.5 of PART 3.

11 HOW TO USE THE RULES

UHX-14 rules have been developed on the basis that the floating TS HE could be treated as a fixed TS, with some modifications due to the floating TS which is free to move axially inside the shell as outlined previously.

11.1 Stationary TS

The ST TS is designed in the same way as the fixed TSs rules of UHX-13, with the following modifications:

- the floating TS is free to move axially, which implies K_J=0 and J=0,
- there is no axial differential thermal expansion between tubes and shell, which implies $T_{t,m}$=0, $T_{s,m}$=0 and γ=0,
- the equivalent pressure depends on the HE type and is defined in Section 7,
- Coefficients Q_{Z1}, Q_{Z2} and U do not apply.

11.2 Floating TS

The FL TS needs some adaptations as it differs from the ST TS:

- no attached shell,
- channel diameters (A, C, D_s, G_s,G_1) are smaller than the shell diameters,
- channel thickness t_c may differ from the ST channel thickness
- TS configuration is different from the ST TS configuration.

The FL TS shall be designed in the same way as the ST TS, using:

- its own geometrical data (A, C, D_s, G_s,G_1, t_c),
- its own TS configuration,
- a_s=a_c which is needed for the calculations of Q_1, ω_s* and ω_c*.

So as to maintain a minimum of symmetry between the two TS, the FL TS shall have the same material and the same design temperature as the ST TS , which implies that the FL TS material properties are those of the ST TS (E_c, ν_c, S_c, $S_{y,c}$), and the same thickness as the ST TS.

11.3 Calculation Procedure

Like for the fixed TS HE, the calculation procedure can be summarized as follows:

- Set the data listed in Section 3.2(a)
- Calculate the design coefficients listed in Section 3.2(b) (x_s,x_t ; ρ_s,ρ_c)
- Calculate first characteristic parameter X_a and coefficients Q_1,Q_2 ; ω_s*,ω_c*
- Calculate the equivalent pressure P_e, and second characteristic parameter Q_3
- Calculate the maximum stresses in TS, tubes, shell and channel and limit their values to the maximum allowable stress limits.

Because of the complexity of the procedure, it is likely that users will computerize the solution.
A Mathcad calculation sheet is provided for the immersed floating head TS HE defined in PTB-4 Example E4.18.8.

The stationary TS is gasketed with shell and channel (configuration d) and the floating TS is gasketed with the channel, not extended as a flange (configuration C). The data are shown in the sheet and the calculations follow strictly steps 1 to 10 of UHX-14.5 calculation procedure. The calculation sheet is provided both for the ST and the FL TS. See Annex W and Annex X for UHX-14-Example E4.18.8 (PTB-4 2013 Edition) Stationary and Floating TS respectively.

11.4 Calculation Using a Fixed TS HE Software

The ST TS can be calculated using a fixed TS HE software, such as the Mathcad software used in Annex V, provided that the other TS is free to move axially by using a bellows of rigidity close to 0, which simulates an externally sealed FL HE. As shown by Section 7.2-Note 2 of PART 3, $P_e = \left(1-\rho_s^2\right)P_s - P_t$, which is effectively the equivalent pressure formula for this type of HE. For immersed floating head HE and internally sealed HE, it is necessary to replace P_e respectively by $P_e = P_s - P_t$ and $P_e = \left(1-\rho_s^2\right)\left(P_s - P_t\right)$ in the software.

REFERENCES—PART 4

[1] GARDNER "Heat Exchanger Tubesheet Design-2", ASME Journal of Applied Mechanics, Vol. 19,Trans. ASME Vol. 74, 1948.

[2] K.A.G. MILLER "The design of TS HEs", Proceedings of the Institution of Mechanical Engineers", London, Vol. 18, 1952.

[3] GALLETLY "Optimum design of Thin Circular Plates on an Elastic Foundation", Proceedings of the Institution of Mechanical Engineers", London Vol. 173, 1952.

[4] GARDNER "Tubesheet design: a basis for standardization." ASME publication of 1st ICPVT conference–Delft. Part 1 Design and Analysis-1969.

[5] SOLER "Mechanical Design of Heat Exchangers" – Arcturus publishers -1984 – 1047 pages.

PART 5
ANALYTICAL TREATMENT OF U-TUBE TUBESHEET HEAT EXCHANGERS

1 SCOPE

PART 5, devoted to U-tube TS HEs (Figure 51), provides the technical basis for the determination of the stresses acting on the TS, shell and channel and their relationships with the design rules of UHX-12.

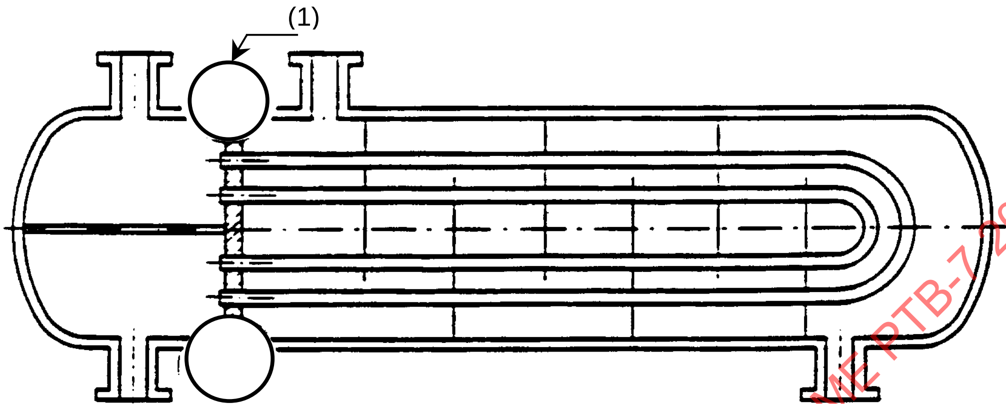


Figure 51 — U-tube Tubesheet Heat Exchangers

(1) see TS configurations in Figure 52

2 HISTORICAL BACKGROUND

The first rules for the design of U-tube TSs were published in Appendix AA of Section VIII, Division 1 in 1982. They were based on the 1969 Gardner method [1] to cover configurations a (integral construction) and b (gasketed construction), with some improvements:

- use of an effective pitch p* to account for unperforated diametral lanes,
- use of an effective tube diameter d* to account for the tube expansion depth ratio ρ,
- derivation of a direct formula to determine the tubesheet thickness.

Later on, Urey Miller [2] , as a member of the ASME Special Working Group on Heat Transfer Equipment, developed a more refined analytical approach accounting for the unperforated rim for configurations b and e (tubesheet extended as a flange), which were not covered before. The method was adopted in ASME in 1992.

This new set of rules was not totally satisfactory for the following reasons:

- Three different set of rules, based on different analytical approaches, were proposed to cover configurations a, b, d and e.
- Configurations c and f (gasketed tubesheet not extended as a flange) were not covered.
- Rule for configuration "d" covered only the case were the tubesheet was not extended as a flange, with gaskets both sides of same diameter,
- Rule for configuration "a" used the same formula, corrected by a TEMA coefficient F which did not account properly for the degree of restraint of the tubesheet by the shell and channel.

Accordingly, Osweiller in 2002 [3] developed a more refined and unique approach to cover the six tubesheet configurations. This approach is based on Urey Miller's method mentioned above, with the following improvements:

- treatment of configurations c and f where the tubesheet is not extended as a flange.
- accounting for local pressures acting on shell and channel, when integral with the tubesheet.
- use of Poisson's ratio ν in all formulas, rather than using ν=0.3, which leads to inexact coefficients.
- derivation of more condensed formulas providing rules consistent with fixed tubesheet rules.

This method was published for the first time in Nonmandatory Appendix AA of Section VIII Division 1. In 2003 it was published in a new part UHX of Section VIII Division 1 “Rules for Shell and Tubes Heat Exchangers” which became mandatory in 2004.

Soler [4] in 1984 developed a similar method accounting for the unperforated rim and the TS-shell-channel connection.

3 GENERAL

3.1 TS Configurations (UHX-12.1)

(a) **The stationary TS** is attached to the shell and the channel by welding (integral TS) or by bolting (gasketed TS) in accordance with the following 6 configurations (see Figure 3752):

- Configuration a: tubesheet integral with shell and channel;
- Configuration b: tubesheet integral with shell and gasketed with channel, extended as a flange;
- Configuration c: tubesheet integral with shell and gasketed with channel, not extended as a flange;
- Configuration d: tubesheet gasketed with shell and channel extends as a flange or not;
- Configuration e: tubesheet gasketed with shell and integral with channel, extended as a flange;
- Configuration f: tubesheet gasketed with shell and integral with channel, not extended as a flange.

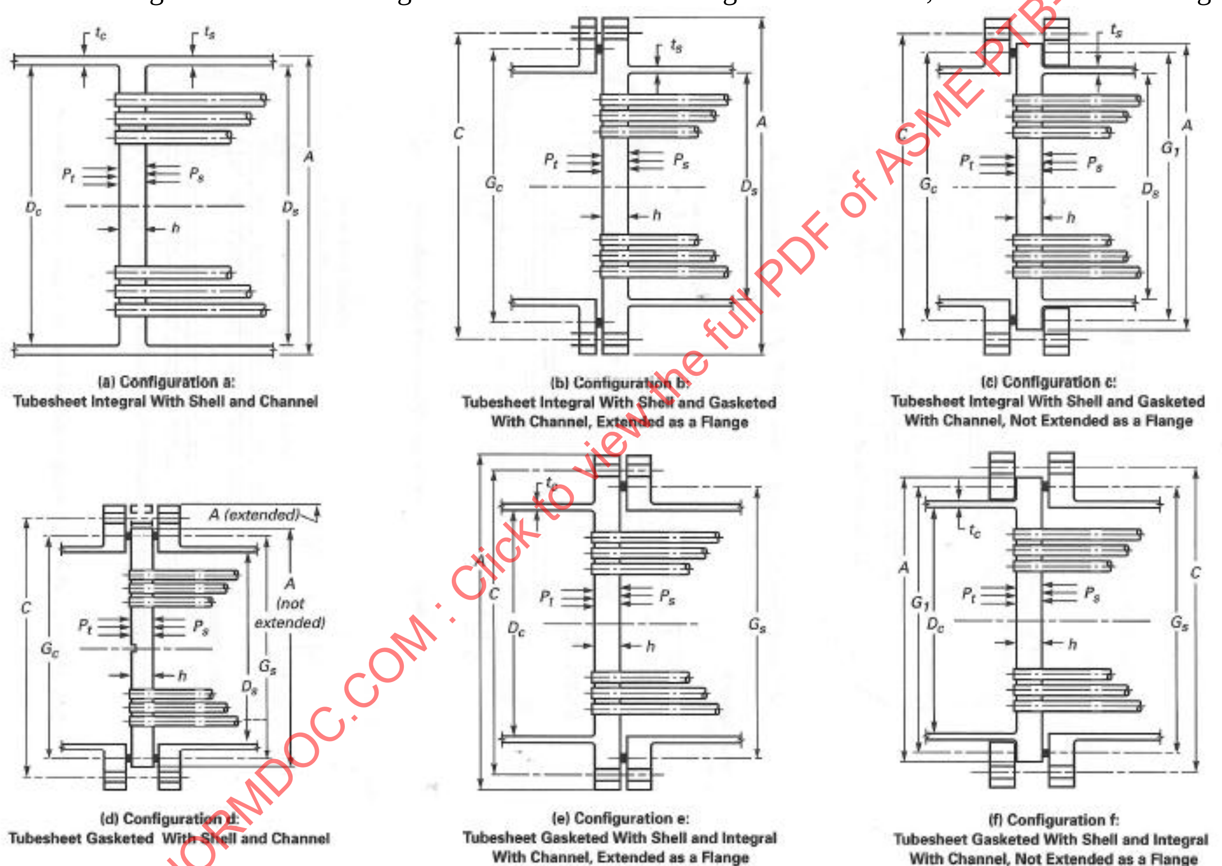


Figure 52 — TS Configurations

3.2 Notations

(a) **Data for the design of the HE are as follows** (UHX-12.3)

Symbols D_o, E^*, h_g', μ, μ^* and ν^* are defined in Section 2 of PART 2.

A = outside diameter of tubesheet

a_c = radial channel dimension

a_o = equivalent radius of outer tube limit circle

a_s = radial shell dimension

C = bolt circle diameter

D_c = inside channel diameter

D_s = Inside shell diameter
E = modulus of elasticity for tubesheet material at T
E_c = modulus of elasticity for channel material at T_c
E_s = modulus of elasticity for shell material at T_s
G_c = diameter of channel gasket load reaction
G_s = diameter of shell gasket load reaction
G_1 = midpoint of contact between flange and tubesheet
h = tubesheet thickness
N_t = number of tubes
P_s = shell side design pressure. For shell side vacuum use a negative value for P_s.
P_t = tube side design pressure. For tube side vacuum use a negative value for P_t.
Notation P_c, instead of P_t, is used throughout the analytical development so as to maintain the symmetry of the equations involving the shell (subscript s) and the channel (subscript c).
S = allowable stress for tubesheet material at T
S_c = allowable stress for channel material at T_c
S_s = allowable stress for shell material at T_s
$S_{y,c}$ = yield strength for channel material at T_c
$S_{y,s}$ = yield strength for shell material at T_s
$S_{y,t}$ = yield strength for tube material at T_t
S_{PS} = allowable primary plus secondary stress for tubesheet material at T
$S_{PS,c}$ = allowable primary plus secondary stress for channel material at T_c
$S_{PS,s}$ = allowable primary plus secondary stress for shell material at T_s
T = tubesheet design temperature
T_c = channel design temperature
T_c = channel design temperature
T_s = shell design temperature
t_c = channel thickness
t_s = shell thickness
t_t = nominal tube wall thickness
W_s, W_c = shell or channel flange design bolt load for the gasket seating condition
W^* = tubesheet effective bolt load determined in accordance with UHX-8
ν_c = Poisson's ratio of channel material
ν_s = Poisson's ratio of shell material

(b) **Design coefficients** (UHX-12.5.1 to 5)
The following coefficients, specific to each component of the HE, will be used in the analytical treatment. They complete the data given above.

(1) **Perforated TS**

- Equivalent diameter of outer tube limit circle (see Section 4.3(a) of PART 2): $D_o = 2\,r_o + d_t$
- Equivalent radius of outer tube limit circle: $a_o = \dfrac{D_o}{2}$

TS coefficients:

- Ligament efficiency: $\mu^* = \dfrac{p^* - d^*}{p^*}$
- Effective tube hole diameter d* and effective pitch p* are defined in Section 4.3(c) and (d) of PART 2
- Effective elastic constants E* and ν* are given in Section 5.6 of PART 2 as a function of μ^* and h/p (triangular or square pitch).

- Bending stiffness: $D^* = \dfrac{E^* \cdot h^3}{12\left(1 - \nu^{*2}\right)}$
- Effective tube side pass partition groove depth given in Section 4.3(f) of PART 2: h'_g
- Effective pressure acting on tubesheet: P_e

(2) **Shell**

- Radial shell dimension: a_s $\rho_s = \dfrac{a_s}{a_o}$
- Integral configurations (a, b and c): $a_s = D_s / 2$
- Gasketed configuration (d, e and f): $a_s = G_s / 2$
- Mean shell radius: $a'_s = \dfrac{D_s + t_s}{2}$
- Shell coefficient: $\beta_s = \dfrac{\sqrt[4]{12\left(1 - \nu_s^2\right)}}{\sqrt{\left(D_s + t_s\right) t_s}}$
- Bending stiffness: $k_s = \beta_s \dfrac{E_s \cdot t_s^3}{6\left(1 - \nu_s^2\right)}$

(3) **Channel**

- Radial channel dimension: a_c $\rho_c = \dfrac{a_c}{a_o}$
- Integral configuration (a): $a_c = D_c / 2$
- Gasketed configurations (b, c and d): $a_c = G_c / 2$
- Mean channel radius: $a'_c = \dfrac{D_c + t_c}{2}$
- Channel coefficient: $\beta_c = \dfrac{\sqrt[4]{12\left(1 - \nu_c^2\right)}}{\sqrt{\left(D_c + t_c\right) t_c}}$
- Bending stiffness: $k_c = \beta_c \cdot \dfrac{E_c \cdot t_c^3}{6\left(1 - \nu_c^2\right)}$

(4) **Unperforated rim**

- D_o = internal diameter
- A = external diameter
- Diameter ratio: $K = A / D_o$

3.3 Loading Cases (UHX-12.4)

The normal operating condition of the HE is achieved when the tube side pressure P_t and shell side pressure P_s act simultaneously. However, a loss of pressure is always possible. Accordingly, for safety reasons, the designer must always consider the cases where P_s=0 and P_t=0 for the normal operating condition(s).

The designer must also consider the start-up condition(s), the shut-down condition(s) and the upset condition(s), if any, which may govern the design.

For each of these conditions, ASME, TEMA and CODAP used to consider the following pressure loading cases.

- Loading Case 1: Tube side pressure P_t acting only (P_s = 0).
- Loading Case 2: Shell side pressure P_s acting only (P_t = 0).
- Loading Case 3: Tube side pressure P_t and shell side pressure P_s acting simultaneously.

ASME 2013 Edition provides the detail of the pressure "design loading cases" to be considered for each operating condition specified by the user (normal operating conditions, startup conditions, the shutdown conditions,…).A table (table UHX-12.4-1) provides the values to be used for the design pressures P_s and P_t in the formulas, accounting for their maximum and minimum values.

As the calculation procedure is iterative, a value *h* is assumed for the tubesheet thickness to calculate and check that the maximum stresses in tubesheet, shell and channel are within the maximum permissible stress limits.

3.4 Design Assumptions (UHX-12.2)

A U-tube TS HE is a complex structure and several assumptions are necessary to derive a 'design by rules' method.

Most of them could be eliminated, but the analytical treatment would lead to 'design by analysis' method requiring the use of a computer.

The design assumptions are as follows.

(a) HE

- The analytical treatment is based on the theory of elasticity applied to the thin shells of revolution.
- The HE is axi-symmetrical.

(b) TS

- The tubesheet is circular.
- The tubesheet is uniformly perforated over a nominally circular area, in either equilateral triangular or square patterns. This implies that the TS is fully tubed (no large untubed window)
- Radial displacement at the mid-surface of the TS is ignored
- Temperature gradient through TS thickness is ignored
- Shear deformation and transverse normal strain in the TS are ignored
- The unperforated rim of the TS is treated as a rigid ring without distortion of the cross section

(c) Shell and channel

- Shell and channel are cylindrical with uniform diameters and thicknesses
- Shell and channel centerlines are the same

(d) Weights and pressures drops

- Weights and pressures drops are ignored
- Pressures P_s and P_t are assumed uniform

3.5 Basis of Analytical Treatment

3.5.1 General

Comparison of Figure 3651 and Figure 5136 of PART 4 shows that the U-tube TS HE can be considered as an immersed floating head TS HE where the floating TS does not exist and the tubes do not play the role of an elastic foundation.

Accordingly, the U-tube HE could be designed as an immersed floating head TS HE where the tubes have no axial rigidity (see Annex U). However the analytical treatment presented below is based on the approach developed by Urey Miller [2] as explained in Section 2.

Although the design is less complex, many geometrical, mechanical and material properties are involved in the design as shown in Section 3.2(a) which lists the extensive input data.

As for a floating TS HE, the analysis includes the effects of the shell and tube side pressures, the stiffening effect of the unperforated ring at the tubesheet edge and the stiffening effect of the integrally attached channel or shell to the tubesheet. When the tubesheet is gasketed with the shell or the channel, the bolt load causes an additional moment in the tubesheet which is included in the total stress in the tubesheet in addition to the moments caused by pressure.

The analysis is based on classical discontinuity analysis methods to determine the moments and forces that the tubesheet, shell and channel must resist. These components are treated using the elastic theory of thin shells of revolution.

The main steps of the U-tube TS design follow the analytical treatment of floating TS HEs.

(a) Tubesheet is disconnected from the shell and channel. Shear load V_a and moment M_a are applied at the tubesheet edge as shown in Figure 53.

(b) Perforated tubesheet is replaced by an equivalent solid circular plate of diameter D_o and effective elastic constants E^* (effective modulus of elasticity) and ν^* (effective Poisson's ratio) depending on the ligament efficiency μ^* of the tubesheet. This equivalent solid plate is treated by the theory of thin circular plates subjected to pressures P_s and P_t and relevant applied loads to determine the maximum stresses.

(c) Unperforated tubesheet rim is treated as a rigid ring whose cross section does not change under loading.

(d) Connection of the tubesheet with shell and channel accounts for the edge displacements and rotations of the 3 components.

(e) Shell and channel are subject to shell side and tube side pressures P_s and P_t and edge loads V_a and M_a to determine the maximum stresses.

(f) Maximum stresses in tubesheet shell and channel are determined and limited to the appropriate allowable stress-based classifications of Section VIII Division 2 Part 4.

3.5.2 Free Body Diagram

Figure 53 shows, for a TS integral both sides (configuration a), the free body diagram of the component parts (perforated tubesheet, unperforated tubesheet rim, shell, channel). The figure details the relevant discontinuity forces (V_a, V_s, Q_s, V_c, Q_c) and moments (M_a, M_s, M_c, M_R) applied on each component, together with edge displacements.

In this figure:
Forces (V_a, V_s, Q_s, V_c, Q_c) and moments (M_a, M_s, M_c, M_R) are per unit of circumferential length

The following subscripts are used:

- s for shell,
- c for channel,
- R for unperforated rim

No subscript is used for the perforated TS

Notation P_c instead of P_t (tube side pressure) is used throughout the analytical development so as to maintain the symmetry of the equations involving the shell (subscript s) and the channel (subscript c).

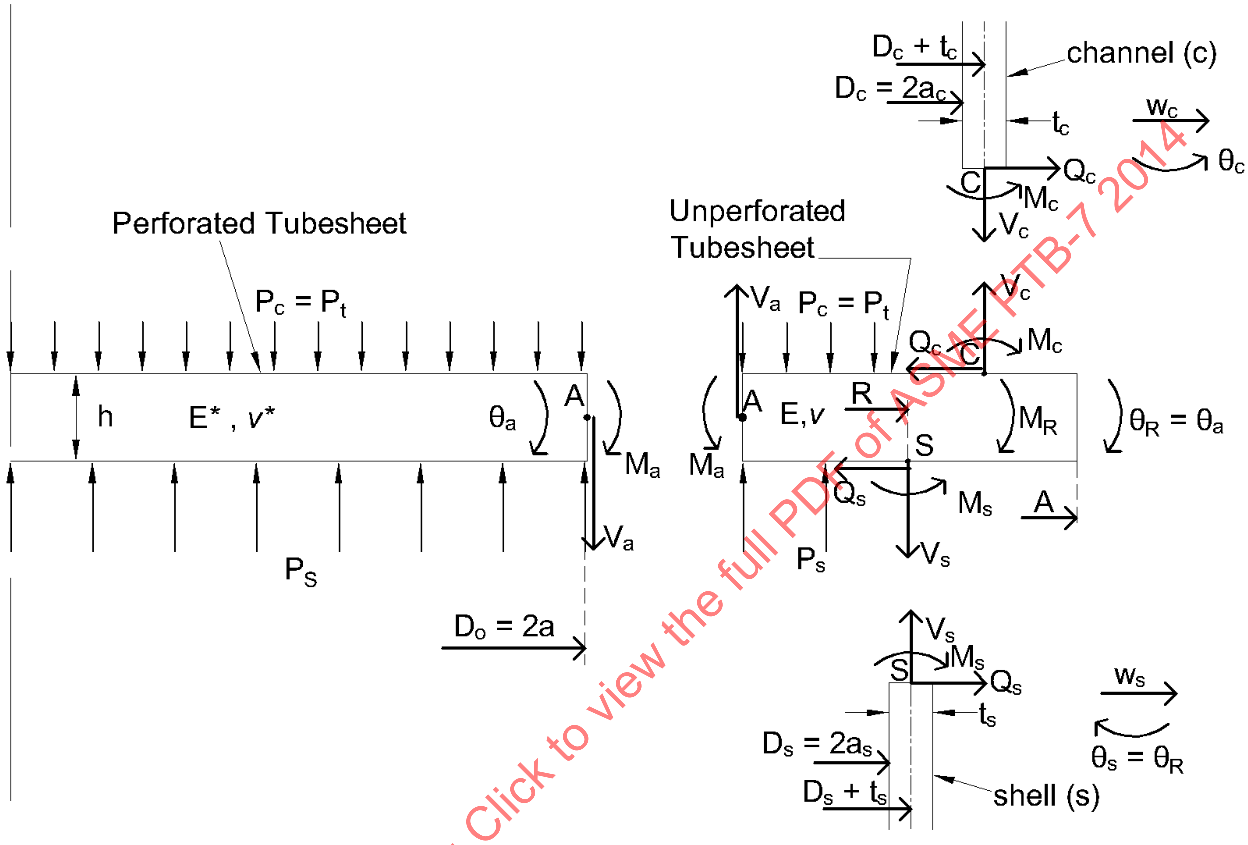


Figure 53 — Free Body Diagram of the Analytical Model for the TS

4 TREATMENT OF THE PERFORATED TUBESHEET

The perforated tubesheet is treated as an equivalent solid circular plate of radius a_o with effective elastic constants E^* and ν^*, subjected to an equivalent pressure P_e and edge loads V_a and M_a at periphery. The equilibrium equation of the plate is written: $-2\,\pi\, a_o\, V_a + \pi\, a_o^2 P_e = 0$

which leads to: $V_a = \frac{a_o}{2} P_e$ [IV-1]

The rotation of the plate at radius a_o is given by: $\theta_a = \frac{12\,(1-\nu^*)}{E^*\, h^3}\left[a_o\, M_a + \frac{a_o^3}{8}\, P_e\right]$ [IV-2]

The radial bending moment at radius *r* of such a plate is given by the classical formula:

$$M(r) = M_a + (3+\nu^*)\,\frac{a_0^2}{16}\left[1-\left(\frac{r}{a_o}\right)^2\right] P_e$$

The maximum bending moment appears

- either at center (r=0): $M(r) = M_a + (3+\nu^*)\,\frac{a_0^2}{16}\, P_e$
- or at periphery (r=a_o): $M(r) = M_a$

5 TREATMENT OF THE UNPERFORATED RIM

5.1 Edge Loads Applied on Shell and Channel at their Connection to the TS

The following equations are developed for integral shell and channel.

(a) **The edge loads Q_s and M_s applied on the shell write (see Section 6.1(a) of PART 3):**

$$\left\{ \boxed{\begin{aligned} M_s &= +k_s\left(1+\frac{t_s'}{2}\right)\theta_s + \beta_s\, k_s\, \delta_s\, P_s \\ Q_s &= -\beta_s\, k_s\left(1+t_s'\right)\theta_s - 2\beta_s^2\, k_s\, \delta_s\, P_s \end{aligned}} \right\}$$ [V.1a] for an integral shell (configurations a, b, c)

When the shell is not integral with the TS (configurations d, e, f), $k_s=0$ and $\delta s=0$ lead to: $M_s = 0$ and $Q_s = 0$.

Note: These formulas are valid for a shell of sufficient length. Annex J provides the minimum length above which these formulas can be applied.

(b) **The edge loads Q_c and M_c applied on the channel write (see Section 6.1(b) of PART 3):**

$$\left\{ \boxed{\begin{aligned} M_c &= +k_c\left(1+\frac{t_c'}{2}\right)\theta_c + \beta_c\, k_c\, \delta_c\, P_c \\ Q_c &= -\beta_c\, k_c\left(1+t_c'\right)\theta_c - 2\beta_c^2\, k_c\, \delta_c\, P_c \end{aligned}} \right\}$$ [V.1b] for an integral channel (configurations, a e, f)

When the channel is not integral with the TS (configurations b, c, d), $k_c = 0$ and $\delta_c = 0$ lead to: $M_c = 0$ and $Q_c = 0$

Note 1: These formulas are valid for a channel of sufficient length. Annex J provides the minimum length above which these formulas can be applied.

Note 2: These formulas are valid for a cylindrical channel. If the channel is hemispherical, it must be attached directly to the TS (configurations a, b or c), without any cylindrical section between the head and the TS. Annex K provides the relevant formulas for that case. Only coefficient δ_c is affected:

$$\boxed{\delta_c = \frac{D_c^2}{4\,E_c\,t_c}\left(\frac{1-\nu_c}{2}\right)}$$

5.2 Equilibrium of the Unperforated Solid Rim

(a) **due to axial loads** $\begin{cases} a_s = D_s / 2 \\ a_s' = a_s + \dfrac{t_s}{2} \end{cases}$ $\quad \begin{cases} a_c = D_c / 2 \\ a_c' = a_c + \dfrac{t_c}{2} \end{cases}$

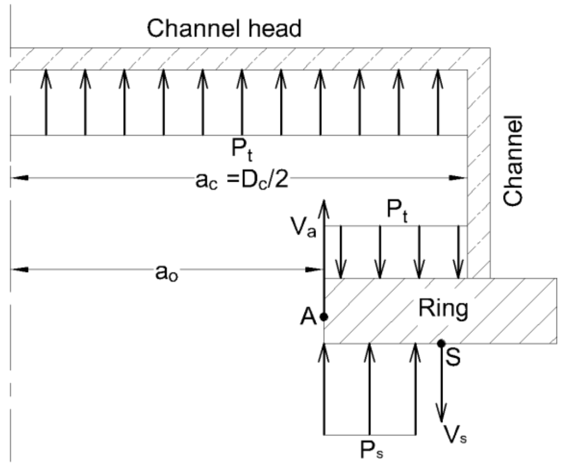


Figure 54 — Ring Equilibrium of the TS

The axial equilibrium of the ring is written: (see Figure 54):

$$2\pi a_s' V_s + \pi\left(a_c^2 - a_o^2\right) P_c = 2\pi a_c' V_c + 2\pi a_o V_a + \pi\left(a_s^2 - a_o^2\right) P_s$$

The axial equilibrium of the shell is written: $2\pi a_s' V_s = \pi a_s^2 P_s$

The axial equilibrium of the channel is written: $2\pi a_c' V_c = \pi a_c^2 P_c$

This leads to: $\boxed{V_a = \frac{a_o}{2}\left(P_s - P_c\right)}$

Comparison with equation [IV-1] shows that $\boxed{P_e = P_s - P_t}$

as for an immersed floating head HE.

(b) **due to applied moments**

Equilibrium of moments applied to the ring relative to the axis located at radius a_o enables to determine the moment M_R (see Figure 53).

R = radius at center of ring $= \dfrac{A + 2a_o}{4}$

$$RM_R = -\left[a_o M_a\right] + \left.\begin{matrix}\left[a_c' M_c - a_c' Q_c \dfrac{h}{2}\right] + \left[M\left(P_c\right) - a_c' V_c \left(a_c' - a_o\right)\right] \\ -\left[a_s' M_s - a_s' Q_s \dfrac{h}{2}\right] + \left[M\left(P_s\right) - a_s' V_s \left(a_s' - a_o\right)\right]\end{matrix}\right\}$$

$M(P_c) =$ moment due to pressure P_c acting on the ring $= \left(a_c^2 - a_o^2\right)\left(\dfrac{a_c + a_o}{2} - a_o\right)\dfrac{P_c}{2}$

$M(P_s) =$ moment due to pressure P_s acting on the ring $= \left(a_s^2 - a_o^2\right)\left(\dfrac{a_s + a_o}{2} - a_o\right)\dfrac{P_s}{2}$

From Annex L:

$$\left[a_c' M_c - a_c' Q_c \frac{h}{2}\right] = a_c' k_c \left(1 + t_c' + \frac{t_c'^2}{2}\right)\theta_c + a_o \omega_c P_c \qquad \boxed{\omega_c = \rho_c \beta_c k_c \delta_c \left(1 + h\beta_c\right)} \qquad t_c' = h\beta_c$$

$$\left[a_s' M_s - a_s' Q_s \frac{h}{2}\right] = a_s' k_s \left(1 + t_s' + \frac{t_s'^2}{2}\right)\theta_s + a_o \omega_s P_s \qquad \boxed{\omega_s = \rho_s \beta_s k_s \delta_s (1 + h\beta_s)} \qquad t_s' = h\beta_s$$

$$\left[M(P_c) - a_c' V_c (a_c' - a_o)\right] = \frac{P_c}{4}\left[(a_c^2 - a_o^2)(a_c - a_o) - 2a_c^2(a_c' - a_o)\right] = \frac{P_c}{4}(a_c - a_o)(-a_c^2 - a_o^2) = \frac{-P_c}{4} a_o^3\left[(\rho_c - 1)(\rho_c^2 + 1)\right]$$

$$\left[M(P_s) - a_s' V_s (a_s' - a_o)\right] = \frac{P_s}{4}\left[(a_s^2 - a_o^2)(a_s - a_o) - 2a_s^2(a_s' - a_o)\right] = \frac{P_s}{4}(a_s - a_o)(-a_s^2 - a_o^2) = \frac{P_s}{4} a_o^3\left[(\rho_s - 1)(\rho_s^2 + 1)\right]$$

$$\left.\begin{array}{l}[R\,M_R] = -a_o M_a + \dfrac{P_s}{4} a_o^3\left[(\rho_s - 1)(\rho_s^2 + 1)\right] - \dfrac{P_c}{4} a_o^3\left[(\rho_c - 1)(\rho_c^2 + 1)\right] \\ \qquad - \left[a_s' k_s \left(1 + t_s' + \dfrac{t_s'^2}{2}\right) + a_c' k_c \left(1 + t_c' + \dfrac{t_c'^2}{2}\right)\right]\theta_a + a_o(\omega_c P_c - \omega_s P_s)\end{array}\right\} \quad \text{[V.2b']}$$

(c) **Rotation of rigid ring**

$K = \dfrac{A}{D_o} \quad \theta_R = \dfrac{12}{E h^3}\dfrac{R\,M_R}{Ln\,K}$ leads to, with $\theta_R = \theta_a$:

$$R\,M_R = \left[\frac{E h^3}{12} Ln\,K\right]\theta_R = \left[\frac{E h^3}{12} Ln\,K\right]\theta_a$$

Replacing RM_R by its expression [V.2b'] permits to calculate the TS rotation θ_a:

$$\left.\begin{array}{l}\overbrace{\left[\dfrac{E h^3}{12} Ln\,K + a_s' k_s \left(1 + t_s' + \dfrac{t_s'^2}{2}\right) + a_c' k_c \left(1 + t_c' + \dfrac{t_c'^2}{2}\right)\right]}^{C1}\theta_a = -a_o M_a \\ + P_s \dfrac{a_o^3}{4}\left[(\rho_s - 1)(\rho_s^2 + 1)\right] - P_c \dfrac{a_o^3}{4}\left[(\rho_c - 1)(\rho_c^2 + 1)\right] + a_o\left[\omega_c P_c - \omega_s P_s\right]\end{array}\right\}$$

$$C_1 = \frac{h^3}{12}\left[\underbrace{\frac{6}{h^3}(D_s + t_s) k_s \left(1 + t_s' + \frac{t_s'^2}{2}\right)}_{\lambda_s} + \underbrace{\frac{6}{h^3}(D_c + t_c) k_c \left(1 + t_c' + \frac{t_c'^2}{2}\right)}_{\lambda_c} + E\,Ln\,K\right] = \frac{h^3}{12}\left[\lambda_s + \lambda_c + E\,Ln\,K\right]$$

$$\boxed{\lambda_s = \frac{6}{h^3}(D_s + t_s) k_s \left(1 + t_s' + \frac{t_s'^2}{2}\right)} \qquad \boxed{\lambda_c = \frac{6}{h^3}(D_c + t_c) k_c \left(1 + t_c' + \frac{t_c'^2}{2}\right)}$$

Note: if $\lambda_s + \lambda_c$ is high (>3) the TS can be considered as clamped

if $\lambda_s + \lambda_c$ is low (<1) the TS can be considered as simply supported

$$\frac{h^3}{12}\left[\lambda_s + \lambda_c + ELnK\right]\theta_a = -a_o M_a + P_s \frac{a_o^3}{4}\left[(\rho_s - 1)(\rho_s^2 + 1)\right] - P_c \frac{a_o^3}{4}\left[(\rho_c - 1)(\rho_c^2 + 1)\right] + a_o\left[\omega_c P_c - \omega_s P_s\right]$$

Where the TS rotation θ_a is given by [IV-2]: $\theta_a = \dfrac{12\,(1 - \nu^*)}{E^*\,h^3}\left[a_o M_a + \dfrac{a_o^3}{8} P_e\right]$ which permits to calculate M_a:

$$\frac{h^3}{12}[\lambda_s+\lambda_c+ELnK]\frac{12\ (1-\nu^*)}{E^*\ h^3}\left[a_o M_a+\frac{a_o^3}{8}\ P_e\right]+a_o M_a=P_s\frac{a_o^3}{4}\left[(\rho_s-1)(\rho_s^2+1)\right]-P_c\frac{a_o^3}{4}\left[(\rho_c-1)(\rho_c^2+1)\right]+a_o[\omega_c P_c-\omega_s P_s]$$

$$\underbrace{[\lambda_s+\lambda_c+ELnK]\frac{(1-\nu^*)}{E^*}}_{F}\left[M_a+\frac{a_o^2}{8}\ P_e\right]+M_a=P_s\frac{a_o^2}{4}\left[(\rho_s-1)(\rho_s^2+1)\right]-P_c\frac{a_o^2}{4}\left[(\rho_c-1)(\rho_c^2+1)\right]+[\omega_c P_c-\omega_s P_s]$$

$$[1+F]\ M_a=-F\frac{a_o^2}{8}\ P_e+\underbrace{P_s\frac{a_o^2}{4}\left[(\rho_s-1)(\rho_s^2+1)\right]-P_c\frac{a_o^2}{4}\left[(\rho_c-1)(\rho_c^2+1)\right]}_{M_{TS}}+[\omega_c P_c-\omega_s P_s]$$

In this equation:

M_{TS} is the the moment due to pressures P_s and P_t acting on the rigid ring:

$$\boxed{M_{TS}=P_s\frac{a_o^2}{4}\left[(\rho_s-1)(\rho_s^2+1)\right]-P_c\frac{a_o^2}{4}\left[(\rho_c-1)(\rho_c^2+1)\right]}$$

F denotes the degree of restrain of the TS by the shell and channel when they are integral with the TS (configuration a):

$$\boxed{F=[\lambda_s+\lambda_c+ELnK]\frac{(1-\nu^*)}{E^*}}$$

Note:

If there is no bending support from the channel $(k_c=0)$ *and the shell* $(k_s=0)$*:* $\lambda_s=1$ *and* $\lambda_c=1$ *lead to F close to zero:*

$M_a=M_{TS}$ and the tubesheet is almost simply supported. If additionally there is no unperforated rim $(K=1, \rho_s=0, \rho_c=0)$*:*

F = 0 and $M_a=0$: the tubesheet is fully simply supported.

If there is a high bending support from the channel $(k_c=\infty)$ *and the shell* $(k_s=\infty)$*:* $\lambda_c=\infty$ *and* $\lambda_s=\infty$ *lead to* $F=\infty$.

The tubesheet is fully clamped: $M_a=-\frac{D_o^2}{32}(P_s-P_c)$.

Finally, for configuration a, M_a is written: $$M_a=\frac{M_{TS}+(\omega_c P_c-\omega_s P_s)-F\left(\frac{a_o^2}{8}\ P_e\right)}{1+F}\qquad\text{[V.2c]}$$

(d) **Generic equation covering the 6 configurations a to f**

Equation [VI.2d] of PART 3 shows that a term $\frac{a_o}{2\pi}\left[W_c\ \gamma_{bc}-W_s\ \gamma_{bs}\right]$, which accounts for the flange bolt loads W_s and W_c, must be added in equation [V.2c] to obtain the generic equation for the 6 configurations.

Rules of UHX-12 cover the case where $C_s=C_c=C$. Accordingly, equation [V.2c] giving M_a becomes:

$$M_a=\frac{\overbrace{M_{TS}+(\omega_c P_c-\omega_s P_s)+\frac{\gamma_b}{2\pi}[W_c-W_s]}^{M^*}-F\left(\frac{a_o^2}{8}\ P_e\right)}{1+F}$$

$$M_a = \frac{\overbrace{M_{TS} + (\omega_c P_c - \omega_s P_s) + \frac{\gamma_b}{2\pi} W^*}^{M^*} - F\left(\frac{a_o^2}{8} \mathrm{P}_e\right)}{1+F}$$

$$\boxed{M^* = M_{TS} + (\omega_c P_c - \omega_s P_s) + \frac{\gamma_b}{2\pi}[W_c - W_s]}$$

- When the shell is integral with the TS (configurations a, b, c): W_s=0
- When the channel is integral with the TS (configurations a, e, f): W_c=0
- When the shell and the channel are gasketed with the TS, the highest bolting moment controls and W_c- W_s must be replaced by: W_{max}=MAX (W_s,W_c) and W* = W_{max}

UHX-12 covers these configurations through the TS effective bolt load W* which is explicated in UHX-12.5.6 for each configuration a to f.

$$\boxed{M^* = M_{TS} + (\omega_c P_c - \omega_s P_s) + \frac{\gamma_b}{2\pi} W^*}$$

$$\boxed{M_{TS} = P_s \frac{a_o^2}{4}\left[(\rho_s - 1)(\rho_s^2 + 1)\right] - P_c \frac{a_o^2}{4}\left[(\rho_c - 1)(\rho_c^2 + 1)\right]}$$

$$\boxed{F = [\lambda_s + \lambda_c + ELnK]\frac{(1-\nu^*)}{E^*}}$$

$$\boxed{M_a = \frac{M^* - F\left(\frac{a_o^2}{8} \mathrm{P}_e\right)}{1+F}}$$

The 1st term accounts for the moments due to pressures P_s and P_c and bolting loads W_s and W_c acting on the unperforated rim. The second term accounts for the moment due to pressures P_s and P_c acting on the perforated TS.

This formula applies as follows for the 6 configurations, depending on coefficients λ_s,λ_c, ω_s,ω_c , γ_b.

- For configuration a: λ_s , λ_c and ω_s , ω_c are calculated from the HE data, $\gamma_b = 0$
- For configuration b: λ_s is calculated, λ_c=0, ω_s is calculated, ω_c=0, $\gamma_b = \frac{G_c - C}{D_o}$
- For configuration c: λ_s is calculated, λ_c=0, ω_s is calculated, ω_c=0, $\gamma_b = \frac{G_c - G_1}{D_o}$
- For configuration d: λ_s=0 λ_c=0, ω_s=0, ω_c=0, $\gamma_b = \frac{G_c - G_s}{2}$
- For configuration e: λ_s=0, λ_c is calculated, ω_s=0, ω_c is calculated, $\gamma_b = \frac{C - G_s}{D_o}$
- For configuration f: λ_s=0, λ_c is calculated, ω_s=0, ω_c is calculated, $\gamma_b = \frac{G_1 - G_s}{D_o}$

6 STRESSES IN THE HEAT-EXCHANGER COMPONENTS

6.1 Stresses in the Tubesheet

(a) ***Bending stress***

as explained in Section 4, the maximum bending moment M(r) appears:

either at periphery ($r=a_o$): $M(r)=M_a$, noted M_P in UHX-12 rules $$M_P = \frac{M^* - F\left(\frac{a_o^2}{8}\,P_e\right)}{1+F}$$

or at center (r=0): $$M_o = M_P + (3+\nu^*)\,\frac{a_0^2}{16}\,P_e$$

The maximum moment $M = MAX\left[|M_P|,|M_o|\right]$

leads to the maximum bending stress in the TS: $$\sigma = \frac{6M}{\left(h-h_g^{'}\right)^2}$$

(b) ***Shear stress***

The shear stress is maximum at periphery: $\tau = \frac{V_a}{\mu h}$ $$\tau = \frac{a_o}{2\mu h}P_e$$

Note: ASME and TEMA rules provide the same formula, but they use the equivalent diameter D_L corresponding to the perimeter of the outermost ligaments, instead of the equivalent diameter D_o of outer tube limit circle. The equivalent diameter D_L is calculated from the perimeter of the tube layout, $C_P = \pi D_L$, and the area $A_P = \pi D_L^2/4$ enclosed by this perimeter. This leads to $D_L = 4A_P/C_P$

D_L is always lower than D_o and leads to a lower TS shear stress: $\tau = \frac{1}{4\mu}\frac{D_L}{h}P_e$

The ASME formula should be used when the shear stress controls the design, generally in high-pressure cases.

6.2 Stresses in the Shell and Channel

(a) **Axial membrane stress in the shell** is given by the classical formula:

$$\sigma_{s,m} = \frac{a_s^2}{(D_s+t_s)\,t_s}P_s$$

For the channel: $$\sigma_{c,m} = \frac{a_c^2}{(D_c+t_c)\,t_c}P_c$$

(c) **Bending stress**

The bending moment M_s in the shell at its connection with the TS exists only when the shell is integral with the TS (configurations a, b, c). As explained in Annex J, the shell must have a minimum length $l_{s,min} = 1.8\sqrt{D_s\,t_s}$ adjacent to the TS.

[V.1a]: $M_s = k_s\left[\beta_s\delta_s P_s + \left(1+\frac{t_s^{'}}{2}\right)\theta_s\right]$ with $\theta_s = \theta_a$ given by [V.5]:

$$\theta_a = \frac{12(1-\nu^*)}{E^* h^3}\left[a_o M_a + \frac{a_o^3}{8} P_e\right]$$

$$M_s = k_s\left[\beta_s \delta_s P_s + \frac{12(1-\nu^*)}{E^*}\frac{a_o}{h^3}\left(1+\frac{t_s'}{2}\right)\left(M_a + \frac{a_o^2}{8} P_e\right)\right]$$

The shell bending stress $\sigma_{s,b}$ is written: $\sigma_{s,b} = 6\frac{M_s}{t_s^2}$

$$\sigma_{s,b} = \frac{6}{t_s^2} k_s\left[\beta_s \delta_s P_s + 6\frac{1-\nu^*}{E^*}\frac{D_o}{h^3}\left(1+\frac{h\,\beta_s}{2}\right)\left(M_p + \frac{D_o^2}{32}(P_s - P_t)\right)\right]$$

The channel bending stress is obtained in the same way, noting that $\theta_c = -\theta_a$:

$$\sigma_{c,b} = \frac{6}{t_c^2} k_c\left[\beta_c \delta_c P_c - 6\frac{1-\nu^*}{E^*}\frac{D_o}{h^3}\left(1+\frac{h\,\beta_c}{2}\right)\left(M_p + \frac{D_o^2}{32}(P_s - P_t)\right)\right]$$

6.3 Determination of Stresses using the Fixed TS Rules

The TS, shell and channel stresses determined here above can also been obtained from the fixed TS analysis of PART 3 as shown in Annex T.

7 DETERMINATION OF THE ALLOWABLE STRESS LIMITS

The determination of the allowable stress limits developed in Section 9 of PART 3 for the design loading cases apply. However, the allowable stress limit for the TS has been upgraded to 2S for the following reasons.

Comparison of TEMA formula with classical plate formula (see Section 10) shows that TEMA allows the bending stress in the equivalent solid plate to be about 2S, instead of 1.5 S recommended by ASME Section VIII Div. 2. Nevertheless, for about 40 years, TEMA formula did not lead to failures in U-tube tubesheets. It is likely that a value of allowable stress of 2S could be used without affecting the safety margin. This value was also recommended by GARDNER in 1969 [1] and has been used in most European codes (BS 5500, CODAP) for about 20 years. This is also justified by limit load analysis applied to circular plates, which leads to 1.9S if the tubesheet is simply supported and 2.1S if the tubesheet is clamped.

8 ADDITIONAL RULES

8.1 Effect of Plasticity at the Tubesheet-Shell-Channel Joint (UHX-12.5)

In the same way as for fixed TS HE rules, an elastic-plastic analysis is proposed when integral shell and/or channel are overstressed. The concept is explained in Section 10.2(a) of PART 3.

The procedure for the shell applies when $1.5S_s < |\sigma_s| \le S_{PS}$

The elastic-plastic method is simplified: the reduced modulus of elasticity E_s* for the shell is based on the degree of overstress in the shell: $E_s^* = E_s \sqrt{1.5S_s / \sigma_s}$.

Quantities affected by the elastic-plastic procedure are those which are involved in the TS-shell-channel joint and which involve E_s, namely k_s which affects λ_s leading to new value for F.

δ_s is not affected because it is used only in $\omega_s = \rho_s \beta_s k_s \delta_s (1 + h\beta_s)$ in which E_s is cancelled out by the product $k_s\delta_s$. Same procedure applies to the channel by replacing subscript "s" by subscript "c".

If |σ| ≤ 1.5*S*, the design is acceptable. Otherwise, the HE geometry must be reconsidered.
For Example E4.18.4 given in Section 9, the results for the elastic calculation are as follows for the controlling loading case 2:
Stiffening coefficient F=0.96, σ =38175<2S=40000, σ_c =56955>1.5S_c=30000, but lower than $S_{PS,c}$=65000
Similar results for the channel.

For the elastic-plastic calculation: the stiffening coefficient F decreases to 0.85, and
σ =39838<2S=40000

In this example the elastic-plastic calculation leads to an increase of the TS stress, which happens in most cases.

9 HOW TO USE THE RULES

Similar to the fixed TS HE, the calculation procedure can be summarized as follows:

- Set the data listed in Section 3.2(a)
- Calculate the shell design coefficients (ρ_s, k_s, λ_s, δ_s, ω_s), and the channel design coefficients (ρ_c, k_c, λ_c, δ_c, ω_c)
- Calculate coefficient F which represents the degree of restrain of the TS by the shell and channel
- Calculate moments M_{TS} and M*
- Calculate the maximum stresses in TS, shell and channel and limit their values to the maximum allowable stress limits.

Because of the complexity of the procedure, it is likely that users will computerize the solution. A Mathcad calculation sheet is provided for The U-Tube TS HE defined in PTB-4 Example E4.18.4.The TS is gasketed with shell and integral with channel (configuration e). The data are shown in the sheet and the calculations follow strictly the steps 1 to 11 of UHX-12.5 calculation procedure. The elastic-plastic procedure is used. See Annex Y for UHX-12-Example E4.18. (PTB-4 2013 Edition).

The U-Tube HE can also be calculated using floating TS HE software, as shown in Annex U.

10 COMPARISON WITH TEMA RULES

10.1 TEMA Formula

The original TEMA formula for the determination of the TS thickness in bending: $T = F\frac{G}{2}\sqrt{\frac{P}{S}}$ is

based on flat circular plates formula: $h_o = C\frac{G}{2}\sqrt{\frac{P}{\mu^* \, \Omega \, S}}$

Which is written, for μ*=0.5 and Ω=1.5: $h_o = \frac{C}{\sqrt{0.75}}\frac{G}{2}\sqrt{\frac{P}{S}}$

where coefficient $\frac{C}{\sqrt{0.75}}$ is equal to:

- 1.285 for simple support, whereas TEMA uses F=1.25
- for clamping, which is the value used in TEMA (F=1.0)

The TEMA formula was modified later to introduce a ligament efficiency η: $\boxed{T = F\frac{G}{3}\sqrt{\frac{P}{\eta S}}}$,

The ligament efficiency η is based on the mean width of the ligament (see Section 4.2 of PART 2):

For Triangular pitch: $\eta = 1 - \frac{0.907}{(p/d_t)^2}$ For Square pitch: $\eta = 1 - \frac{0.785}{(p/d_t)^2}$

TEMA mean ligament efficiency η (usually around 0.5) is greater than ASME ligament efficiency μ* (usually around 0.3) which is based on the minimum width of the ligament. Coefficient 3 in new TEMA formula has been tailored so that old and new formulasgive approximately the same results.

10.2 Numerical Comparisons

Numerical comparisons have been performed using the four U-tube tubesheet heat exchangers treated in PTB-4 (2013 Edition) Examples E4.18.1 thru E4.18.4. Table 3 shows the results obtained by ASME and TEMA for the tubesheet bending thickness. Examples 1 thru 4 in Table 3 correspond to Examples E4.18.1 thru E4.18.4 respectively.

In the 4 examples, TEMA considers the tubesheet as simply supported (TEMA coefficient *F* = 1.25). However UHX-12 method shows that in Example 1 the tubesheet is almost clamped (UHX-12 coefficient F=9.4), due to the high bending rigidities of the shell and channel as compared to the tubesheet bending rigidity.

Thicknesses obtained by UHX are slightly less than TEMA thicknesses, except for Example 3 where the ASME ligament efficiency is much smaller than TEMA ligament efficiency.

Table 3 — Comparison of TEMA and ASME TS Thicknesses for 4 U-tube HEs

EXAMPLE			LIGAMENT EFFICIENCIES			TEMA T (using η)	ASME h (using μ^*)
N°	Config.	Pitch	μ (ASME) (TEMA)	μ^* (ASME)	η (TEMA)		
1	a	square	0.25	0.35	0.56	**0.61**	**0.52**
2	d	triang.	0.17	0.28	0.37	**1.27**	**1.28**
3	d	triang.	0.20	0.24	0.42	**3.43**	**4.15**
4	e	square	0.25	0.39	0.56	**3.91**	**3.46**

REFERENCES—PART 5

[1] Karl GARDNER "Tubesheet design: a basis for standardisation". ASME publication of 1st ICPVT conference-Delft. Part 1 Design and Analysis-1969.

[2] Urey MILLER "U-Bundle TS extended as flange and integral with shell or channel" – Unpublished report.

[3] Francis Osweiller " New common design rules for U-Tube HEs in ASME, CODAP and UPV Codes" – 2002 PVP Conference (Vancouver) Pressure Vessel and Piping Codes and Standards volume p.229-240.

[4] Alan SOLER "Mechanical Design of Heat Exchangers" – Arcturus publishers -1984 – 1047 pages.

PART 6
SUMMARY AND CONCLUSIONS

1 SUMMARY AND CONCLUSIONS

(a) The purpose of this document is to justify and provide technical criteria for the rules of Part UHX of ASME Section VIII Division 1, 2013 Edition, devoted to the design of U-tube, Fixed and Floating head Tubesheet Heat Exchangers. The document is structured in 5 PARTS.

- PART 1: "Introduction" explains how the UHX rules are structured and provides the heat exchangers types, the loading cases, and the tubesheet configurations covered.
- PART 2: "Tubesheet Characteristics" provides the technical basis of UHX-11 which covers the ligament efficiencies and the effective elastic constants of the tubesheet.
- PART 3: "Fixed Tubesheets Heat Exchangers" provides the technical basis of UHX-13.
- PART 4: "Floating Tubesheet Heat Exchangers" provides the technical basis of UHX-14.
- PART 5: "U-tube Tubesheet Heat Exchangers" provides the technical basis of UHX-12.

PARTS 3, 4 and 5 are independent and structured in the same way (Scope, Historical background, Notations, Tubesheet Configurations, Loading cases, Design assumptions, and Analytical treatment). Floating and U-tube tubesheet heat exchangers are treated as simplified cases of fixed tubesheet heat exchangers. Accordingly, this type of heat exchanger is treated first in PART 3 which forms the main part of this document.

(b) The analytical treatment is based on classical discontinuity analysis methods to determine the moments and forces that the tubesheet, tubes, shell and channel must resist. These components are treated using the elastic theory of thin shells of revolution.
The heat exchanger is assumed to be a symmetrical unit with identical tubesheets on both ends and the unperforated rim is considered as a rigid circular ring.
The tubes are assumed to be identical and uniformly distributed throughout the tubesheets. Additional assumptions are necessary to perform the analytical treatment and derive the "design by rules" method of UHX.

(c) Fixed Tubesheets heat exchangers are covered in PART 3. Main steps of the analytical treatment are as follows.

(1) Tubesheet is disconnected from the shell and channel.
(2) Perforated tubesheet is replaced by an equivalent solid circular plate with effective elastic constants E^* and ν^* which depend on the ligament efficiency μ^* of the tubesheet. This equivalent solid circular plate is treated by the theory of thin circular plates subjected to pressure and other applied loads to determine the maximum stresses.
(3) Unperforated tubesheet rim is treated as a rigid ring whose cross section does not change under loading.
(4) Tubes are replaced by an equivalent elastic foundation of modulus k_w.
(5) Connection of the tubesheet to the shell and channel accounts for the edge displacements and rotations of the 3 components.
(6) Shell and channel are treated by the elastic theory of thin shells of revolution subjected to edge loads to determine the maximum stresses.

The treatment provides, at any radius r of the perforated tubesheet:

- the deflection w(r)
- the rotation $\theta(r)$
- the bending stress $\sigma(r)$
- the shear stress $\tau(r)$
- the axial stress in the tubes $\sigma_t(r)$

These quantities, which are not given in UHX rules, allow one to determine how the tubesheet deflects and the radii corresponding to the locations of the maximum bending stress in the tubesheet and the maximum axial stress in the tube bundle.

These stresses depend on two parameters:

- X_a, which represents the ratio of the tube- bundle axial stiffness to the tubesheet bending stiffness.
- Q_3, which accounts for the moments due to tube side and shell side pressures and bolt loads acting on the tubesheet.

A parametric study performed with these two parameters permits the derivation of the formulas for the maximum stress in the tubesheet and in the tubes which are given in UHX-13.

The treatment also determines the loads and displacements acting on the shell and channel at their connection to the tubesheets and the loads applied on the unperforated rim. These are not given in UHX rules which provide only the shell and channel stresses.

Maximum stresses in tubesheet, tubes, shell and channel are determined and limited to the appropriate allowable stress-based classifications of Section VIII Division 2 Appendix 4, 2004 edition.

(d) Floating Tubesheet and U-tube Tubesheet heat exchangers are treated as simplified cases of fixed tubesheets heat exchangers.

(e) This document provides the bases for all UHX formulas, especially the following:

- shell and channel coefficients β, k, λ and δ for the calculation of coefficients ω and ω*
- parameter X_a
- coefficient F for the calculation of coefficients Φ, Q_1, Q_2 and parameter Q_3.
- coefficient J when the shell has an expansion joint.
- coefficient γ_b when the tubesheet has a gasketed flanged connection to the shell or channel.
- equivalent pressures P'_s, P'_t, P_γ, P_w, P_{rim} for the calculation of the effective pressure P_e.
- coefficients F_m and F_t for the calculation of maximum bending stress σ and maximum shear stress τ in the tubesheet
- maximum axial stress $\sigma_t(r)$ and buckling stress limit S_{tb} in the tubes
- axial membrane stress $\sigma_{s,m}$ and axial bending stress $\sigma_{s,b}$ in the shell and similar formulas for the channel.

This document provides the mechanical signification basis for the various coefficients and parameters noted above.

The results of the analytical treatment have shown that all UHX formulas were correct, except for four of them which are mentioned in PART 3. These formulas have been corrected in the 2010 and subsequent editions of UHX.

(f) The basis of the additional UHX rules is provided in this document**.** They concern:

- The effect of different shell thickness adjacent to the tubesheet
- The effect of plasticity at tubesheet-shell-channel joint
- The effect of radial thermal expansion adjacent to the tubesheet
- The calculation procedure for simply supported tubesheets
- The derivation of the tubesheet flange extension
- The case where the heat exchanger has a thin-walled or thick-walled expansion joint

(g) Checking of the results obtained has been made by comparison with FEA and other code rules. FEA results obtained for the axial tube stresses $\sigma_t(r)$ throughout the tubesheet match UHX results with discrepancies of less than 5%. The comparisons were not as close for the tubesheet bending stresses

and these discrepancies are still being evaluated by the Sub-Group on Heat Transfer Equipment. They might be due to the unperforated rim considered as a rigid ring in the analytical treatment.

The French pressure vessel code, CODAP, is based on the same analytical approach as UHX, but additional simplifications have been made (unperforated rim is ignored). When incorporating these simplifications in the UHX method, all CODAP formulas have been retrieved. This demonstrates that both methods are correct and consistent. This consistency extends to TEMA: by implementing the additional TEMA simplifications, all TEMA formulas have been confirmed.

UHX formulas have also been used to simulate circular plates under pressure by ignoring the tubesheet holes, the unperforated rim, the tubes and the connection with the shell and channel. The classical formulas for circular plates subjected to pressure have been obtained.

Thus, applying the relevant simplifications, it has been analytically demonstrated that the UHX-13.3 method leads to CODAP, TEMA and circular plate formulas. This confirms the correctness of the ASME method, which has also been confirmed by FEA comparisons for the tubesheet deflection.

(h) It is thought that justification and documentation of the basis for UHX-rules is important for the SG-HTE members for future reference and developments or if additional confirmation or comparisons are required.

It will be a valuable reference for early career engineers that are using the UHX rules or becoming involved in code developments of such rules in the future.

ANNEX A — VALUES OF EFFECTIVE ELASTIC CONSTANTS FROM VARIOUS AUTHORS

Table 4 — Comparison of Effective Elastic Constants E* and ν* Values by Various Theoretical Methods for Plane Stress Problem

(a) Triangular pitch

μ	effective elastic constants	DIRECT METHOD		UNDIRECT METHOD
		Meijers [5]	Grigolyuk and Fil'shtinskii [11]	Slot — O'Donnell [7]
0,1	E^*_Δ/E	0,048 2	0,048 2	0,048 2
0,2		0,146 2	0,146 1	0,146 2
0,3		0,267 5		0,267 5
0.4		0.397	0,396 8	0,396 3
0,5		0,529 1		0,529 1
0,7		0,789 5		0,789 5
0,8		0.899	0,899 6	0,898 6
0,1	ν^*_Δ (ν = 0,3)	0,684 4	0,683 4	0,684 3
0,2		0,488 6	0,488 7	0,488 9
0,3		0,384 1		0,384 1
0,4		0,337	0,337 2	0,337 4
0,5		0,319 4		0,319 4
0,7		0,307 0		0,307 0
0,8		0,303	0,303 6	0,303 3

(b) Square pitch

μ	effective elastic constants	DIRECT METHOD		UNDIRECT METHOD
		Meijers [5]	Grigolyuk and Fil'shtinskii [11]	Bailey — Hicks [6] Slot — O'Donnell [7]
0.1	E_p^*/E	0.1857	0.1856	0.1857
0.2		0.3000	0.2999	0.3000
0.3		0.4069		0.4069
0.4			0.5116	0.5117
0.5		0.6168		0.6168
0.7		0.8244		0.8244
0.8			0.9139	0.9139
0.1	ν_p^* (ν = 0.3)	0.0956	0.0965	0.0956
0.2		0.1506	0.1509	0.1506
0.3		0.1975		0.1974
0.4			0.2365	0.2365
0.5		0.2665		0.2665
0.7		0.2974		0.2975
0.8			0.3000	0.3010

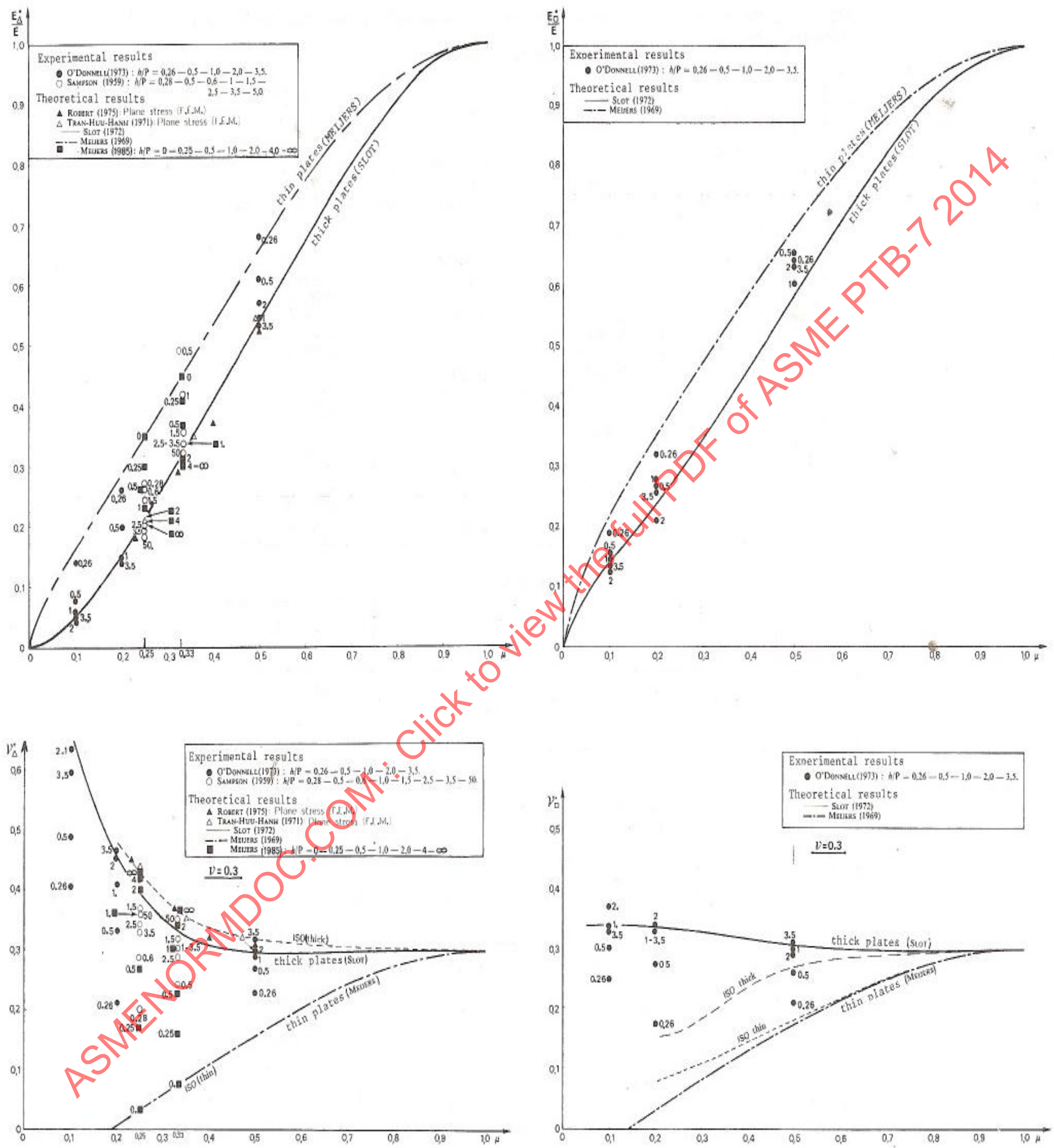


Figure 55 — Synthesis of E*/E and ν∗ Values from [1], Provided by Various Authors for Triangular and Square Pattern

ANNEX B — VALUES OF EFFECTIVE ELASTIC CONSTANTS FOR THE FULL RANGE OF M (0.1≤M*≤1.0)

1 Introduction

This Annex provides the curves, numerical values and polynomials to calculate the effective elastic constants E*/E and ν* for the full range of the ligament efficiency μ* (0.1≤μ*≤1.0).

2 Curves (From [13])

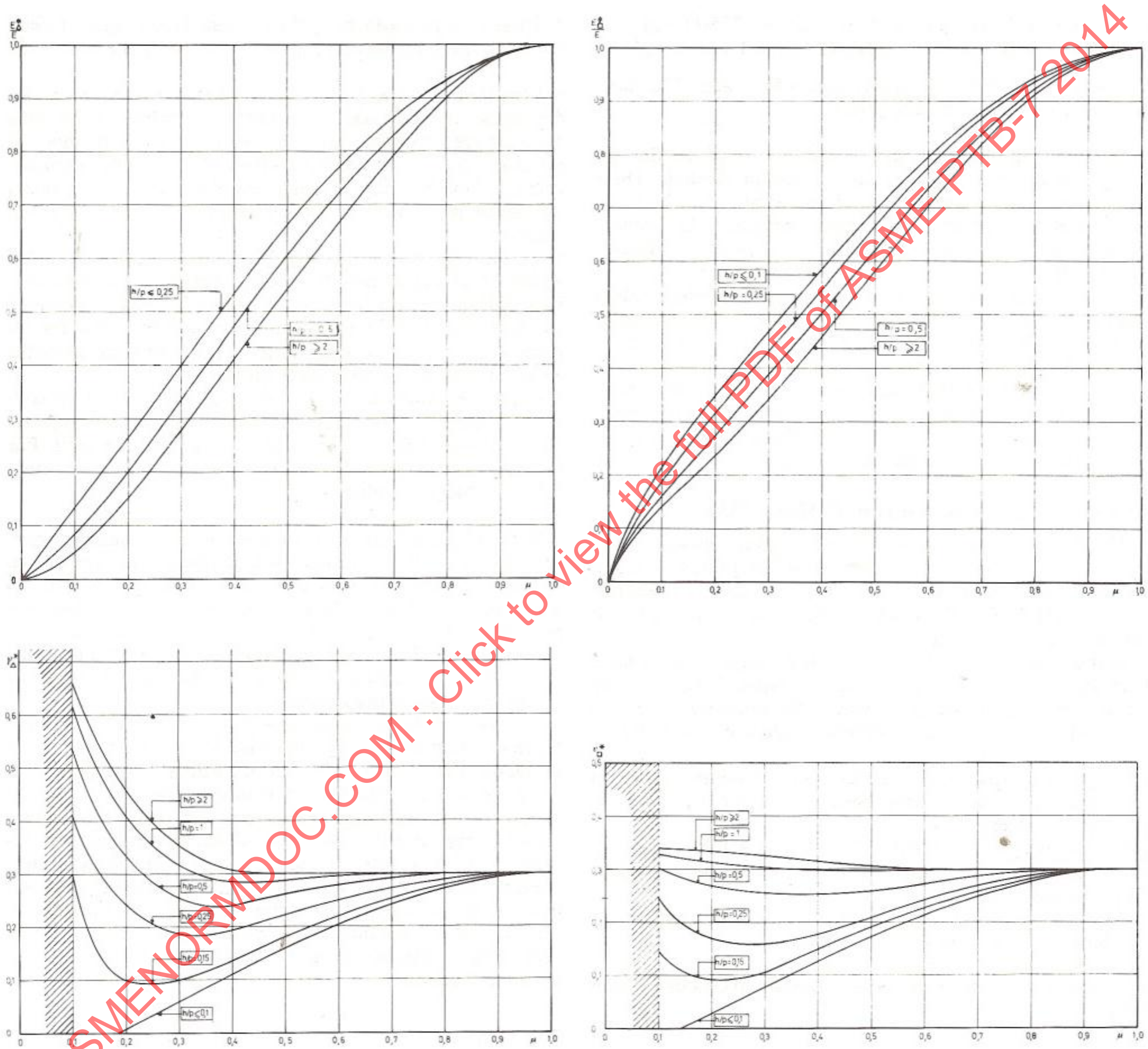


Figure 56 — Curves of Effective Elastic Constants for the Full Range of μ* (0.1≤μ*≤1.0)

3 Numerical Values (From [13])

h/p	μ*	E*/E
0.1	0.0	0.0
0.1	0.02	0.05
0.1	0.05	0.096
0.1	0.1	0.16
0.1	0.15	0.223
0.1	0.2	0.285
0.1	0.22	0.310
0.1	0.25	0.348
0.1	0.28	0.3858
0.1	0.3	0.411
0.1	0.32	0.436
0.1	0.35	0.473
0.1	0.38	0.510
0.1	0.4	0.535
0.1	0.42	0.559
0.1	0.45	0.596
0.1	0.48	0.6318
0.1	0.5	0.656
0.1	0.52	0.678
0.1	0.55	0.712
0.1	0.58	0.7448
0.1	0.6	0.767
0.1	0.626	0.792
0.1	0.64	0.806
0.1	0.657	0.823
0.1	0.675	0.84
0.1	0.7	0.863
0.1	0.719	0.879
0.1	0.75	0.904
0.1	0.777	0.9222
0.1	0.8	0.937
0.1	0.828	0.9523
0.1	0.85	0.964
0.1	0.866	0.9708
0.1	0.882	0.9772
0.1	0.9	0.984
0.1	0.918	0.990
0.1	0.938	0.994
0.1	0.95	0.996
0.1	0.977	0.999
0.1	1.0	1.0

h/p	μ*	E*/E
0.25	0.0	0.0
0.25	0.1	0.123
0.25	0.15	0.181
0.25	0.2	0.243
0.25	0.25	0.308
0.25	0.3	0.374
0.25	0.35	0.439
0.25	0.4	0.505
0.25	0.45	0.568
0.25	0.5	0.632
0.25	0.55	0.690
0.25	0.6	0.747
0.25	0.7	0.847
0.25	0.75	0.892
0.25	0.8	0.930
0.25	0.825	0.9459
0.25	0.85	0.96
0.25	0.9	0.982
0.25	0.95	0.995
0.25	1.0	1.0

h/p	μ*	E*/E
0.5	0.0	0.0
0.5	0.02	0.0122
0.5	0.04	0.0287
0.5	0.05	0.0375
0.5	0.0804	0.0653
0.5	0.1	0.0849
0.5	0.12	0.106
0.5	0.14	0.127
0.5	0.15	0.139
0.5	0.184	0.18
0.5	0.2	0.201
0.5	0.219	0.226
0.5	0.25	0.2678
0.5	0.279	0.307
0.5	0.3	0.3361
0.5	0.318	0.361
0.5	0.35	0.4047
0.5	0.4	0.4753
0.5	0.435	0.521
0.5	0.45	0.541
0.5	0.465	0.561
0.5	0.5	0.6075
0.5	0.527	0.641
0.5	0.55	0.6688
0.5	0.6	0.7272
0.5	0.64	0.769
0.5	0.65	0.7794
0.5	0.664	0.794
0.5	0.7	0.8313
0.5	0.718	0.85
0.5	0.739	0.87
0.5	0.75	0.8812
0.5	0.779	0.906
0.5	0.796	0.919
0.5	0.8	0.922
0.5	0.817	0.935
0.5	0.838	0.949
0.5	0.85	0.9563
0.5	0.856	0.96
0.5	0.877	0.971
0.5	0.895	0.979
0.5	0.9	0.9806
0.5	0.917	0.986
0.5	0.938	0.992
0.5	0.95	0.9944
0.5	0.978	0.999
0.5	1.0	1.0

h/p	μ*	E*/E
2.0	0.0	0.0
2.0	0.0296	0.008
2.0	0.05	0.0172
2.0	0.0596	0.0217
2.0	0.0807	0.0356
2.0	0.1	0.052
2.0	0.12	0.0688
2.0	0.14	0.088
2.0	0.15	0.0994
2.0	0.176	0.129
2.0	0.2	0.1553
2.0	0.22	0.1798
2.0	0.25	0.2165
2.0	0.28	0.255
2.0	0.3	0.2806
2.0	0.333	0.3246
2.0	0.35	0.348
2.0	0.38	0.3862
2.0	0.4	0.412
2.0	0.42	0.4385
2.0	0.45	0.4783
2.0	0.48	0.5181
2.0	0.5	0.5446
2.0	0.52	0.5701
2.0	0.55	0.6084
2.0	0.58	0.6466
2.0	0.6	0.6721
2.0	0.64	0.724
2.0	0.655	0.742
2.0	0.7	0.7996
2.0	0.716	0.82
2.0	0.738	0.845
2.0	0.76	0.869
2.0	0.779	0.888
2.0	0.791	0.9
2.0	0.8	0.908
2.0	0.813	0.92
2.0	0.834	0.938
2.0	0.85	0.95
2.0	0.876	0.966
2.0	0.9	0.978
2.0	0.918	0.986
2.0	0.938	0.992
2.0	0.95	0.996
2.0	0.978	0.999
2.0	1.0	1.0

Table 5 — Values of Curves ν* as a Function of μ* for Ratios h/p=0.1, 0.15, 0.25, 0.5, 1.0 and 2.0 for Triangular pattern

h/p	μ*	ν*	h/p	μ*	ν*	h/p	μ*	ν*	h/p	μ*	ν*	h/p	μ*	ν*	h/p	μ*	ν*
0.1	0.0	-0.096	**0.15**	0.1	0.3025	**0.25**	0.1	0.4135	**0.5**	0.1	0.5397	**1.0**	0.1	0.6181	**2.0**	0.05	0.8078
0.1	0.05	-0.0672	**0.15**	0.1198	0.2216	**0.25**	0.1	0.4135	**0.5**	0.1191	0.4828	**1.0**	0.1179	0.5675	**2.0**	0.1	0.6606
0.1	0.1	-0.038	**0.15**	0.1267	0.2025	**0.25**	0.1103	0.3817	**0.5**	0.1305	0.453	**1.0**	0.1362	0.5224	**2.0**	0.15	0.5445
0.1	0.15	-0.0158	**0.15**	0.1345	0.1818	**0.25**	0.1182	0.3628	**0.5**	0.139	0.4335	**1.0**	0.146	0.5031	**2.0**	0.18	0.49
0.1	0.184	0.0001	**0.15**	0.1401	0.1685	**0.25**	0.1292	0.34	**0.5**	0.1546	0.4029	**1.0**	0.1575	0.4783	**2.0**	0.2	0.4575
0.1	0.2	0.0069	**0.15**	0.155	0.141	**0.25**	0.1389	0.3224	**0.5**	0.1675	0.3813	**1.0**	0.1669	0.4627	**2.0**	0.22	0.4308
0.1	0.22	0.0166	**0.15**	0.1682	0.1229	**0.25**	0.1517	0.3027	**0.5**	0.1807	0.3616	**1.0**	0.1779	0.445	**2.0**	0.25	0.3956
0.1	0.25	0.0312	**0.15**	0.1748	0.1165	**0.25**	0.16	0.2904	**0.5**	0.1963	0.3405	**1.0**	0.1923	0.422	**2.0**	0.28	0.3693
0.1	0.28	0.0458	**0.15**	0.1829	0.1103	**0.25**	0.1719	0.2741	**0.5**	0.2	0.336	**1.0**	0.2	0.4109	**2.0**	0.3	0.354
0.1	0.3	0.056	**0.15**	0.1911	0.1052	**0.25**	0.1844	0.2614	**0.5**	0.218	0.3152	**1.0**	0.2174	0.389	**2.0**	0.31	0.347
0.1	0.32	0.0656	**0.15**	0.2	0.1011	**0.25**	0.194	0.2505	**0.5**	0.2276	0.3062	**1.0**	0.2259	0.3792	**2.0**	0.321	0.3405
0.1	0.35	0.0809	**0.15**	0.21	0.0983	**0.25**	0.2	0.2449	**0.5**	0.2384	0.2964	**1.0**	0.2379	0.3677	**2.0**	0.333	0.3344
0.1	0.38	0.0961	**0.15**	0.218	0.0965	**0.25**	0.2043	0.2409	**0.5**	0.25	0.2869	**1.0**	0.247	0.3585	**2.0**	0.35	0.3263
0.1	0.4	0.106	**0.15**	0.2291	0.0952	**0.25**	0.2174	0.2296	**0.5**	0.2601	0.2786	**1.0**	0.25	0.3559	**2.0**	0.36	0.3224
0.1	0.42	0.1162	**0.15**	0.2382	0.0947	**0.25**	0.2288	0.2211	**0.5**	0.2718	0.271	**1.0**	0.2593	0.3479	**2.0**	0.38	0.3166
0.1	0.45	0.1311	**0.15**	0.2498	0.0943	**0.25**	0.2394	0.2143	**0.5**	0.2822	0.265	**1.0**	0.2684	0.34	**2.0**	0.4	0.312
0.1	0.48	0.146	**0.15**	0.25	0.0943	**0.25**	0.25	0.2083	**0.5**	0.3	0.2562	**1.0**	0.2787	0.3318	**2.0**	0.42	0.3082
0.1	0.5	0.156	**0.15**	0.2604	0.0946	**0.25**	0.2593	0.2034	**0.5**	0.3198	0.2486	**1.0**	0.3	0.3185	**2.0**	0.45	0.3033
0.1	0.52	0.1653	**0.15**	0.2708	0.0958	**0.25**	0.2711	0.198	**0.5**	0.3323	0.2452	**1.0**	0.2916	0.3235	**2.0**	0.48	0.3005
0.1	0.55	0.1793	**0.15**	0.2787	0.0971	**0.25**	0.2797	0.1951	**0.5**	0.3456	0.2417	**1.0**	0.3051	0.3154	**2.0**	0.5	0.2994
0.1	0.6	0.203	**0.15**	0.29	0.0991	**0.25**	0.2899	0.1914	**0.5**	0.35	0.241	**1.0**	0.3191	0.3082	**2.0**	0.52	0.299
0.1	0.63	0.2165	**0.15**	0.3	0.1015	**0.25**	0.3	0.1887	**0.5**	0.3587	0.2397	**1.0**	0.3337	0.302	**2.0**	0.55	0.2986
0.1	0.65	0.2245	**0.15**	0.3102	0.1038	**0.25**	0.3104	0.1868	**0.5**	0.3692	0.2394	**1.0**	0.3482	0.2969	**2.0**	0.6	0.2988
0.1	0.67	0.2325	**0.15**	0.3196	0.1067	**0.25**	0.3199	0.186	**0.5**	0.3804	0.2395	**1.0**	0.35	0.2963	**2.0**	0.65	0.3
0.1	0.7	0.243	**0.15**	0.3303	0.1099	**0.25**	0.3295	0.1854	**0.5**	0.3914	0.24	**1.0**	0.3604	0.293	**2.0**	0.7	0.3
0.1	0.72	0.2501	**0.15**	0.3394	0.1131	**0.25**	0.3386	0.1851	**0.5**	0.4	0.241	**1.0**	0.3792	0.2893	**2.0**	0.75	0.3
0.1	0.75	0.2597	**0.15**	0.35	0.1172	**0.25**	0.35	0.1857	**0.5**	0.4129	0.243	**1.0**	0.4	0.2877	**2.0**	0.8	0.3
0.1	0.8	0.274	**0.15**	0.3591	0.1204	**0.25**	0 .3604	0.1873	**0.5**	0.4198	0.2448	**1.0**	0.4188	0.2866	**2.0**	0.85	0.3

h/p	μ*	ν*	h/p	μ*	ν*	h/p	μ*	ν*	h/p	μ*	ν*	h/p	μ*	ν*	h/p	μ*	ν*
0.1	0.83	0.2815	**0.15**	0.3795	0.1289	**0.25**	0.3704	0.1891	**0.5**	0.4394	0.2499	**1.0**	0.4399	0.2865	**2.0**	0.9	0.3
0.1	0.85	0.2859	**0.15**	0.4	0.1376	**0.25**	0.3805	0.1911	**0.5**	0.45	0.2525	**1.0**	0.45	0.2867	**2.0**	0.95	0.3
0.1	0.875	0.2906	**0.15**	0.42	0.1464	**0.25**	0.3908	0.193	**0.5**	0.5	0.2635	**1.0**	0.5	0.2893	**2.0**	1.0	0.3
0.1	0.9	0.295	**0.15**	0.45	0.1604	**0.25**	0.4	0.1955	**0.5**	0.55	0.273	**1.0**	0.55	0.2935			
0.1	0.92	0.2974	**0.15**	0.5	0.1834	**0.25**	0.4099	0.198	**0.5**	0.6	0.2813	**1.0**	0.6	0.2976			
0.1	0.95	0.2996	**0.15**	0.55	0.2038	**0.25**	0.4197	0.201	**0.5**	0.65	0.289	**1.0**	0.65	0.2997			
0.1	1.0	0.3	**0.15**	0.6	0.2228	**0.25**	0.4395	0.2067	**0.5**	0.7	0.295	**1.0**	0.7	0.3			
			0.15	0.65	0.2413	**0.25**	0.45	0.2097	**0.5**	0.75	0.2997	**1.0**	0.75	0.3			
			0.15	0.7	0.2571	**0.25**	0.5	0.2246	**0.5**	0.8	0.3	**1.0**	0.8	0.3			
			0.15	0.75	0.2709	**0.25**	0.55	0.2395	**0.5**	0.85	0.3	**1.0**	0.85	0.3			
			0.15	0.8	0.2824	**0.25**	0.6	0.2526	**0.5**	0.9	0.3	**1.0**	0.9	0.3			
			0.15	0.8237	0.2872	**0.25**	0.65	0.2653	**0.5**	0.95	0.3	**1.0**	0.95	0.3			
			0.15	0.85	0.2923	**0.25**	0.7	0.276	**0.5**	1.0	0.3	**1.0**	1	0.3			
			0.15	0.8827	0.2971	**0.25**	0.75	0.2851									
			0.15	0.9	0.2993	**0.25**	0.8	0.2923									
			0.15	0.9018	0.2995	**0.25**	0.85	0.2977									
			0.15	0.95	0.3	**0.25**	0.9	0.3									
			0.15	1.0.0	0.3	**0.25**	0.95	0.3									
						0.25	1.0	0.3									

Table 6 — Values of Curves ν* as a function of μ* for ratios h/p=0.1, 0.15, 0.25, 0.5, 1.0 and 2.0 for square pattern

h/p	μ*	ν*	h/p	μ*	ν*	h/p	μ*	ν*	h/p	μ*	ν*	h/p	μ*	ν*	h/p	μ*	ν*
0.1	0.1	-0.022	**0.15**	0.1	0.1465	**0.25**	0.1	0.2471	**0.5**	0.1	0.3027	**1.0**	0.1	0.3283	**2.0**	0.05	0.339
0.1	0.142	0.0003	**0.15**	0.1082	0.1357	**0.25**	0.1183	0.2201	**0.5**	0.1122	0.2958	**1.0**	0.1376	0.3216	**2.0**	0.1	0.34
0.1	0.2	0.031	**0.15**	0.1179	0.1267	**0.25**	0.1278	0.2099	**0.5**	0.1193	0.2925	**1.0**	0.15	0.3196	**2.0**	0.125	0.3403
0.1	0.22	0.0408	**0.15**	0.1281	0.1184	**0.25**	0.1384	0.2002	**0.5**	0.1294	0.2881	**1.0**	0.1784	0.3151	**2.0**	0.15	0.34
0.1	0.25	0.0555	**0.15**	0.1379	0.1124	**0.25**	0.15	0.192	**0.5**	0.15	0.2809	**1.0**	0.1984	0.3125	**2.0**	0.175	0.3387
0.1	0.28	0.0702	**0.15**	0.15	0.1053	**0.25**	0.1609	0.185	**0.5**	0.1775	0.273	**1.0**	0.2	0.3123	**2.0**	0.2	0.337
0.1	0.3	0.08	**0.15**	0.1579	0.1016	**0.25**	0.1772	0.1767	**0.5**	0.2	0.2678	**1.0**	0.2383	0.3082	**2.0**	0.22	0.3354
0.1	0.32	0.0896	**0.15**	0.1694	0.0978	**0.25**	0.1888	0.1722	**0.5**	0.2383	0.2611	**1.0**	0.25	0.307	**2.0**	0.25	0.333
0.1	0.35	0.104	**0.15**	0.1788	0.0951	**0.25**	0.2	0.1685	**0.5**	0.25	0.2594	**1.0**	0.279	0.304	**2.0**	0.28	0.33
0.1	0.38	0.1184	**0.15**	0.1901	0.093	**0.25**	0.2104	0.165	**0.5**	0.2784	0.256	**1.0**	0.3	0.3025	**2.0**	0.3	0.328

h/p	μ*	ν*		h/p	μ*	ν*		**h/p**	μ*	ν*		**h/p**	μ*	ν*		**h/p**	μ*	ν*		**h/p**	μ*	ν*
0.1	0.4	0.128		**0.15**	0.1984	0.092		**0.25**	0.2307	0.1602		**0.5**	0.3	0.2542		**1.0**	0.3396	0.3005		**2.0**	0.32	0.3256
0.1	0.42	0.137		**0.15**	0.2	0.0919		**0.25**	0.2391	0.1589		**0.5**	0.3187	0.253		**1.0**	0.35	0.3002		**2.0**	0.35	0.322
0.1	0.45	0.1505		**0.15**	0.2099	0.0915		**0.25**	0.25	0.158		**0.5**	0.3388	0.2524		**1.0**	0.4	0.2992		**2.0**	0.38	0.3184
0.1	0.48	0.164		**0.15**	0.2186	0.0914		**0.25**	0.2529	0.1577		**0.5**	0.35	0.2522		**1.0**	0.4386	0.2986		**2.0**	0.4	0.316
0.1	0.5	0.173		**0.15**	0.2292	0.0917		**0.25**	0.263	0.1573		**0.5**	0.3581	0.2521		**1.0**	0.45	0.2984		**2.0**	0.42	0.314
0.1	0.52	0.1814		**0.15**	0.239	0.0926		**0.25**	0.273	0.1572		**0.5**	0.3786	0.252		**1.0**	0.4792	0.298		**2.0**	0.45	0.3106
0.1	0.55	0.194		**0.15**	0.25	0.0937		**0.25**	0.281	0.1572		**0.5**	0.3975	0.252		**1.0**	0.5	0.298		**2.0**	0.48	0.3076
0.1	0.58	0.2066		**0.15**	0.2591	0.0953		**0.25**	0.2945	0.1573		**0.5**	0.4	0.252		**1.0**	0.5198	0.298		**2.0**	0.5	0.306
0.1	0.6	0.215		**0.15**	0.2681	0.0971		**0.25**	0.3	0.1576		**0.5**	0.4181	0.2524		**1.0**	0.55	0.2984		**2.0**	0.52	0.3044
0.1	0.65	0.233		**0.15**	0.2788	0.0999		**0.25**	0.3105	0.1582		**0.5**	0.4309	0.2533		**1.0**	0.6	0.299		**2.0**	0.55	0.3028
0.1	0.7	0.25		**0.15**	0.2917	0.1036		**0.25**	0.3222	0.1592		**0.5**	0.45	0.2544		**1.0**	0.65	0.2995		**2.0**	0.58	0.3018
0.1	0.75	0.2644		**0.15**	0.3	0.1065		**0.25**	0.3316	0.1604		**0.5**	0.4802	0.2577		**1.0**	0.7	0.3		**2.0**	0.6	0.301
0.1	0.8	0.277		**0.15**	0.3181	0.1144		**0.25**	0.3399	0.1622		**0.5**	0.5028	0.261		**1.0**	0.75	0.3		**2.0**	0.65	0.3002
0.1	0.85	0.2873		**0.15**	0.3286	0.1188		**0.25**	0.35	0.1644		**0.5**	0.5232	0.2643		**1.0**	0.8	0.3		**2.0**	0.7	0.3
0.1	0.9	0.2956		**0.15**	0.3395	0.1235		**0.25**	0.3697	0.1687		**0.5**	0.5	0.2606		**1.0**	0.85	0.3		**2.0**	0.75	0.3
0.1	0.95	0.2995		**0.15**	0.35	0.1283		**0.25**	0.4	0.1762		**0.5**	0.55	0.268		**1.0**	0.9	0.3		**2.0**	0.8	0.3
0.1	0.98	0.2999		**0.15**	0.4	0.1498		**0.25**	0.4121	0.1799		**0.5**	0.6	0.275		**1.0**	0.95	0.3		**2.0**	0.85	0.3
0.1	1.0	0.3		**0.15**	0.45	0.1698		**0.25**	0.4255	0.1843		**0.5**	0.64	0.28		**1.0**	1.0	0.3		**2.0**	0.9	0.3
				0.15	0.5	0.1903		**0.25**	0.4386	0.1882		**0.5**	0.65	0.2812						**2.0**	0.95	0.3
				0.15	0.55	0.2092		**0.25**	0.45	0.1924		**0.5**	0.7	0.287						**2.0**	1.0	0.3
				0.15	0.6	0.2267		**0.25**	0.4798	0.203		**0.5**	0.72	0.289								
				0.15	0.65	0.2427		**0.25**	0.5	0.21		**0.5**	0.74	0.291								
				0.15	0.7	0.2578		**0.25**	0.55	0.2276		**0.5**	0.75	0.2919								
				0.15	0.7196	0.263		**0.25**	0.6	0.2441		**0.5**	0.78	0.2948								
				0.15	0.741	0.2682		**0.25**	0.62	0.25		**0.5**	0.8	0.296								
				0.15	0.75	0.2702		**0.25**	0.6461	0.257		**0.5**	0.82	0.2972								
				0.15	0.7799	0.2773		**0.25**	0.65	0.2579		**0.5**	0.84	0.2982								
				0.15	0.8	0.2816		**0.25**	0.6786	0.2644		**0.5**	0.85	0.2986								
				0.15	0.821	0.2856		**0.25**	0.7	0.2692		**0.5**	0.88	0.2995								
				0.15	0.8316	0.2875		**0.25**	0.7194	0.2729		**0.5**	0.9	0.3								
				0.15	0.85	0.2904		**0.25**	0.7404	0.2771		**0.5**	0.95	0.3								
				0.15	0.8787	0.2944		**0.25**	0.75	0.2788		**0.5**	1.0	0.3								
				0.15	0.9	0.2965		**0.25**	0.7794	0.2841												
				0.15	0.92	0.2981		**0.25**	0.8	0.2872												
				0.15	0.94	0.2994		**0.25**	0.8204	0.29												
				0.15	0.95	0.2997		**0.25**	0.84	0.2922												
				0.15	1.0.0	0.3		**0.25**	0.85	0.2932												

h/p	μ*	ν*		h/p	μ*	ν*		**h/p**	μ*	ν*		**h/p**	μ*	ν*		**h/p**	μ*	ν*		**h/p**	μ*	ν*
								0.25	0.8785	0.2958												
								0.25	0.9	0.2971												
								0.25	0.9206	0.2983												
								0.25	0.94	0.2995												
								0.25	0.95	0.2996												
								0.25	1.0	0.3												

4 Polynomials

Table 1: E*/E Coefficients for Triangular Tube Pattern

μ^*	α_0	α_1	α_2	α_3	α_4
0.10	0.002	1.239	0.573	-0.878	0.063
0.50	-0.003	0.728	1.895	-2.051	0.434
2.00	-0.009	0.484	1.957	-1.442	0.016

Table 2: E*/E Coefficients for Square Tube Pattern

μ^*	α_0	α_1	α_2	α_3	α_4
0.10	0.017	2.112	-2.759	3.368	-1.747
0.25	0.013	1.840	-2.111	2.893	-1.642
0.50	0.011	1.567	-1.648	2.747	-1.682
2.00	0.012	1.278	-1.135	2.587	-1.749

Table 3: ν* Coefficients for Triangular Tube Pattern

μ^*	β_0	β_1	β_2	β_3	β_4	β_5	β_6	β_7
0.10	-0.089	0.450	0.267	-0.325	-0.002	0	0	0
0.15	1.232	-16.249	93.259	-284.35	507.628	-528.539	296.515	-69.197
0.25	0.771	-4.78	13.226	-11.952	-7.503	23.725	-17.848	4.66
0.50	0.989	-6.399	23.448	-53.158	87.214	-96.808	61.016	-16.003
1.00	1.01	-5.142	13.806	-17.444	10.436	-2.366	0	0
2.00	1.018	-4.52	10.821	-12.221	6.421	-1.21	0	0

Table 4: ν* Coefficients for Square Tube Pattern

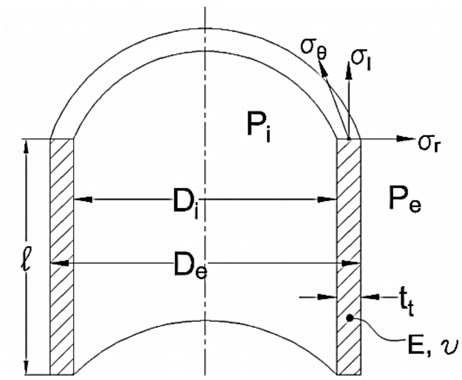
μ^*	β_0	β_1	β_2	β_3	β_4	β_5	β_6	β_7
0.10	-0.067	0.451	0.191	-0.273	0	0	0	0
0.15	0.337	-2.905	11.728	-19.521	15.421	-4.761	0	0
0.25	0.434	-2.632	8.552	-11.617	7.451	-1.889	0	0
0.50	0.400	-1.602	8.636	-29.539	62.382	-74.962	46.713	-11.728
1.00	0.348	-0.201	-0.180	2.124	-4.358	4.145	-1.954	0.374
2.00	0.354	-0.286	2.27	-10.35	22.733	-25.581	14.349	-3.188

Polynomials of Effective Elastic Constants for the full range of μ* (0.1≤μ*≤1.0) initially used for Appendix AA of ASME Section VIII-Div. 1

ANNEX C — POISSON'S RATIO IN TUBES AND SHELL

(a) **Deformation of a shell of length l**; diameters D_i ,D_e ; pressures P_i, P_e:

$$\varepsilon = \frac{1}{E}\left[\sigma_l - \nu\left(\sigma_\theta + \sigma_r\right)\right] = \varepsilon_l + \varepsilon_\nu$$



Deformation due to Poisson's ratio: $\varepsilon_\nu = \dfrac{\delta_l(\nu)}{l} = -\dfrac{\nu}{E}\left(\sigma_\theta + \sigma_r\right)$

$$\sigma_\theta + \sigma_r = 2\,\frac{P_i\,D_i^2 - P_e\,D_e^2}{D_e^2 - D_i^2} = \frac{\pi}{2}\,\frac{P_i\,D_i^2 - P_e\,D_e^2}{s} \qquad s = \frac{\pi}{4}\left(D_e^2 - D_i^2\right)$$

$$\delta_l(\nu) = -\frac{\pi}{2}\,\frac{l}{E\,s}\left[P_i\,D_i^2 - P_e\,D_e^2\right]\nu = -\frac{\pi}{2\,k}\left[P_i\,D_i^2 - P_e\,D_e^2\right]\nu \qquad k = \frac{E\,s}{l}$$

(b) **Application to the tubes**: $D_e = d_t \qquad D_i = d_t - 2\,t_t \qquad P_i = P_t \qquad P_e = P_s$

$$\delta_t(\nu_t) = -\frac{\pi}{2\,k_t}\left[P_t\left(d_t - 2\,t_t\right)^2 - P_s\,d_t^2\right]\nu_t$$

$$\begin{cases} x_t = 1 - N_t\,\dfrac{\left(d_t - 2\,t_t\right)^2}{D_o^2} \\ x_s = 1 - N_t\,\dfrac{d_t^2}{D_o^2} \end{cases} \quad \begin{cases} 1 - x_t = N_t\,\dfrac{\left(d_t - 2\,t_t\right)^2}{D_o^2} \\ 1 - x_s = N_t\,\dfrac{d_t^2}{D_o^2} \end{cases} \quad .. \quad \begin{cases} \left(d_t - 2\,t_t\right)^2 = \left(1 - x_t\right)\dfrac{D_o^2}{N_t} \\ d_t^2 = \left(1 - x_s\right)\dfrac{D_o^2}{N_t} \end{cases}$$

$$\delta_t(\nu_t) = -\frac{\pi\,D_o^2}{2\,k_t\,N_t}\left[\left(1 - x_t\right)P_t - \left(1 - x_s\right)P_s\right]\nu_t \qquad k_w = \frac{N_t\,k_t}{\pi\,a_o^2} = 4\,\frac{N_t\,k_t}{\pi\,D_o^2}$$

$$\boxed{\delta_t(\nu_t) = -\frac{2}{k_w}\left[\left(1 - x_t\right)P_t - \left(1 - x_s\right)P_s\right]\nu_t}$$

(c) **Application to the shell**: $D_i = D_s \qquad P_i = P_s \qquad P_e = 0 \qquad k_s^{'} = \dfrac{\pi\,E_s\,t_s\left(D_s + t_s\right)}{l}$

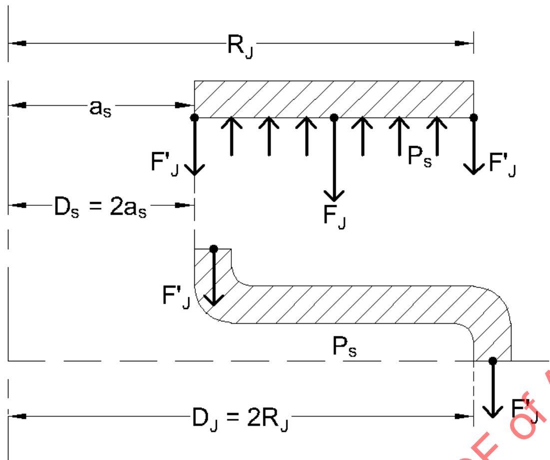
$$\boxed{\delta_s(\nu_s) = -\frac{\pi}{2\,k_s^{'}}\left[P_s\,D_s^2\right]\nu_s = \frac{-l\,D_s^2\,P_s}{2\,E_s\,t_s\left(D_s + t_s\right)}\,\nu_s}$$

$$k_{\mathrm{w}}\ \delta_{\mathrm{s}}\left(\nu_s\right) = -\frac{\pi}{2\,k_{\mathrm{s}}^{'}}\ \frac{N\ k_{\mathrm{t}}}{\pi\left(D_o^2/4\right)}\ P_{\mathrm{s}}\ D_s^2\ \nu_s = -\frac{2\,\nu_{\mathrm{s}}}{k_{\mathrm{s,t}}}\left(\frac{D_{\mathrm{s}}}{D_o}\right)^2 P_{\mathrm{s}}$$

ANNEX D — SHELL PRESSURE ACTING ON THE EXPANSION JOINT SIDEWALLS

Axial force acting on the sidewall of one half of the expansion joint due to internal shell pressure P_s:

$$F_J = P_s\,\pi\left(R_J^2 - R_s^2\right) \qquad R_J = \frac{D_J}{2} \qquad R_s = \frac{D_s}{2}$$



The shell pressure acting on the sidewall of the expansion joint creates an axial force. It is assume that half of this force is resisted by the expansion joint, and the other half of it is carried out by the shell. Accordingly, the axial force acting on the shell is:

$$F_J^{'} = \frac{F_J}{2} = \frac{\pi}{2}\left(R_J^2 - R_s^2\right)P_s = \frac{\pi}{8}\left(D_J^2 - D_s^2\right)P_s$$

Axial displacement of the half-joint of rigidity k_J, due to F'_J:

$$\delta_J\left(P_s\right) = \frac{F_J^{'}}{k_J} = \frac{\pi}{8}\,\frac{D_J^2 - D_s^2}{k_J}\,P_s$$

Expansion joint rigidity: $K_J = \dfrac{k_J}{2}$

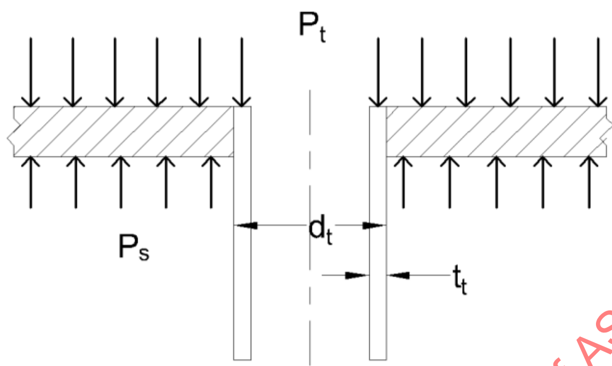
$$\boxed{\delta_J\left(P_s\right) = \frac{\pi}{16}\,\frac{D_J^2 - D_s^2}{K_J}\,P_s}$$

Note: this formula is still valid if the shell is extended as a flange (D_s should not be replaced by G_s)

ANNEX E — DIFFERENTIAL PRESSURE ACTING ON THE EQUIVALENT SOLID PLATE

(a) **Force due to pressure P_s** acting on the TS of radius a_o:

$$P_s\left[\pi\, a_o^2 - N_t\, \pi\left(\frac{d_t}{2}\right)^2\right]$$

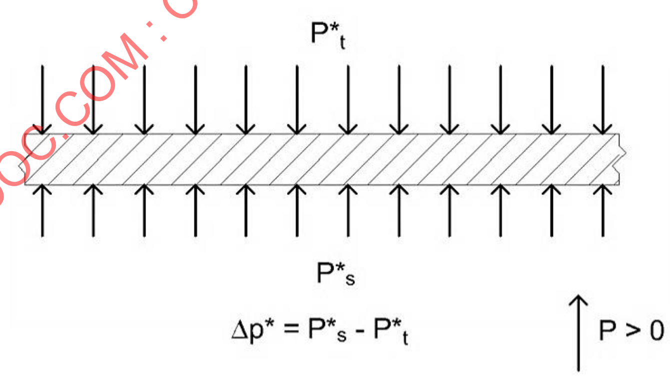


This force leads to a uniform pressure P_s^* acting on the equivalent solid plate given by:

$$P_s^*\,\pi\, a_o^2 = P_s\left[\pi\, a_o^2 - N_t\, \pi\left(\frac{d_t}{2}\right)^2\right]$$

$$P_s^* = P_s\left[1 - N_t\left(\frac{d_t}{2\,a_o}\right)^2\right] = P_s\, x_s \qquad D_o = 2\,a_o$$

TS coefficient relating to the tubes on shell side: $\boxed{x_s = 1 - N_t\left(\frac{d_t}{D_o}\right)^2}$



(b) **Force due to pressure P_t** acting on the TS of radius a_o:

$$P_t\left[\pi\, a_o^2 - N_t\, \pi\left(\frac{d_t - 2\,t_t}{2}\right)^2\right]$$

This force leads to a uniform pressure P_t^* acting on the equivalent solid plate given by:

$$P_t^*\,\pi\, a_o^2 = P_t\left[\pi\, a_o^2 - N_t\, \pi\left(\frac{d_t - 2\,t_t}{2}\right)^2\right]$$

$$P_t^* = P_t \left[1 - N_t \left(\frac{d_t - 2\,t_t}{2a_o} \right)^2 \right] = P_t\; x_t$$

TS coefficient relating to the tubes on tube side: $\boxed{x_t = 1 - N_t \left(\frac{d_t - 2\,t_t}{D_o} \right)}$

(c) **Differential pressure acting on the equivalent solid plate:**

$$\boxed{\Delta p^* = x_s\; P_s - x_t\; P_t}$$

ANNEX F — SOLUTION OF DIFFERENTIAL EQUATION W(X)

The deflection of the equivalent solid plate is governed by a differential equation of 4th order:

$$\frac{d^4w}{dr^4}+\frac{2}{r}\frac{d^3w}{dr^3}-\frac{1}{r^2}\frac{d^2w}{dr^2}+\frac{1}{r^3}\frac{dw}{dr}=\frac{q(r)}{D^*}=\frac{Q-k_w\,w(r)}{D^*}$$

$$\frac{d^4w}{dr^4}+\frac{2}{r}\frac{d^3w}{dr^3}-\frac{1}{r^2}\frac{d^2w}{dr^2}+\frac{1}{r^3}\frac{dw}{dr}+\frac{k_w}{D^*}w(r)=\frac{Q}{D^*}$$

A change of variable is necessary to solve the differential equation: $\boxed{x=kr}$

$$\frac{k_w}{D^*}=k^4 \qquad \boxed{k=\sqrt[4]{\frac{k_w}{D^*}}} \qquad 0\le r\le a_0 \;\Rightarrow\; \boxed{0\le x\le X_a} \qquad \boxed{X_a=k\,a_o}$$

$$\frac{d\,w}{d\,r}=\frac{d\,w}{d\,x}\frac{d\,x}{d\,r}=k\frac{d\,w}{d\,x} \qquad \frac{d^2\,w}{d\,r^2}=k^2\frac{d^2\,w}{d\,x^2} \qquad \frac{d^3\,w}{d\,r^3}=k^3\frac{d^3\,w}{d\,x^3} \qquad \frac{d^4\,w}{d\,r^4}=k^4\frac{d^4\,w}{d\,x^4}$$

$$\frac{d^4w}{dx^4}+\frac{2}{x}\frac{d^3w}{dx^3}-\frac{1}{x^2}\frac{d^2w}{dx^2}+\frac{1}{x^3}\frac{dw}{dx}+w(x)=\frac{Q}{k^4\,D^*}=\frac{Q}{k_w}$$

- The homogeneous solution is:

$$w(x)=A\,ber(x)+B\,bei(x)+C\,ker(x)+D\,kei(x)$$

where:

ber(x), bei(x), ker(x), kei(x) are Kelvin functions of order 0, which will be noted berx, beix, kerx, keix

A, B, C, D are integration constants

$$berx=\sum_0^\infty(-1)^n\frac{(x/2)^{4n}}{\left[(2n)!\right]^2}=1-\frac{(x/2)^4}{(2!)^2}+\cdots$$

$$beix=\sum_1^\infty(-1)^{2n-1}\frac{(x/2)^{4n-2}}{\left[(2n-1)!\right]^2}=\frac{(x/2)^2}{(1!)^2}-\frac{(x/2)^6}{(3!)^2}+\cdots$$

$$kerx=-ln(x/2)berx+\frac{\pi}{4}beix+[1]-\left[\frac{(x/2)^4}{(2!)^2}(1+\frac{1}{2})\right]+\left[\frac{(x/2)^8}{(4!)^2}(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4})\right]-...$$

$$keix=-ln(x/2)beix-\frac{\pi}{4}berx+\left[(x/2)^2\right]-\left[\frac{(x/2)^6}{(3!)^2}(1+\frac{1}{2}+\frac{1}{3})\right]+\left[\frac{(x/2)^{10}}{(5!)^2}(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5})\right]-...$$

$$\text{For } x=0:\begin{cases} ber(0)=1 & bei(0)=0 & ker(0)=\rightarrow\infty & kei(0)=-\frac{\pi}{4}\\ ber'(0)=1 & bei'(0)=0 & ker'(0)=\rightarrow\infty & kei'(0)=0\\ ber''(0)=0 & bei''(0)=\frac{1}{2} & ker''(0)=\rightarrow\infty & kei''(0)=\rightarrow\infty\end{cases}$$

The particular solution with 2nd member is: $w(x)=\frac{Q}{k_w}$

General solution: $w(x)=A\,berx+B\,beix+C\,kerx+D\,keix+\frac{Q}{k_w}$

Determination of constants C and D. Bending moment is written:

$$M_r(r) = -D^* k^2 \left(\frac{d^2 w}{dx^2} + \frac{\nu^*}{x} \frac{dw}{dx} \right) = -D^* k^2 \begin{bmatrix} (Aber''x + Bbei''x + C\ker''x + Dkei''x) \\ + \frac{\nu^*}{x}(Aber'x + Bbei'x + C\ker'x + Dkei'x) \end{bmatrix}$$

The bending moment $M_r(0)$ at center of TS must remain finite: → C=0 D=0

Finally, the solution is written: $$w(x) = A\,berx + B\,beix + \frac{Q}{k_w}$$

ANNEX G — COEFFICIENTS Z_D, Z_V, Z_W, Z_M; Q_M, Q_V; Q_A, Q_B; F_M, F_T

(a) **Purpose**

This Annex provides the equations for coefficients $Z_d(x)$, $Z_v(x)$, $Z_w(x)$, $Z_m(x)$; $Q_m(x)$, $Q_v(x)$, $Q_\alpha(x)$, $Q_\beta(x)$; $F_m(x)$, $F_t(x)$ and their values for:

- x=0 or close to 0 for a given value of X_a.
- $x=X_a$
- $x=X_a$ when X_a is close to 0

Notations are taken from UHX-13:

- *ber*, *bei* are Kelvin functions
- *f(x)* represents the value of function *f* for *x*.
- *f(X_a)* is always noted "*f*": $f = f(X_a)$. Therefore:

$ber = ber(X_a)$; $ber' = ber'(X_a)$

$bei = bei(X_a)$; $bei' = bei'(X_a)$

$Z_a = Z_a(X_a)$

$Z_m = Z_m(X_a)$; $Z_v = Z_v(X_a)$

$Z_d = Z_d(X_a)$; $Z_w = Z_w(X_a)$

$\psi_1 = \psi_1(X_a)$; $\psi_2 = \psi_2(X_a)$

(b) **Kelvin Functions**

$$ber(x) = \sum_{n=0}^{n=m-1} (-1)^n \frac{(x/2)^{4n}}{\left[(2n)!\right]^2} = 1 - \frac{(x/2)^4}{(2!)^2} + \frac{(x/2)^8}{(4!)^2} - \frac{(x/2)^{12}}{(6!)^2} + \ldots \approx 1 - \frac{x^4}{2^6} \quad \text{for x close to 0}$$

$$bei(x) = \sum_{n=1}^{n=m} (-1)^{n-1} \frac{(x/2)^{4n-2}}{\left[(2n-1)!\right]^2} = \frac{(x/2)^2}{(1!)^2} - \frac{(x/2)^6}{(3!)^2} + \frac{(x/2)^{10}}{(5!)^2} - \ldots \approx \frac{x^2}{2^2} \quad \text{for x close to 0}$$

$$ber'(x) = \sum_{n=1}^{n=m} (-1)^n \frac{(2n)(x/2)^{4n-1}}{\left[(2n)!\right]^2} = -\frac{2(x/2)^3}{(2!)^2} + \frac{4(x/2)^7}{(4!)^2} - \frac{6(x/2)^{11}}{(6!)^2} + \ldots \approx -\frac{x^3}{2^4}$$

for x close to 0

$$bei'(x) = \sum_{n=1}^{n=m} (-1)^{n-1} \frac{(2n-1)(x/2)^{4n-3}}{\left[(2n-1)!\right]^2} = \frac{(x/2)^1}{(1!)^2} - \frac{3(x/2)^5}{(3!)^2} + \frac{5(x/2)^9}{(5!)^2} - \ldots \approx \frac{x}{2}$$

for x close to 0

$$ber''(x) = \sum_{n=1}^{n=m} (-1)^n \frac{(n)(4n-1)(x/2)^{4n-2}}{\left[(2n)!\right]^2} = -\frac{3(x/2)^2}{(2!)^2} + \frac{14(x/2)^6}{(4!)^2} - \frac{33(x/2)^{10}}{(6!)^2} + \ldots \approx -\frac{3x^2}{2^4}$$

for x close to 0

$$bei''(x) = \sum_{n=1}^{n=m} (-1)^{n-1} \frac{(2n-1)(4n-3)(x/2)^{4n-4}}{2\left[(2n-1)!\right]^2} = \frac{1}{2} - \frac{15(x/2)^4}{2(3!)^2} + \frac{45(x/2)^8}{2(5!)^2} - \ldots \approx \frac{1}{2}$$

for x close to 0

(c) **Coefficients $\Psi_1(x)$, $\Psi_2(x)$, Z_a**

$$\psi_1(x) = bei(x) + \frac{1-\upsilon^*}{x} ber'(x) \approx \frac{3+\upsilon^*}{2^4} x^2 \quad \text{for x close to 0}$$

$$\Psi_1(X_a) \approx \frac{3+\upsilon^*}{2^4} X_a{}^2 \quad \text{for } X_a \text{ close to 0}$$

$$\psi_2(x) = ber(x) - \frac{1-\nu^*}{x} bei'(x) \quad \approx \frac{1+\upsilon^*}{2} \quad \text{for x close to 0}$$

$$\Psi_2(X_a) \approx \frac{1+\upsilon^*}{2} \quad \text{for } X_a \text{ close to 0}$$

$$Z_a = bei' \cdot \Psi_2 - ber' \cdot \Psi_1 = \left(ber \cdot bei' - ber' \cdot bei\right) - \frac{1-\upsilon^*}{X_a}\left(ber'^2 + bei'^2\right)$$

$$Z_a \approx \frac{1+\upsilon^*}{4} X_a \quad \text{for } X_a \text{ close to 0}$$

(d) **Coefficients $Z_d(x)$, $Z_v(x)$, $Z_w(x)$, $Z_m(x)$**

$$Z_d(x) = \frac{\psi_2 \cdot ber(x) + \psi_1 \cdot bei(x)}{X_a^3 Z_a} = \frac{\left(ber \cdot ber(x) + bei \cdot bei(x)\right) + \frac{1-\upsilon^*}{X_a}\left(ber' \cdot bei(x) - bei' \cdot ber(x)\right)}{X_a^3 Z_a}$$

$$\approx \frac{4\Psi_2 + \Psi_1 x^2}{4X_a^2 Z_a} \approx \frac{\Psi_2}{X_a^3 Z_a} \quad \text{for x close to 0} \qquad Z_d(0) = \frac{\Psi_2}{X_a^3 Z_a}$$

$$Z_d = \frac{\psi_2 \cdot ber + \psi_1 \cdot bei}{X_a^3 Z_a} = \frac{\left(ber^2 + bei^2\right) + \frac{1-\upsilon^*}{X_a}\left(ber' \cdot bei - ber \cdot bei'\right)}{X_a^3 Z_a} \approx \frac{2}{X_a^4} \quad \text{for } X_a \text{ close to 0}$$

$$Z_v(x) = \frac{\psi_2 \cdot ber'(x) + \psi_1 \cdot bei'(x)}{X_a^2 Z_a} = \frac{\left(ber \cdot ber'(x) + bei \cdot bei'(x)\right) + \frac{1-\upsilon^*}{X_a}\left(ber' \cdot bei'(x) - bei' \cdot ber'(x)\right)}{X_a^2 Z_a}$$

$$\approx \frac{8\Psi_1 x - \Psi_2 x^3}{16X_a^2 Z_a} \approx \frac{\Psi_1}{2X_a^2 Z_a} x \quad \text{for x close to 0} \qquad Z_v(0) = 0$$

$$Z_v = \frac{\psi_2 \cdot ber' + \psi_1 \cdot bei'}{X_a^2 Z_a} = \frac{ber \cdot ber' + bei \cdot bei'}{X_a^2 Z_a} \approx \frac{1}{4\left(1+\upsilon^*\right)} \quad \text{for } X_a \text{ close to 0}$$

$$Z_w(x) = \frac{ber' \cdot ber(x) + bei' \cdot bei(x)}{X_a^2 Z_a}$$

$$\approx \frac{4ber' + x^2 bei'}{4X_a^2 Z_a} \approx \frac{ber'}{X_a^2 Z_a} \quad \text{for x close to 0} \qquad Z_w(0) = \frac{ber'}{X_a^2 Z_a}$$

$$Z_w = \frac{ber \cdot ber' + bei \cdot bei'}{X_a^2 Z_a} = Z_v \approx \frac{1}{4\left(1+\upsilon^*\right)} \quad \text{for } X_a \text{ close to 0}$$

$$Z_m(x) = \frac{ber' \cdot ber'(x) + bei' \cdot bei'(x)}{X_a Z_a}$$

$$\approx \frac{8x\,bei' - x^3 ber'}{16X_a Z_a} \approx \frac{x\,bei'}{2X_a Z_a} = \frac{x}{\left(1+\upsilon^*\right)X_a} \quad \text{for x close to 0} \qquad Z_m(0) = 0$$

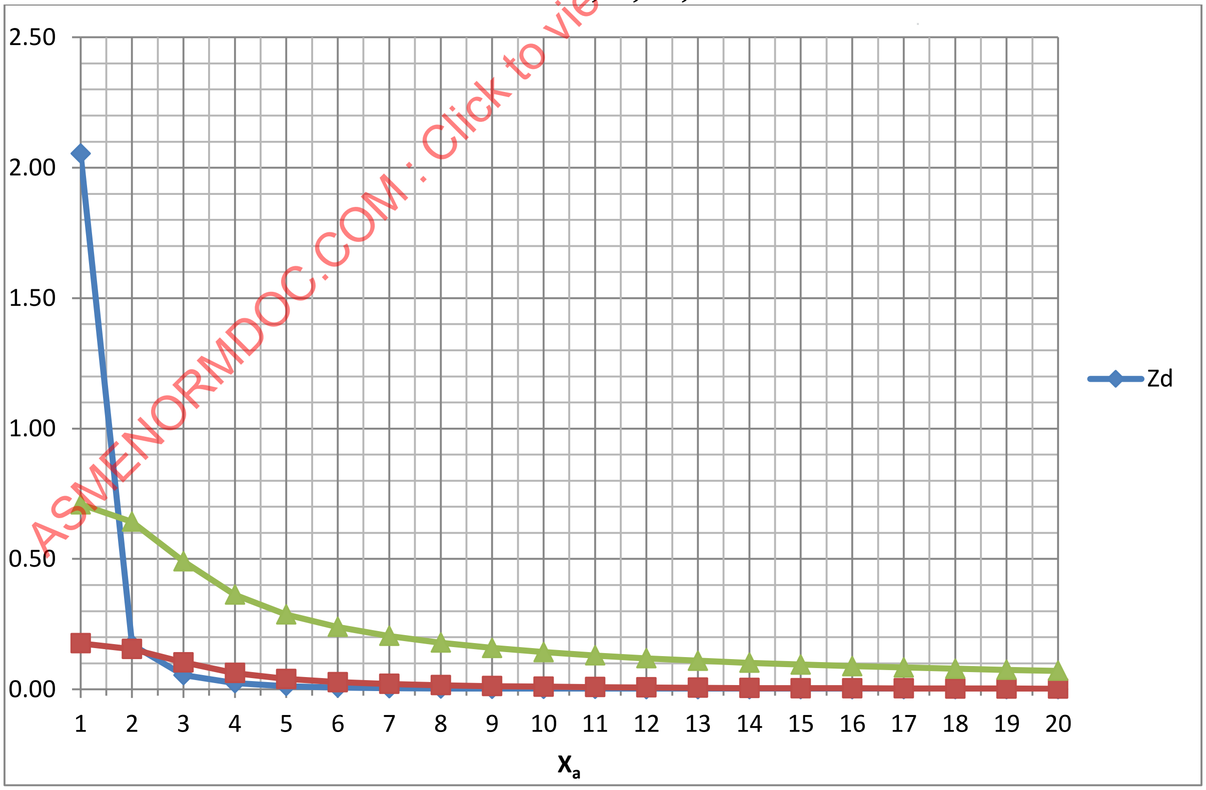
$$Z_m = \frac{ber'^2 + bei'^2}{X_a Z_a} \approx \frac{1}{1+\upsilon^*} \quad \text{for } X_a \text{ close to 0}$$

The numerical values and graph of Z_d, Z_v, Z_w, Z_m as a function of X_a for $1 \le X_a \le 20$ are provided below.

Coefficients Z_d, Z_v, Z_w, Z_m

Xa	Z_d	$Z_v=Z_w$	Z_m
1	2.054458	0.176798	0.709032
2	0.171868	0.154393	0.642542
3	0.054471	0.103768	0.491113
4	0.023791	0.062263	0.362879
5	0.012079	0.039878	0.286907
6	0.006924	0.027776	0.238824
7	0.004334	0.020476	0.204651
8	0.002889	0.015704	0.178926
9	0.002021	0.012418	0.158913
10	0.001468	0.010062	0.142915
11	0.001100	0.008317	0.129837
12	0.000845	0.006988	0.118947
13	0.000663	0.005954	0.109738
14	0.000530	0.005133	0.101851
15	0.000430	0.004471	0.095019
16	0.000354	0.003929	0.089046
17	0.000295	0.003480	0.083778
18	0.000248	0.003103	0.079098
19	0.000211	0.002785	0.074912
20	0.000180	0.002513	0.071147

Table Zd,Zv,Zw,Zm



(e) **Coefficients $Q_m(x)$, $Q_v(x)$ and $Q_\alpha(x)$, $Q_\beta(x)$**

$$Q_m(x) = \frac{bei' \cdot \psi_2(x) - ber' \cdot \psi_1(x)}{Z_a} = \frac{\left(bei' \cdot ber(x) - ber' \cdot bei(x)\right) - \frac{1-\upsilon^*}{X_a}\left(ber' \cdot ber'(x) + bei' \cdot bei'(x)\right)}{Z_a}$$

$$\approx \frac{8\left(1+\upsilon^*\right)bei' - (3+\upsilon^*)x^2 ber'}{16 Z_a} \approx \frac{\left(1+\upsilon^*\right)bei'}{2Z_a} \quad \text{for x close to 0} \qquad Q_m(0) = \frac{\left(1+\upsilon^*\right)}{2Z_a} bei'$$

$$Q_m = \frac{bei' \cdot \psi_2 - ber' \cdot \psi_1}{Z_a} = \frac{\left(ber \cdot bei' - ber' \cdot bei\right) - \frac{1-\upsilon^*}{X_a}\left(ber'^2 + bei'^2\right)}{Z_a} = \frac{Z_a}{Z_a} = 1$$

$$Q_v(x) = \frac{\psi_1 \cdot \psi_2(x) - \psi_2 \cdot \psi_1(x)}{X_a Z_a} = \frac{ber \cdot \psi_2(x) - bei \cdot \psi_1(x) + \frac{1-\upsilon^*}{X_a}\left(ber' \cdot \Psi_2(x) - bei' \cdot \Psi_1(x)\right)}{X_a Z_a}$$

$$\approx \frac{8\left(1+\upsilon^*\right)\Psi_1 - \left(3+\upsilon^*\right)\Psi_2 x^2}{16 X_a Z_a} \approx \frac{\left(1+\upsilon^*\right)\Psi_1}{2X_a Z_a} \quad \text{for x close to 0} \qquad Q_v(0) = \frac{\left(1+\upsilon^*\right)}{2X_a Z_a}\Psi_1$$

$$Q_v = Q_v(X_a) = \frac{\psi_1 \cdot \psi_2 - \psi_2 \cdot \psi_1}{X_a Z_a} = 0$$

$$Q_\alpha(x) = \frac{ber' \cdot bei'(x) - bei' \cdot ber'(x)}{Z_a}$$

$$\approx \frac{8x ber' + x^3 bei'}{16 Z_a} = \frac{ber'}{2Z_a}x \quad \text{for x close to 0} \qquad Q_\alpha(0) = 0$$

$$Q_\alpha = Q_\alpha(X_a) = \frac{ber' \cdot bei' - bei' \cdot ber'}{Z_a} = 0$$

$$Q_\beta(x) = \frac{\psi_2 \cdot bei'(x) - \psi_1 \cdot ber'(x)}{Z_a} = \frac{\left(ber \cdot bei'(x) - bei \cdot ber'(x)\right) + \frac{1-\upsilon^*}{X_a}\left(ber' \cdot ber'(x) - bei' \cdot bei'(x)\right)}{Z_a}$$

$$\approx \frac{8\Psi_2 x + \Psi_1 x^3}{16 Z_a} \approx \frac{\Psi_2}{2Z_a}x \quad \text{for x close to 0} \qquad Q_\beta(0) = 0$$

$$Q_\beta = Q_\beta(X_a) = \frac{\psi_2 \cdot bei' - \psi_1 \cdot ber'}{Z_a} = \frac{Z_a}{Z_a} = 1$$

(f) **Coefficients $F_m(x)$ and $F_t(x)$**

$$F_m(x) = \frac{Q_3 \cdot Q_m(x) + Q_v(x)}{2} = \frac{\left(\Psi_1 + Q_3 X_a bei'\right)\Psi_2(x) - \left(\Psi_2 + Q_3 X_a ber'\right)\Psi_1(x)}{2X_a Z_a}$$

$$F_m(0) = \frac{Q_3\, Q_m(0) + Q_v(0)}{2} = \frac{1}{2}\left[Q_3 \frac{1+\upsilon^*}{2Z_a} bei' + \frac{1+\upsilon^*}{2Z_a\, X_a}\Psi_1\right] = \frac{1+\upsilon^*}{4Z_a}\left[Q_3\, bei' + \frac{\Psi_1}{X_a}\right] = \frac{Q_3}{2} + \frac{3+\upsilon^*}{16}$$

$$F_m(X_a) = \frac{Q_3 Q_m(X_a) + Q_v(X_a)}{2} = \frac{Q_3}{2}$$

Note - When F_m is maximum at periphery:

$$F_m = MAX = \left[\left|F_m(x)\right|\right] = \left|F_m(X_a)\right| = \frac{Q_3}{2} = \frac{Q_3}{2} \ \ if\ Q_3 > 0$$

$$\frac{-Q_3}{2} \ \ if\ Q_3 < 0$$

$$F_Q(x) = Q_3 \cdot Q_\alpha(x) + Q_\beta(x) = \frac{(\Psi_2 + Q_3\, ber')bei'(x) - (\Psi_1 + Q_3\, bei')ber'(x)}{Z_a}$$

$$F_Q(0) = Q_3\, Q_\alpha(0) + Q_\beta(0) = \frac{Q_3\, ber' + \Psi_2}{2Z_a} x \quad \text{for x close to 0} \quad \rightarrow F_Q(0) = 0$$

$$F_Q(X_a) = Q_3 \cdot Q_\alpha(X_a) + Q_\beta(X_a) = 1$$

$$F_t(x) = [Q_3 \cdot Z_w(x) + Z_d(x)]\frac{X_a^4}{2} = \frac{(\Psi_1 + Q_3 X_a bei')bei(x) + (\Psi_2 + Q_3 X_a ber')ber(x))}{2Z_a} X_a$$

$$F_t(0) = \frac{Q_3\, Q_m(0) + Q_v(0)}{2} = \frac{X_a^{\ 4}}{2}\left[Q_3 \frac{ber'}{X_a^{\ 2}\, Z_a} + \frac{\Psi_2}{X_a^3\, Z_a}\right] = \frac{X_a^{\ 2}}{2Z_a}\left[Q_3\, ber' + \frac{\Psi_2}{X_a}\right]$$

$$F_t(X_a) = \left[\frac{Q_3}{4(1+\upsilon^*)} + \frac{2}{X_a^{\ 4}}\right]\frac{X_a^{\ 4}}{2} = \frac{Q_3}{1+\upsilon^*}\frac{X_a^{\ 4}}{8} + 1 = 1$$

(g) **Parameter Q_3 when X_a=0**

$$Q_1 = \frac{(\rho_s - 1) - \Phi\, Z_v}{1 + \Phi\, Z_m} = \frac{1}{4}\frac{(\rho_s - 1)4(1+\upsilon^*) - \Phi}{(1+\upsilon^*) + \Phi} = \frac{(\rho_s - 1) - \frac{F}{4}}{1+F}$$

$$Q_2 = \frac{\overbrace{(\omega_s^*\, P_s - \omega_c^*\, P_c) + \left(W^* \frac{\gamma_b}{2\pi}\right)}^{N}}{1 + \Phi\, Z_m} = \frac{N}{1+F}$$

$$Q_3 = Q_1 + \frac{2}{a_0^2\, P_e} Q_2 = \frac{(\rho_s - 1) - \frac{F}{4}}{1+F} + \frac{2}{a_0^2\, P_e}\frac{N}{1+F} = \boxed{\frac{(\rho_s - 1) + \frac{2N}{a_0^2\, P_e} - \frac{F}{4}}{1+F}}$$

ANNEX H — RADIAL DISPLACEMENT AND ROTATION OF THE SHELL AT ITS CONNECTION WITH THE RING

1 Radial Displacement Due to Internal Pressure P_s

Due to internal pressure P_s, the circumference of the shell internal wall increases from $2\pi a_s$ to $2\pi a'_s$, thus its elongation is written: $2\pi(a'_s - a_s) = 2\pi\, w_s(P_s)$ where $w_s(P_s)$ is the radial displacement of the shell due to pressure P_s.

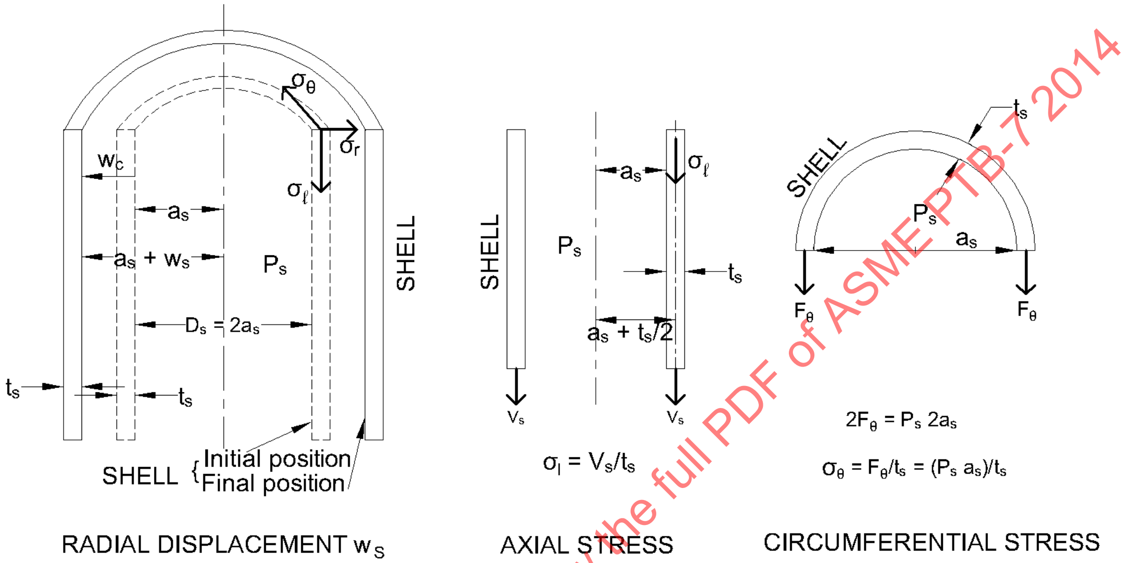


Figure 57 — Radial Displacement due to Internal Pressure

Shell strain: $\varepsilon_s(P_s) = \dfrac{2\pi w_s(P_s)}{2\pi a_s} = \dfrac{w_s(P_s)}{a_s} = \dfrac{1}{E_s}\left[\sigma_\theta - \nu_s(\sigma_l + \sigma_r)\right] = \dfrac{1}{E_s}\left[\dfrac{a_s P_s}{t_s} - \nu_s \dfrac{V_s}{t_s}\right]$

where: $\sigma_\theta = \dfrac{a_s P_s}{t_s}$ is the circumferential stress in the shell

$\sigma_l = \dfrac{V_s}{t_s}$ is the longitudinal stress in the shell subjected to the longitudinal force V_s

$\sigma_r = -P_s$ is the radial stress in the shell, which can be neglected compared to σ_θ.

Shell radial displacement due to internal pressure:

$$\boxed{w_s(P_s) = \frac{a_s^2}{E_s\, t_s}\left[P_s - \frac{V_s}{a_s}\nu_s\right]} \quad \text{[A-VI.1a-1]}$$

Note: For a U-tube or floating TS HE, V_s is known:

$$2\pi a'_s V_s = \pi a_s^2 P_s \quad \Rightarrow \quad V_s = \frac{a_s^2 P_s}{2a'_s} \quad \Rightarrow \quad \sigma_l = \frac{a_s^2 P_s}{2a'_s t_s}$$

$$w_s(P_s) = \frac{a_s^2}{E_s t_s}\left[P_s - \frac{a_s P_s}{a'_s}\frac{\nu_s}{2}\right] = \frac{a_s^2}{E_s t_s}\left(1 - \frac{a_s}{a'_s}\frac{\nu_s}{2}\right)P_s = \delta_s P_s$$

where: $\delta_s = \frac{a_s^2}{E_s\, t_s}\left(1 - \frac{a_s}{a_s^{'}}\frac{\nu_s}{2}\right) = \frac{D_s^2}{4\, E_s\, t_s}\left(1 - \frac{D_s}{D_s + t_s}\frac{\nu_s}{2}\right)$

2 Radial Displacement and Rotation Due to Edge Loads Q_s and M_s

Formulas provided in this Annex are taken from Appendix 4.2 of ASME Section VIII-Div. 2 [11] .

$$w_s\left(Q_s\,, M_s\right) = \frac{Q_s}{2\,\beta_s^3\, D_s} + \frac{M_s}{2\,\beta_s^2\, D_s} \qquad \theta_s\left(Q_s\,, M_s\right) = \frac{Q_s}{2\,\beta_s^2\, D_s} + \frac{M_s}{2\,\beta_s\, D_s}$$

$$\beta_s = \frac{\sqrt[4]{12\left(1-\nu_s^2\right)}}{\sqrt{\left(D_s + t_s\right)t_s}} \qquad D_s = \frac{E_s\, t_s^3}{12\left(1-\nu_s^2\right)} \qquad k_s = 2\,\beta_s\, D_s = \beta_s\,\frac{E_s\, t_s^3}{6\left(1-\nu_s^2\right)}$$

$$w_s\left(Q_s\,, M_S\right) = \frac{Q_s}{\beta_s^2\, k_s} + \frac{M_s}{k_s} \qquad \theta_s\left(Q_s\,, M_s\right) = \frac{Q_s}{\beta_s\, k_s} + \frac{2\, M_s}{k_s}$$

3 Radial Displacement Due to Internal Pressure and Edge Loads

$$w_s = \frac{Q_s}{\beta_s^2\, k_s} + \frac{M_s}{\beta_s\, k_s} + \delta_s\, P_s \qquad \theta_s = \frac{Q_s}{\beta_s\, k_s} + \frac{2\, M_s}{k_s}$$

With: $\delta_s = \frac{a_s^2}{E_s\, t_s}\left(1 - \frac{a_s}{a_s^{'}}\frac{\nu_s}{2}\right)$ where V_s is unknown

At this point of the development, it appears that it is not possible to get a solution if V_s is unknown. Thus, the classical shell formula:

$$\sigma_l = \frac{a_s^2 P_s}{2(a_s + t_s/2)t_s} = \frac{D_s}{D_s + t_s}\frac{P_s D_s}{2t_s}$$ is used, which leads to:

$$\delta_s = \frac{D_s^2}{4\, E_s\, t_s}\left(1 - \frac{D_s}{D_s + t_s}\frac{\nu_s}{2}\right)P_s$$

Note: In UHX-13, t_s has been neglected compared to D_s and: $\delta_s = \frac{D_s^2}{4\, E_s\, t_s}\left(1 - \frac{\nu_s}{2}\right)P_s$

4 Channel

Above formulas apply to the channel by replacing subscript s by subscript c, except that the axial force and longitudinal stress in the channel are known:

$V_c = \frac{a_c\, P_c}{2}$ $\qquad \sigma_l = \frac{a_c\, P_c}{2\, t_c}$ which leads to: $\delta_c = \frac{D_c^2}{4\, E_c\, t_c}\left[1 - \frac{D_c}{D_c + t_c}\frac{\upsilon_c}{2}\right]$

Note: In UHX-13, t_c has been neglected compared to D_c and: $\delta_c = \frac{D_c^2}{4\, E_c\, t_c}\left[1 - \frac{\nu_c}{2}\right]$

ANNEX I — SHELL-TO-RING CONNECTION IN RADIAL DIRECTION

Radial force $Q_r(a_o)$ acting at TS periphery, due to Q_s and Q_c (see Figure 58 below): $Q_r(a_o) = Q_s + Q_c$

TS radial displacement $w_r(a_o)$ at mid-thickness, at point A, due to $Q_r(a_o)$: $w_r(a_o) = \frac{a_o}{h}\frac{1-\nu^*}{E^*}Q_R$

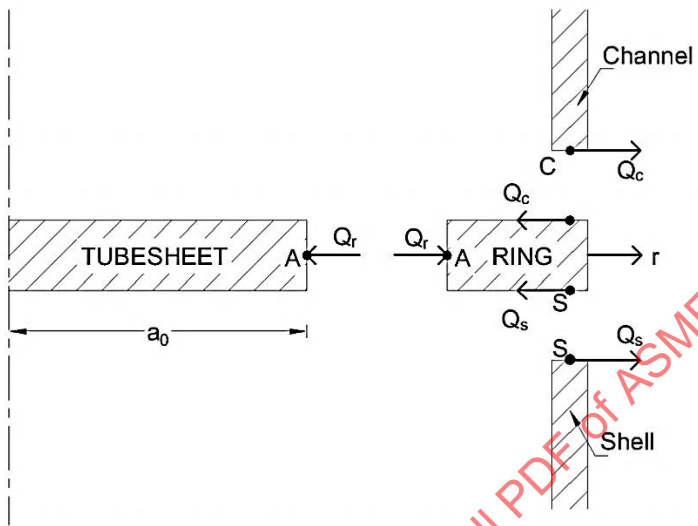


Figure 58 — Radial Force at Tubesheet Periphery

Ring radial displacement w_R at point S (see Figure 59 below):

$$t_g(\theta_R) = \frac{-w_R}{h/2} \approx \theta_R \quad \Rightarrow \quad w_R = -\frac{h}{2}\theta_R$$

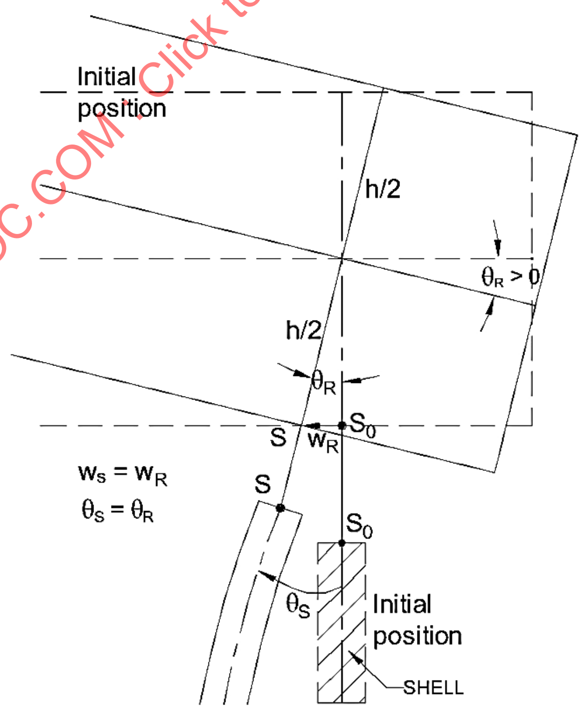


Figure 59 — Ring Radial Displacement

Continuity of displacements at ring-shell connection (point S):

$$w_R = w_s \qquad \theta_R = \theta_s \quad \Rightarrow \quad w_s = -\frac{h}{2}\theta_s$$

Total radial displacement w_T of the ring at point S: $w_T = w_r(a_o) + w_R = w_r(a_o) + w_s$

At this point of the development, it appears that it is necessary to ignore the radial displacement at mid thickness $w_r(a_o)$ to get a solution.

Compatibility of shell-ring displacements is written: $\boxed{w_s = -\frac{h}{2}\theta_s}$

ANNEX J — MINIMUM LENGTH OF SHELL AND CHANNEL WHEN INTEGRAL WITH THE TS

The bending moment M(x) located at the distance x of the edge of the shell (point S in Figure 17), submitted to moment M_s is given by (see Appendix 4.2 of [11]):

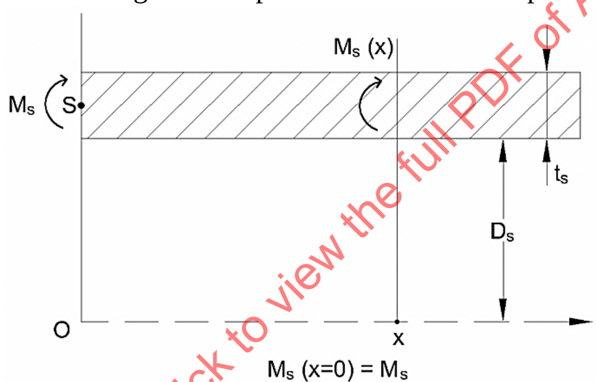
$$M_s(x) = \frac{Q_s}{\beta_s} f_4(\beta x) + M_s f_3(\beta x) \qquad \text{with} \quad \beta_s = \frac{\sqrt[4]{12\left(1-\nu_s^2\right)}}{\sqrt{\underbrace{\left(D_s + t_s\right)}_{D_{s,m}} t_s}} = \frac{1.817}{\sqrt{\left(D_{s,m}\right) t_s}} \qquad for\ \nu = 0.3$$

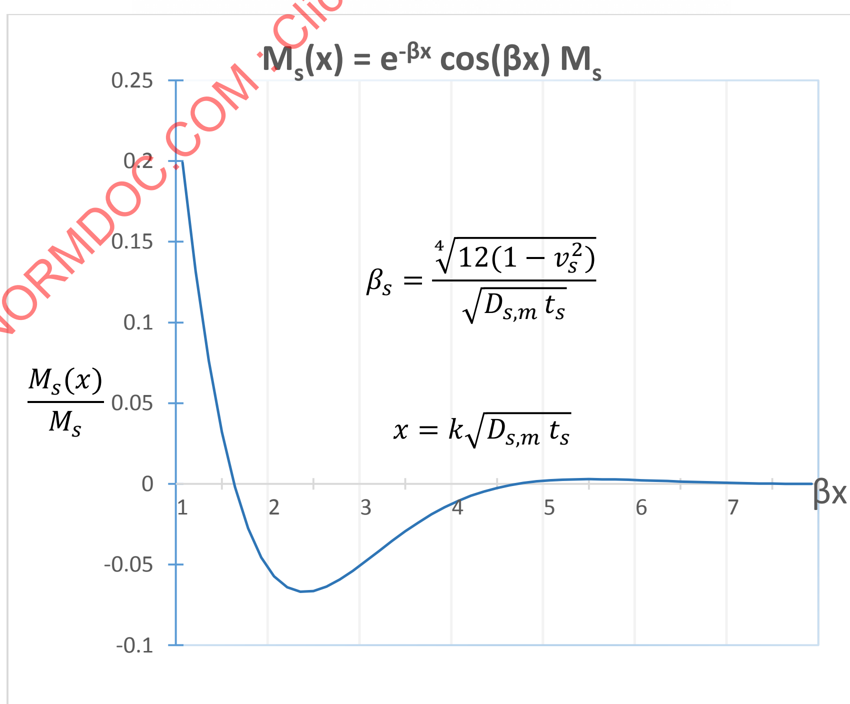
Neglecting the radial displacement of the shell submitted to edge loads M_s and Q_s, given by Section H.2 of Annex H, the following relationship is obtained: $Q_s = -\beta_s M_s$ which leads to:

$$M_s(x) = \left[f_3(\beta x) - f_4(\beta x) \right] M_s \qquad with: \quad \left. \begin{aligned} f_3(\beta x) &= e^{-\beta x}\left[\cos(\beta x) + \sin(\beta x)\right] \\ f_4(\beta x) &= e^{-\beta x}\left[\sin(\beta x)\right] \end{aligned} \right\} f_3(\beta x) - f_4(\beta x) = e^{-\beta x}\cos(\beta x)$$

which leads to: $M_s(x) = e^{-\beta x} \cos(\beta x)\, M_s$

The moment $M_s(x)$ decreases according to a damped sinusoidal curve represented on figure below.





The length x can be written as: $x = k\sqrt{D_{s,m}\, t} \quad \Rightarrow \quad \beta x = \beta k \sqrt{D_{s,m}\, t} = 1.817\, k$

For $\text{k} = 1.0 \quad x = 1.0\sqrt{D_{s,m}\, t_s} \quad \text{M}_s(x) = 0.040\,\text{M}_s$

For $\text{k} = 1.4 \quad x = 1.4\sqrt{D_{s,m}\, t} \quad \text{M}_s(x) = 0.065\,\text{M}_s$

For $\text{k} = 1.8 \quad x = 1.8\sqrt{D_{s,m}\, t} \quad \text{M}_s(x) = 0.040\,\text{M}_s$

For $\text{k} = 2.0 \quad x = 2.0\sqrt{D_{s,m}\, t} \quad \text{M}_s(x) = 0.020\,\text{M}_s$

For $\text{k} = 2.5 \quad x = 2.5\sqrt{D_{s,m}\, t} \quad \text{M}_s(x) = 0.002\,\text{M}_s$

The value $x = 1.8\sqrt{D_m\, t}$ has been retained as the remaining bending moment at this distance of the shell edge is only 4%.

Therefore, when they are integral with the TS, the shell and the channel must have a minimum length adjacent to the TS of $l_{s,\min} = 1.8\sqrt{D_s\, t_s}$ and $l_{c,\min} = 1.8\sqrt{D_c\, t_c}$.

ANNEX K — FORMULAS FOR A HEMISPHERICAL CHANNEL WHEN INTEGRAL WITH THE TS

The formulas given in Annex H are valid for a cylindrical channel. If the channel is hemispherical and attached directly to the TS (configurations a, b or c), without any cylindrical section between the head and the TS (Figure 60), similar formulas can be developed, based on Appendix 4.3 of Section VIII-Div. 2 [11] .

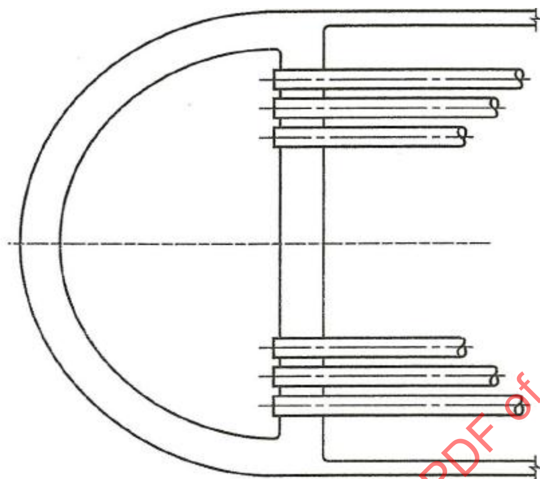


Figure 60 — Hemispherical Head

1 Radial Displacement Due to Internal Pressure P_c

Channel strain:

$$\varepsilon_c\left(P_c\right)=\frac{2\pi w_c\left(P_c\right)}{2\pi a_c}=\frac{w_c\left(P_c\right)}{a_c}=\frac{1}{E_c}\left[\sigma_\theta-\nu_c\left(\sigma_l+\sigma_r\right)\right]=\frac{1}{E_c}\left[\frac{a_c\,P_c}{2t_c}-\nu_c\,\frac{a_c^2P_c}{2(a_c+t_c/2)t_c}\right]$$

where: $\sigma_\theta=\dfrac{a_c\,P_c}{t_c}$ is the circumferential stress in the channel

$\sigma_l=\dfrac{a_c^2P_c}{2(a_c+t_c/2)t_c}$ is the longitudinal stress in the channel

$\sigma_r=-P_c$ is the radial stress in the channel, which can be neglected compared to σ_θ.

Channel radial displacement due to internal pressure:

$$w_c\left(P_c\right)=\underbrace{\frac{a_c^2P_c}{E_c\,t_c}\left[\frac{1-\frac{a_c}{a'_c}\nu_c}{2}\right]}_{\delta_c}=\delta_c\,P_c \qquad \text{where:}\quad \delta_c=\frac{a_c^2}{E_c\,t_c}\left[\frac{1-\frac{a_c}{a'_c}\nu_c}{2}\right]=\frac{D_c^2}{4\,E_c\,t_c}\left[\frac{1-\frac{D_c}{D_c+t_c}\nu_c}{2}\right]$$

2 Radial Displacement and Rotation Due to Edge Loads Q_s and M_s

Radial displacement w_c and rotation θ_c formulas are taken from Clause 4-332(b) of Appendix 4.3 of Section VIII Div. 2 [11] :

$$w_c = \frac{2\beta_c R_m^2}{E_c t_c} Q_c + \frac{2\beta_c^2 R_m^2}{E_c t_c} M_c \qquad \theta_c = \frac{2\beta_c^2 R_m^2}{E_c t_c} Q_c + \frac{4\beta_c^3 R_m^2}{E_c t_c} M_c \qquad with: R_m = \frac{D_c + t_c}{2}$$

$$\frac{2\beta_c R_m^2}{E_c t_c} = \frac{1}{2\beta_c^3} \frac{4\beta_c^4 R_m^2}{E_c t_c} = \frac{1}{2\beta_c^3} \frac{12(1-\upsilon_c^2)}{E_c t_c^3} = \frac{1}{2\beta_c^3} \frac{1}{D_c} = \frac{1}{2\beta_c^3 D_c} = \frac{1}{\beta_c^2 k_c}$$

$$\frac{2\beta_c^2 R_m^2}{E_c t_c} = \beta_c \frac{2\beta_c R_m^2}{E_c t_c} = \frac{1}{2\beta_c^2 D_c} = \frac{1}{\beta_c k_c}$$

$$\frac{4\beta_c^3 R_m^2}{E_c t_c} = 2\beta_c \frac{2\beta_c^2 R_m^2}{E_c t_c} = 2\beta_c \frac{1}{2\beta_c^2 D_c} = \frac{1}{\beta_c D_c} = \frac{2}{k_c}$$

$$w_c = \frac{Q_c}{2\beta_c^3 D_c} + \frac{M_c}{2\beta_c^2 D_c} = \frac{Q_c}{\beta_c^2 k_c} + \frac{M_c}{\beta_c k_c} \qquad \theta_c = \frac{Q_c}{2\beta_c^2 D_c} + \frac{M_c}{\beta_c D_c} = \frac{Q_c}{\beta_c k_c} + \frac{2M_c}{k_c}$$

These formulas are the same as for a cylindrical channel (see Section 4 of Annex H)

3 Radial Displacement Due to Internal Pressure and Edge Loads

$$\boxed{w_c = \frac{Q_c}{\beta_c^2 k_c} + \frac{M_c}{\beta_c k_c} + \delta_c P_c} \qquad \boxed{\theta_c = \frac{Q_c}{\beta_c k_c} + \frac{2M_c}{k_c}} \qquad \text{with: } \boxed{\delta_c = \frac{D_c^2}{4E_c t_c}\left(\frac{1}{2} - \frac{D_c}{D_c + t_c}\frac{\nu_c}{2}\right)}$$

Accordingly, the formulas are the same as for a cylindrical channel, provided in Section 4 of Annex H,

$$\delta_s = \frac{D_c^2}{4E_c t_c}\left(1 - \frac{D_c}{D_c + t_c}\frac{\nu_c}{2}\right) P_c \quad \text{is replaced by} \quad \boxed{\delta_c = \frac{D_c^2}{4E_c t_c}\left(\frac{1}{2} - \frac{D_c}{D_c + t_c}\frac{\nu_c}{2}\right)}$$

Note: In UHX-13, t_c has been neglected compared to D_c: $$\boxed{\delta_c = \frac{D_c^2}{4E_c t_c}\left[\frac{1}{2} - \frac{\nu_c}{2}\right]}$$

ANNEX L — EQUILIBRIUM OF RING SUBJECTED TO EDGE MOMENTS

(a) **For configuration a**

Axial equilibrium of forces

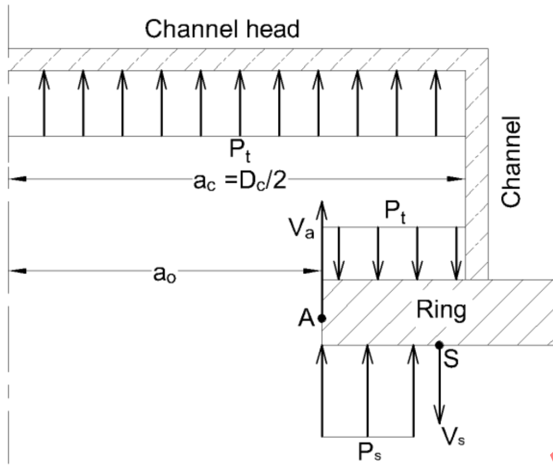


Figure 61 — Configuration a

Axial force V_s in the shell:

[VI.2a-1] $$2\,\pi\,a_s'\,V_s + \pi\left(a_c^2 - a_o^2\right)P_c = 2\,\pi\,a_c'\,V_c + 2\,\pi\,a_o\,V_a + \pi\left(a_s^2 - a_o^2\right)P_s$$

Axial force V_c in the channel:

[VI.2a-2] $$2\,\pi\,a_c'\,V_c = \pi\,a_c^2\,P_c \quad \Rightarrow \quad a_c'\,V_c = \frac{a_c^2}{2}\,P_c$$

Axial equilibrium of the ring is written: [VI.2a-3] $$\boxed{a_s'\,V_s = a_o\,V_a + \frac{a_o^2}{2}\,P_c + \frac{a_s^2 - a_o^2}{2}\,P_s}$$

Axial equilibrium of moments

[VI.1a] $$\begin{cases} M_s = +\left[k_s\left(1+\dfrac{t_s'}{2}\right)\right]\theta_s + \left[\beta_s\,k_s\right]\delta_s\,P_s \\ Q_s = -\left[\beta_s\,k_s\left(1+t_s'\right)\right]\theta_s - \left[2\,\beta_s^2\,k_s\right]\delta_s\,P_s \end{cases}$$

[VI.1b] $$\begin{cases} M_c = +\left[k_c\left(1+\dfrac{t_c'}{2}\right)\right]\theta_c + \left[\beta_c\,k_c\right]\delta_c\,P_c \\ Q_c = -\left[\beta_c\,k_c\left(1+t_c'\right)\right]\theta_c - \left[2\beta_c^2\,k_c\right]\delta_c\,P_c \end{cases}$$

From Figure 17 and Figure 61, ring equilibrium is written:

$$\left[R\,M_R\right]_a = -\overbrace{a_o\,M_a}^{A0} + \overbrace{\left[a_c'\,M_c - a_c'\,Q_c\,\frac{h}{2}\right]}^{A1c} + \overbrace{\left[M\left(P_c\right) - a_c'\,V_c\left(a_c' - a_o\right)\right]}^{A2c}$$
$$- \underbrace{\left[a_s'\,M_s - a_s'\,Q_s\,\frac{h}{2}\right]}_{A1s} - \underbrace{\left[M\left(P_s\right) - a_s'\,V_s\left(a_s' - a_o\right)\right]}_{A2s} \qquad \text{[VI.2b]}$$

where: R = radius at center of ring $= \dfrac{A+2\,a_0}{4}$

$M(P_c) =$ moment due to pressure P_c acting on the ring: $\left(a_c^2-a_o^2\right)\left(\dfrac{a_c+a_o}{2}-a_o\right)\dfrac{P_c}{2}$

$M(P_s) =$ moment due to pressure P_s acting on the ring: $\left(a_s^2-a_o^2\right)\left(\dfrac{a_s+a_o}{2}-a_o\right)\dfrac{P_s}{2}$

Derivations are performed so that the moment M_R can be calculated from V_a, M_a, θa and P_s, P_c.

$$A1c=\left[a_c^{'}\,k_c\left(1+\frac{t_c^{'}}{2}\right)\theta_c+a_c^{'}\,k_c\,\beta_c\,\delta_c\,P_c\right]+\left[a_c^{'}\,\beta_c\,k_c\left(1+t_c^{'}\right)\frac{h}{2}\theta_c+2\,a_c^{'}\,k_c\,\beta_c^2\,\delta_c\,\frac{h}{2}P_c\right]$$

$$=a_c^{'}\,k_c\left[\left(1+\frac{t_c^{'}}{2}\right)+\beta_c\left(1+t_c^{'}\right)\frac{h}{2}\right]\theta_c+a_c^{'}\,k_c\,\beta_c\,\delta_c\left(1+2\,\beta_c\,\frac{h}{2}\right)P_c$$

$$=\;a_c^{'}\,k_c\left(1+t_c^{'}+\frac{t_c^{'2}}{2}\right)\theta_c+a_0\,\rho_c\,\beta_c\,k_c\,\delta_c\left(1+\beta_c\,h\right)P_c$$

$$=\;a_c^{'}\,k_c\left(1+t_c^{'}+\frac{t_c^{'2}}{2}\right)\theta_c+a_o\,\omega_c\,P_c \qquad \boxed{t_c^{'}=h\,\beta_c} \qquad \boxed{\omega_c=\rho_c\,\beta_c\,k_c\,\delta_c\left(1+h\,\beta_c\right)}$$

$$A1s=\;a_s^{'}\,k_s\left(1+t_s^{'}+\frac{t_s^{'2}}{2}\right)\theta_s+a_o\,\omega_s\,P_s \qquad \boxed{t_s^{'}=h\,\beta_s} \qquad \boxed{\omega_s=\rho_s\,\beta_s\,k_s\,\delta_s\left(1+h\,\beta_s\right)}$$

$$A1c\text{ - }A1s=a_c^{'}\,k_c\left(1+t_c^{'}+\frac{t_c^{'2}}{2}\right)\theta_c-a_s^{'}\,k_s\left(1+t_s^{'}+\frac{t_s^{'2}}{2}\right)\theta_s+a_o\left(\omega_s\,P_s-\omega_c\,P_c\right)$$

Accounting for compatibility of shell and channel rotations: $\begin{cases}\theta_s=\theta_a\\ \theta_c=-\theta_a\end{cases}$

$$A1c\text{ - }A1s=-\left[a_s^{'}\,k_s\left(1+t_s^{'}+\frac{t_s^{'2}}{2}\right)+a_c^{'}\,k_c\left(1+t_c^{'}+\frac{t_c^{'2}}{2}\right)\right]\theta_a+a_o\left(\omega_c\,P_c-\omega_s\,P_s\right)$$

$$A2c=\left(a_c^2-a_o^2\right)\left(a_c-a_o\right)\frac{P_c}{4}-a_c^2\,\frac{P_c}{2}\left(a_c^{'}-a_o\right)=\frac{P_c}{4}\left[\left(a_c^2-a_o^2\right)\left(a_c-a_o\right)-2\,a_c^2\left(a_c^{'}-a_o\right)\right]$$

$$A2s=\left(a_s^2-a_o^2\right)\left(a_s-a_o\right)\frac{P_s}{4}-\left[a_o\,V_a+\left(a_s^2-a_o^2\right)\frac{P_s}{2}+a_o^2\,\frac{P_c}{2}\right]\left(a_s^{'}-a_o\right)$$

$$=-a_o\,V_a\left(a_s^{'}-a_o\right)+\frac{P_s}{4}\left[\left(a_s^2-a_o^2\right)\left(a_s-a_o\right)-2\left(a_s^2-a_o^2\right)\left(a_s^{'}-a_o\right)\right]-2\,a_0^2\left(a_s^{'}-a_o\right)\frac{P_c}{4}$$

$$=-a_o\,V_a\left(a_s^{'}-a_o\right)+\frac{P_s}{4}\left(a_s^2-a_o^2\right)\left(a_s+a_o-2\,a_s^{'}\right)-\frac{P_c}{4}\left[2\,a_o^2\left(a_s^{'}-a_o\right)\right]$$

$$A2c-A2s=+a_0\,V_a\left(a_s^{'}-a_o\right)-\frac{P_s}{4}\left(a_s^2-a_o^2\right)\left(a_s+a_o-2\,a_s^{'}\right)+\frac{P_c}{4}\left[\left(a_c^2-a_o^2\right)\left(a_c-a_o\right)-2\,a_c^2\left(a_c^{'}-a_o\right)+2\,a_0^2\left(a_s^{'}-a_o\right)\right]$$

simplification of the calculations: $t_s \ll a_s \rightarrow a'_s=a_s$ $\qquad$ $t_c \ll a_c \rightarrow a'_c=a_c$

$$A2c-A2s=+a_o\,V_a\left(a_s-a_o\right)+\frac{P_s}{4}\,a_o^3\left[\left(\rho_s^2-1\right)\left(\rho_s-1\right)\right]-\frac{P_c}{4}\,a_o^3\left[\left(\rho_c^2+1\right)\left(\rho_c-1\right)-2\left(\rho_s-1\right)\right]$$

Equation [VI.2b] is written for configuration a:

$$[R\,M_R]_a = -a_o\,M_a + a_o^2\,V_a\,(\rho_s - 1) + \frac{P_s}{4}\,a_o^3\left[(\rho_s^2 - 1)(\rho_s - 1)\right] - \frac{P_c}{4}\,a_o^3\left[(\rho_c^2 + 1)(\rho_c - 1) - 2(\rho_s - 1)\right]$$
$$-\left[a_s'\,k_s\left(1 + t_s' + \frac{t_s'^2}{2}\right) + a_c'\,k_c\left(1 + t_c' + \frac{t_c'^2}{2}\right)\right]\theta_a + a_o\,(\omega_c\,P_c - \omega_s\,P_s) \quad \text{[VI.2b']}$$

(b) **For configuration b:**

Axial equilibrium of forces

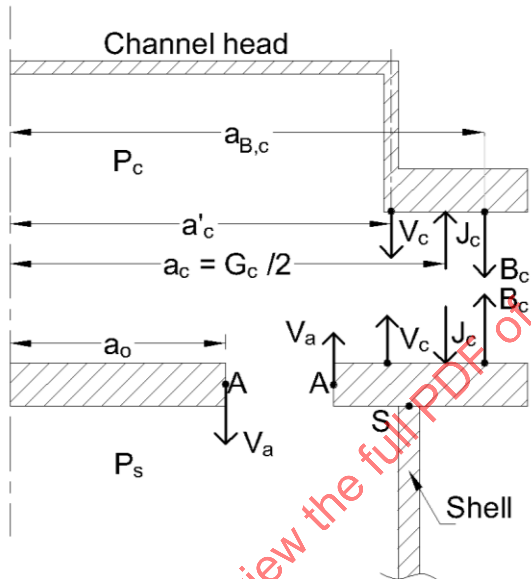


Figure 62 — Configuration b

Equilibrium of the channel:

$$2\,\pi\,a_c'\,V_c = 2\,\pi\,a_{Bc}\,B_c - 2\,\pi\,a_c\,J_c = \pi\,a_c^2\,P_c \;\Rightarrow\; a_{Bc}\,B_c - a_c\,J_c = \frac{a_c^2}{2}\,P_c$$

Axial force V_s in the shell:

$$2\,\pi\,a_s'\,V_s + \pi\left(a_c^2 - a_o^2\right)P_c = 2\,\pi\,a_o\,V_a + \underbrace{2\,\pi\,a_{Bc}\,B_c - 2\,\pi\,a_c\,J_c}_{\pi\,a_c^2\,P_c} + \pi\left(a_s^2 - a_o^2\right)P_s$$

Axial equilibrium of the ring remains unchanged:

$$a_s'\,V_s = a_o\,V_a + \frac{a_o^2}{2}\,P_c + \frac{a_s^2 - a_o^2}{2}\,P_s$$

Where (see Figure 62):

$$\begin{cases} a_c = \textit{mean gasket radius} = \dfrac{G_c}{2} & J_c = \textit{gasket load per unit of circumference} \\ a_{Bc} = \textit{bolt circle radius} = \dfrac{C_c}{2} & B_c = \textit{bolt load per unit of circumference} \end{cases}$$

Axial equilibrium of moments. In equation [VI.2b]:

$$[R\,M_R]_a = \overbrace{-a_o\,M_a}^{A0} + \overbrace{\left[a_c^{'}\,M_c - a_c^{'}\,Q_c\,\frac{h}{2}\right]}^{A1c} + \overbrace{\left[M(P_c) - a_c^{'}\,V_c\,(a_c^{'} - a_o)\right]}^{A2c} - \underbrace{\left[a_s^{'}\,M_s - a_s^{'}\,Q_s\,\frac{h}{2}\right]}_{A1s} - \underbrace{\left[M(P_s) - a_s^{'}\,V_s\,(a_s^{'} - a_o)\right]}_{A2s} \qquad \text{[VI.2b]}$$

- Terms A0, A1s, A2s due to shell remain unchanged.
- Term A1c due to moments applied to the channel disappears, but can be maintained provided that k_c is taken equal to 0: $k_c=0$.
- Term A2c becomes $[A2c]_b$:

  $$[A2c]_b = M(P_c) - a_{Bc}\,B_c\,(a_{Bc} - a_o) + a_c\,J_c\,(a_c - a_o) = M(P_c) + A_B$$

  $$A_B = -a_{Bc}\,B_c\,(a_{Bc} - a_o) + \left(a_{Bc}\,B_c - \frac{a_c^2}{2}\,P_c\right)(a_c - a_o)$$

  Where:

  $$= -a_{Bc}\,B_c\,a_{Bc} + a_{Bc}\,B_c\,a_c - \frac{a_c^2}{2}\,P_c\,(a_c - a_o) = a_{Bc}\,B_c\,(a_c - a_{Bc})\,\frac{a_c^2}{2}\,(a_c - a_o)$$

  $$[A2c]_b = \underbrace{M(P_c) - \frac{a_c^2}{2}\,(a_c - a_o)\,P_c + a_{Bc}\,B_c\,(a_c - a_{Bc})}_{A3\ with:\ a_c^{'}=a_c} = A3 + a_{Bc}\,B_c\,(a_c - a_{Bc})$$

- $A2c_b$ – A2s = A2c – A2s $+a_{Bc}\,B_c\,(a_c - a_{Bc})$ leads to equation $R\,M_R$ for configuration b:

  $$[R\,M_R]_b = [R\,M_R]_a + a_{Bc}\,B_c\,(a_c - a_{Bc})$$

  Using UHX-13 notations:

  $$a_{Bc}\,B_c\,(a_c - a_{Bc}) = \frac{C_c}{2}\,\frac{W_c}{\pi\,C_c}\left(\frac{G_c}{2} - \frac{C_c}{2}\right) = \frac{a_o}{2\,\pi}\,W_c\left(\frac{G_c - C_c}{D_o}\right)$$

  $$a_{Bc}\,B_c\,(a_c - a_{Bc}) = \frac{a_o}{2\,\pi}\,W_c\,\gamma_{bc} \quad \text{where:} \quad \gamma_{bc} = \frac{G_c - C_c}{D_o}$$

  $$[R\,M_R]_b = [R\,M_R]_a + \frac{a_o}{2\,\pi}\,W_c\,\gamma_{bc} \quad \text{with } k_c = 0 \text{ in } [RM_R]_a \text{ equation}$$

(c) **For configuration c:**

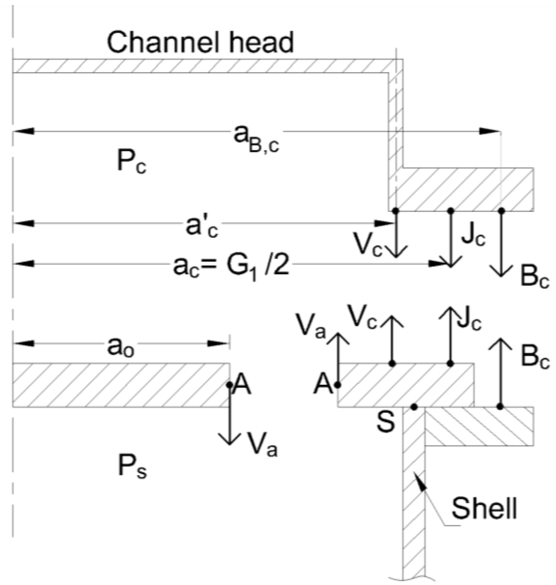


Figure 63 — Configuration c

The same applies but the bolting diameter C_c becomes the midpoint of contact between flange and TS, G_1 (see Figure 63). Thus γ_{bc} becomes: $\gamma_{bc} = \dfrac{G_c - G_1}{D_o}$

(d) **For configuration d:**
Axial equilibrium of forces

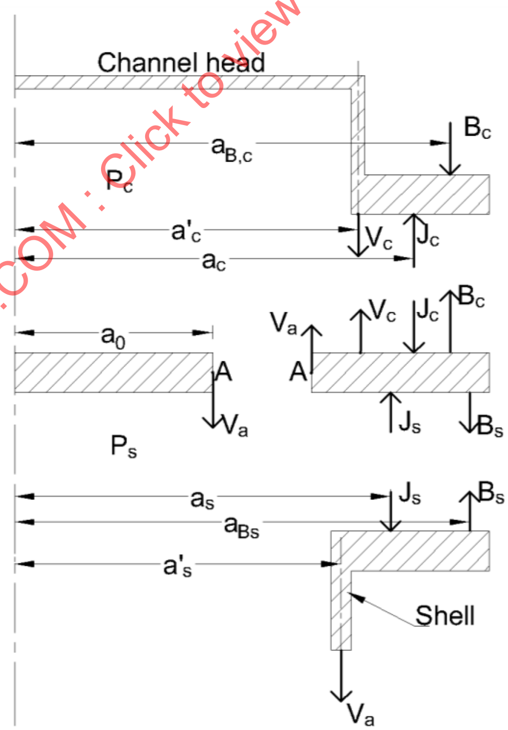


Figure 64 — Configuration d

Axial force V_c in the channel remains unchanged:

$$a_c\, V_c = a_{Bc}\, B_c - a_c\, J_c = \frac{a_c^2}{2} P_c \qquad a_c = \frac{G_c}{2} \qquad a_{Bc} = \frac{C_c}{2}$$

Axial force V_s in the shell is given by, with $a_s = \frac{G_s}{2}$, $a_{Bc} = \frac{C_c}{2}$ and $a_{Bs} = \frac{C_s}{2}$

$$2\,\pi\, a_s'\, V_s + 2\,\pi\, a_s\, J_s = 2\,\pi\, a_{Bs}\, B_s \quad \Rightarrow \quad \boxed{a_s'\, V_s = a_{Bs}\, B_s - a_s\, J_s}$$

Axial equilibrium of moments. Comparing to configuration b:
Terms A0, A1c and $[A2c]_b$ due to the channel remain unchanged.
Term A1s due to moments applied to the shell disappears, but can be maintained provided that k_s is taken equal to 0: k_s=0.
Term A2s becomes $[A2s]_d$, similar to term $[A2c]_b$ of the channel:

$$[\text{A2s}]_\text{d} = \underbrace{M(P_s) - \frac{a_s^2}{2}(a_s - a_o)P_s}_{A3\; avec\; a_s' = a_s} + a_{Bs}\, B_s\,(a_s - a_{Bs})\, P_c = A2s + a_{Bs}\, B_s\,(a_s - a_{Bs})$$

Term: $[A2c]_b - [A2s]_d = A2c + a_{Bc}\, B_c\,(a_c - a_{Bc}) - A2s - a_{Bs}\, B_s\,(a_s - a_{Bs})$

leads to the equation $\left[R\, M_R\right]_d$ for configuration d:

$$[R\, M_R]_d = [R\, M_R]_a + a_{Bc}\, B_c\,(a_c - a_{Bc}) - a_{Bs}\, B_s\,(a_s - a_{Bs})$$

Using UHX-13 notations:

$$a_{Bs}\, B_s\,(a_s - a_{Bs}) = \frac{C_s}{2}\,\frac{W_s}{\pi\, C}\left(\frac{G_s}{2} - \frac{C_s}{2}\right) = \frac{a_o}{2\,\pi} W_s \left(\frac{G_s - C_s}{D_o}\right) = \frac{a_o}{2\,\pi} W_s\, \gamma_{bs}$$

$$[R\, M_R]_d = [R\, M_R]_a + \frac{a_o}{2\,\pi} W_c\, \gamma_{bc} - \frac{a_o}{2\,\pi} W_s\, \gamma_{bs} \;\text{ with: } \begin{cases} k_s = 0 \quad and \quad \gamma_{bs} = \dfrac{G_s - C_s}{D_o} \\ k_c = 0 \quad and \quad \gamma_{bc} = \dfrac{G_c - C_c}{D_o} \end{cases}$$

Finally, the generic equation covering the 4 configurations a, b, c and d is written:

$$\boxed{\begin{aligned} R\, M_R = & - a_o\, M_a + a_o^2 V_a(\rho_s - 1) + P_s\,\frac{a_o^3}{4}\left[(\rho_s^2 - 1)(\rho_s - 1)\right] - P_c\,\frac{a_o^3}{4}\left[(\rho_c^2 + 1)(\rho_c - 1) - 2(\rho_s - 1)\right] \\ & - \left[a_s'\, k_s\left(1 + t_s' + \frac{t_s'^2}{2}\right) + a_c'\, k_c\left(1 + t_c' + \frac{t_c'^2}{2}\right)\right]\theta_a + a_o(\omega_c\, P_c - \omega_s\, P_s) + \frac{a_o}{2\,\pi}\left[W_c\, \gamma_{bc} - W_s\, \gamma_{bs}\right] \end{aligned}} \quad \text{[VI.2d]}$$

In this equation:

- λ_s , λ_c and w_s, w_c are coefficients obtained above. They are known for a given HE.
- W_s and W_c are bolt loads applied on shell and channel when the TS is gasketed (configurations b, c, d).
- Coefficients γ_{bs} and γ_{bc} are defined for each configuration as follows:

  Configuration a: k_s given in VI.1a $\quad k_c$ given in VI.1b $\quad \gamma_{bs} = 0 \quad \gamma_{bc} = 0$

  Configuration b: k_s given in VI.1a $\quad k_c = 0 \quad \gamma_{bs} = 0 \quad \gamma_{bc} = \frac{G_c - C_c}{D_o}$

Configuration c: k_s given in VI.1a $\quad k_c = 0 \quad \gamma_{bs} = 0 \quad \gamma_{bc} = \dfrac{G_c - G_1}{D_0}$

Configuration d: $k_s = 0 \quad k_c = 0 \quad \gamma_{bs} = \dfrac{G_s - C_s}{D_o} \quad \gamma_{bc} = \dfrac{G_c - C_c}{D_o}$

ANNEX M — DIRECT DETERMINATION OF THE EQUIVALENT PRESSURE

Equivalent pressures P'_s, P'_t and P'_γ can be obtained directly by examining the loads applied on the TS as follows.

(a) **Pressures P_s and P_t acting on the perforated TS** (Figure 65)

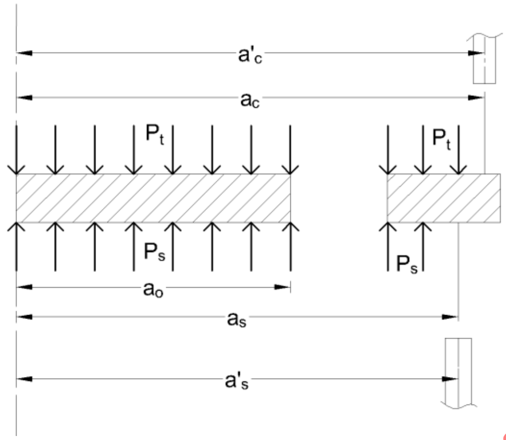


Figure 65 — Pressures Ps and Pt Acting on TS

$$P_{TS}(P_s, P_t) = x_s P_s - x_t P_t \qquad x_s = 1 - N_t\left(\frac{d_t}{D_0}\right)^2 \qquad x_t = 1 - N_t\left(\frac{d_t - 2t_t}{D_0}\right)^2$$

(b) **Pressures P_s and P_t acting on the unperforated TS RIM**

Load acting on the rim:

$$F_{RIM}(P_s, P_t) = \pi(a_s^2 - a_o^2)P_s - \pi(a_c^2 - a_o^2)P_t = \pi a_o^2\left[(\rho_s^2 - 1)P_s - (\rho_c^2 - 1)P_t\right]$$

Part of F_{rim} supported by the tubes: $F_{RIM}(P_s, P_t)\dfrac{N_t K_t}{K_s^*} = F_{RIM}(P_s, P_t)\dfrac{N_t K_t}{JK_s} = \dfrac{F_{RIM}(P_s, P_t)}{JK_{s,t}}$

Equivalent pressure acting on the unperforated RIM:

$$P_{RIM}(P_s, P_t) = -\frac{F_{RIM}(P_s, P_t)}{\pi a_o^2 JK_{s,t}} = -\frac{(\rho_s^2 - 1)P_s - (\rho_c^2 - 1)P_t}{JK_{s,t}}$$

$$P_{RIM}(P_s, P_t) = -\frac{F_{RIM}(P_s, P_t)}{\pi a_o^2 JK_{s,t}} = \boxed{-\frac{(\rho_s^2 - 1)}{JK_{s,t}}P_s + \frac{(\rho_c^2 - 1)}{JK_{s,t}}P_t}$$

(c) **Pressure P_s acting on the joint** (Figure 66)

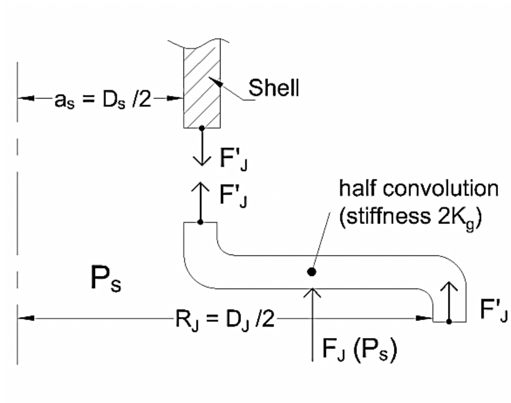


Figure 66 — Pressure P_s Acting on Bellows Joint

Axial load acting on the shell: $F_J'(P_s) = \dfrac{F_J(P_s)}{2} = P_s \dfrac{\pi}{2}\left(R_J^2 - R_s^2\right)$

Axial displacement of the half-shell: $\Delta_J(P_s) = \dfrac{P_s \pi \left(R_J^2 - R_s^2\right)/2}{2K_J}$

Equivalent pressure acting on the perforated TS: using $\dfrac{k_w}{2K_J} = \dfrac{1}{\pi a_o^2}\dfrac{1}{K_{s,t}}\dfrac{1-J}{J}$ and $K_{s,t} = \dfrac{k_s}{N_t k_t}$

$$P_J(P_s) = -k_w\,\Delta_J(P_s) = -\frac{1}{\pi a_o^2}\frac{1-J}{JK_{s,t}}\frac{\pi\left(R_J^2 - R_s^2\right)}{2}P_s = \boxed{-\frac{1-J}{2JK_{s,t}}\frac{\left(D_J^2 - D_s^2\right)}{D_o^2}P_s}$$

(d) **Effect of ν_t due to pressures P_s and P_t acting on the tubes** (Figure 67)

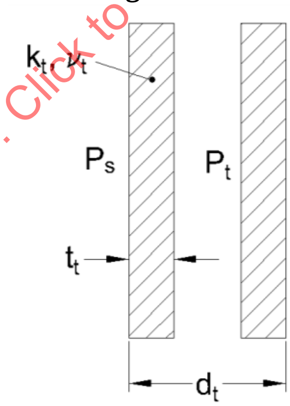


Figure 67 — Effect of ν_t Due to Pressures P_s and P_t

$$\Delta L_T(\nu_t) = \frac{\pi}{2\,k_t}\left[P_s\,d_t^2 - P_t\left(d_t - 2\,t_t\right)^2\right]\nu_t$$

Using $k_w = \dfrac{N_t \cdot k_t}{\pi a_0^2}$:

$$P_T(\upsilon_t) = k_w \Delta L_T(\upsilon_t) = \frac{\pi}{2k_t}\frac{N_t k_t}{\pi a_o^2}\left[P_s d_t^2 - P_t\left(d_t - 2t_t\right)^2\right]\nu_t = \frac{2N_t}{4a_o^2}\left[P_s d_t^2 - P_t\left(d_t - 2t_t\right)^2\right]\nu_t$$

$$\boxed{P_T(\upsilon_t) = 2\left[P_s\,(1-x_s) - P_t\,(1-x_t)\right]\nu_t}$$

(e) **Effect of ν_s due to pressure P_s acting on the shell**:

In the same way as for the tubes: $\Delta L_S\left(\nu_s\right) = -\dfrac{\pi}{2\,k_s}\left[P_s\,D_s^2\right]\nu_s$

Using $k_w = \dfrac{N_t \cdot k_t}{\pi\,a_o^2}$ and $K_{s,t} = \dfrac{k_s}{N_t k_t}$:

$$P_S(\upsilon_s) = -k_w \Delta L_S(\upsilon_s) = \frac{\pi}{2k_s}\frac{N_t k_t}{\pi a_o^2}\left[P_s D_s^2\right]\nu_s = \boxed{\frac{2}{K_{s,t}}\frac{D_s^2}{D_o^2}P_s \nu_s}$$

(f) **Pressure P_t acting on the Channel Head** (Figure 68)

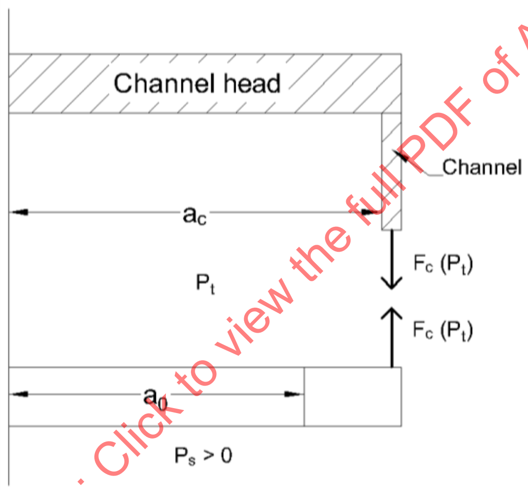


Figure 68 — Pressure P_t Acting on the Channel Head

Load acting at periphery of TS: $F_C(P_t) = \pi a_c^2 P_t$

Part of $F_C(P_t)$ supported by the tubes: $F_C(P_t)\dfrac{N_t K_t}{K_s^*} = F_C(P_t)\dfrac{N_t K_t}{JK_s} = \dfrac{F_C(P_t)}{JK_{s,t}}$

Equivalent pressure acting on the perforated TS:

$$P_C(P_t) = -\frac{F_C(P_t)}{\pi a_o^2 JK_{s,t}} = -\frac{a_c^2}{a_o^2 JK_{s,t}}P_t = \boxed{-\frac{\rho_c^2}{JK_{s,t}}P_t}$$

(g) **Effect of γ due to differential thermal expansion between tubes and shell**

$$\gamma = \left[\alpha_{t,m}(T_{t,m} - T_a) - \alpha_{s,m}(T_{s,m} - T_a)\right]L$$

Equivalent pressure acting on the perforated TS: $P_\gamma = k_w \gamma = \boxed{\dfrac{N_t k_t}{\pi a_o^2}\gamma}$

(h) **The total pressure acting on the TS is written:**

$$P_T = P_{TS}(P_s, P_t) + \mathrm{P}_{RIM}(P_s, P_t) + P_J(P_s) + P(\upsilon_t) + P(\upsilon_s) + P_C(P_t)$$

$$P_T = x_s P_s - x_t P_t + -\frac{\rho_s^2 - 1)}{JK_{s,t}} P_s + \frac{\rho_c^2 - 1}{JK_{s,t}} P_t - \frac{1-J}{2JK_{s,t}} \frac{\left(D_J^2 - D_s^2\right)}{D_o^2} P_s + 2(1-x_s)\nu_t P_s - 2(1-x_t)\nu_t P_t + \frac{2}{K_{s,t}} \frac{D_s^2}{D_o^2} \nu_s P_s$$

$$-\frac{\rho_c^2}{JK_{s,t}} P_t + \frac{N_t k_t}{\pi a_o^2} \gamma$$

P_T can be written: $\boxed{P_T = P_s' - P_t' + P_\gamma}$

with: $\boxed{P_s' = P_s\left[x_s + 2(1-x_s)\nu_t + \frac{2}{K_{s,t}}\left(\frac{D_s}{D_o}\right)^2 \nu_s - \frac{\rho_s^2 - 1}{JK_{s,t}} - \frac{1-J}{2JK_{s,t}} \frac{D_J^2 - D_s^2}{D_o^2}\right]}$

$\boxed{P_t' = P_t\left[x_t + 2(1-x_t)\nu_t + \frac{1}{J\,K_{s,t}}\right]}$ $\boxed{P_\gamma = \left[\frac{N_t\,K_t}{\pi\,a_o^2}\right]\gamma}$

Formulas P'_s, P'_t, P_γ match the formulas obtained in VII.2 above. Formulas P_W and P_{rim} cannot be calculated directly as they involve the edge moments at the TS-shell-channel connection.

ANNEX N — FORMULAS TO BE USED WHEN P_E=0

The equivalent pressure P_e may be equal to 0 in the following loading cases:

- loading case 1 if $P_t=0$
- loading case 2 if $P_s=0$
- loading case 3 if $P_s=P_t$
- loading case 4 if $P_s=P_t$

When $P_e=0$, $V_a=0$ and $Q_3=Q_1+\frac{Q_2}{a_o\,V_a}$ becomes infinity. General formulas depending on V_a and M_a, given in VIII, must be used with $V_a=0$ to determine the stresses. These formulas can be written in the general symbolic form: $F(x)=K\left[M_a\,A(x)+(a_0\,V_a)\,B(x)\right]$

with $M_a=(a_o\,V_a)\,Q_1+Q_2=Q_2$ Thus $\boxed{M_a=Q_2}$ and $F(x)=K\,Q_2\,A(x)$

Accordingly, the equations giving $q(x)$, $w(x)$, $\theta(x)$, $\sigma(x)$, $\tau(x)$, $\sigma_t(x)$ write as follows.

1 Net Effective Pressure: q(x)

$$\boxed{q(x)=\frac{X_a^4}{a_o^2}\,Q_2\,Z_w(x)}$$

Maximum of q(x) is obtained for $Z_w(x)$ maximum which is located either inside the TS ($x<X_a$) or at TS periphery ($x=X_a$).

2 Axial Displacement: w(x)

$$\boxed{w(x)=\frac{Q}{k_w}-\frac{1}{k_w}\,\frac{X_a^4}{a_o^2}\,Q_2\,Z_w(x)}$$

Maximum of w(x) is obtained for $Z_w(x)$ maximum which is located either inside the TS ($x<X_a$) or at TS periphery ($x=X_a$).

3 Rotation: θ(x)

$$\boxed{\theta(x)=\frac{a_o}{D^*}\,Q_2\,Z_m(x)}$$

Maximum of θ(x) is obtained for $Z_m(x)$ maximum which is located either inside the TS ($x<X_a$) or at TS periphery ($x=X_a$).

4 Bending Stress: σ(x)

$$M_r(x)=Q_2\,Q_m(x)$$

$$\boxed{\sigma_r(x)=\frac{6\,Q_2}{\mu^*\,h^2}\,Q_m(x)}$$

Maximum of σ(x) is obtained for $Q_m(x)$. A parametric study performed on $X_a=1,2,3\ldots,20$ shows that the maximum is always located at TS periphery ($x=X_a$) and is equal to 1 (See Annex G):

$$\text{MAX}\left[Q_m(x)\right] = Q_m(X_a) = 1 \quad \text{Thus} \quad \boxed{\sigma = \frac{6\,Q_2}{\mu^*\,h^2}}$$

5 Shear Stress: $\tau(x)$

$$Q(x) = \frac{Q_2}{a_o} Q_\alpha(x)$$

$$\boxed{\tau(x) = \frac{Q_2}{\mu\,a_o\,h} Q_\alpha(x)}$$

Maximum of $\tau(x)$ is obtained for $Q_\alpha(x)$ maximum which is located either inside the TS ($x<X_a$) or at TS periphery ($x=X_a$).

6 Axial Stress in Tubes: $\sigma_t(x)$

$$\sigma_t(x) = \frac{1}{x_t - x_s}\left[\Delta p^* - q(x)\right] \qquad q(x) = \frac{X_a^4}{a_o^2} Q_2\,Z_w(x)$$

$$\boxed{\sigma_t(x) = \frac{1}{x_t - x_s}\left[(x_s\,P_s - x_t\,P_t) - \frac{X_a^4}{a_o^2} Q_2\,Z_w(x)\right]}$$

Maximum and minimum values of $\sigma_t(x)$ are obtained for $Z_w(x)$ maximum or minimum which are located either inside the TS ($x<X_a$) or at TS periphery ($x=X_a$).

ANNEX O — TABULAR AND GRAPHICAL REPRESENTATION OF COEFFICIENT $F_t(x)$

Annex O provides for $1 \leq X_a \leq 20$ and $-0.8 \leq Q_3 \leq +0.8$:

- values and graphs of $F_t(x)$ for $0 \leq x \leq X_a$
- values and graphs of the minimum and maximum of $F_t(x)$: $F_{t,min}$ and $F_{t,max}$
- locations of the minimum and maximum of $F_t(x)$: x_{min} and x_{max}

$F_t(x)$ for X_A=1,5,7,10,20

$F_t(x)$ for X_a= 1, 5 , 7 , 10 , 20 and Q_3 = -0.8 , 0.0 , +0.8 (ν*=0.4)

$X_a = 1$				$X_a = 5$				$X_a = 7$				$X_a = 10$				$X_a = 20$			
x	$Q_3 = -0.8$	$Q_3 = 0$	$Q_3 = +0.8$	x	$Q_3 = -0.8$	$Q_3 = 0$	$Q_3 = +0.8$	x	$Q_3 = -0.8$	$Q_3 = 0$	$Q_3 = +0.8$	x	$Q_3 = -0.8$	$Q_3 = 0$	$Q_3 =+0.8$	x	$Q_3 = -0.8$	$Q_3 = 0$	$Q_3 = +0.8$
2.00E-02	1.038118	0.9676997	0.8972814	1.00E-01	1.032634	-0.6066762	-2.245986	0.14	-0.8756667	-2.81E-02	0.8194348	0.2	-9.87E-02	4.48E-02	1.88E-01	0.4	7.24E-04	4.65E-05	-6.31E-04
4.00E-02	1.038039	0.9677894	0.8975396	2.00E-01	1.046217	-0.6069373	-2.260092	0.28	-0.8633661	-3.16E-02	0.8000731	0.4	-1.09E-01	4.54E-02	2.00E-01	0.8	5.27E-04	5.92E-05	-4.09E-04
0.06	1.037908	0.9679389	0.8979697	0.3	1.06879	-0.6073345	-2.283459	0.42	-0.8426553	-3.75E-02	0.7676076	0.6	-1.26E-01	4.64E-02	2.19E-01	1.2	1.88E-04	7.96E-05	-2.89E-05
0.08	1.037724	0.9681479	0.8985717	0.4	1.100256	-0.607811	-2.315878	0.56	-0.8132234	-4.57E-02	0.72175	0.8	-1.50E-01	4.76E-02	2.46E-01	1.6	-3.06E-04	1.06E-04	5.18E-04
0.1	1.037487	0.9684163	0.8993454	0.5	1.140476	-0.608287	-2.35705	0.7	10 , 20 an	-5.63E-02	0.6621073	1	-1.81E-01	4.90E-02	2.79E-01	2	-9.65E-04	1.37E-04	1.24E-03
0.12	1.037197	0.9687439	0.9002905	0.6	1.189266	-0.6086601	-2.406586	0.84	-0.7263712	-6.91E-02	0.5881936	1.2	-2.17E-01	5.05E-02	3.18E-01	2.4	-1.79E-03	1.68E-04	2.12E-03
0.14	1.036854	0.9691301	0.9014067	0.7	1.246399	-0.6088049	-2.464009	0.98	-0.6677723	-8.42E-02	0.4994436	1.4	-2.60E-01	5.18E-02	3.63E-01	2.8	-2.76E-03	1.93E-04	3.14E-03
0.16	1.036456	0.9695748	0.9026937	0.8	1.311598	-0.6085731	-2.528745	1.12	-0.5981158	-1.01E-01	0.3952311	1.6	-3.06E-01	5.29E-02	4.12E-01	3.2	-3.81E-03	2.04E-04	4.22E-03
0.18	1.036004	0.9700776	0.9041512	0.9	1.384535	-0.6077937	-2.600122	1.26	-0.5165977	-1.21E-01	0.2748903	1.8	-3.57E-01	5.35E-02	4.64E-01	3.6	-4.85E-03	1.91E-04	5.23E-03
0.2	1.035497	0.9706377	0.9057785	1	1.464823	-0.6062728	-2.677369	1.4	-0.4223562	-1.42E-01	0.1377414	2	-4.11E-01	5.33E-02	5.18E-01	4	-5.69E-03	1.40E-04	5.97E-03
0.22	1.034934	0.9712548	0.9075752	1.1	1.552018	-0.6037938	-2.759606	1.54	-0.3144919	-1.66E-01	-0.01688031	2.2	-4.66E-01	5.20E-02	5.70E-01	4.4	-6.10E-03	3.71E-05	6.17E-03
0.24	1.034316	0.9719282	0.9095408	1.2	1.645607	-0.6001173	-2.845841	1.68	-0.1920907	-1.91E-01	-0.189591	2.4	-5.21E-01	4.94E-02	6.19E-01	4.8	-5.71E-03	-1.34E-04	5.44E-03
0.26	1.03364	0.9726573	0.9116746	1.3	1.745007	-0.5949817	-2.93497	1.82	-0.05425141	-2.18E-01	-0.3809201	2.6	-5.72E-01	4.50E-02	6.62E-01	5.2	-4.12E-03	-3.88E-04	3.34E-03
0.28	1.032907	0.9734414	0.913976	1.4	1.849559	-0.5881031	-3.025765	1.96	0.0998848	-2.46E-01	-0.5912681	2.8	-6.18E-01	3.85E-02	6.95E-01	5.6	-7.99E-04	-7.33E-04	-6.68E-04
0.3	1.032115	0.9742795	0.9164441	1.5	1.95852	-0.5791762	-3.116872	2.1	0.2710992	-0.2748804	-8.21E-01	3	-6.56E-01	2.94E-02	7.15E-01	6	4.77E-03	-1.17E-03	-7.10E-03
0.32	1.031264	0.9751709	9.19E-01	1.6	2.071058	-0.5678742	-3.21E+00	2.24	0.4600573	-0.3048178	-1.069693	3.2	-6.80E-01	1.75E-02	7.15E-01	6.4	1.31E-02	-1.67E-03	-1.64E-02
0.34	1.030352	0.9761147	9.22E-01	1.7	2.186247	-0.5538498	-3.29E+00	2.38	6.67E-01	-0.3351072	-1.33748	3.4	-0.6879466	2.12E-03	0.6921918	6.8	2.44E-02	-2.20E-03	-2.88E-02
0.36	1.029379	0.97711	9.25E-01	1.8	2.303055	-0.5367362	-3.38E+00	2.52	0.8930232	-0.3652826	-1.623588	3.6	-0.6736465	-1.70E-02	0.6396902	7.2	3.86E-02	-2.68E-03	-4.40E-02
0.38	1.028343	0.9781556	9.28E-01	1.9	2.420344	-0.5161481	-3.45E+00	2.66	1.137372	-0.3948018	-1.926976	3.8	-0.631919	-4.02E-02	0.5515026	7.6	5.52E-02	-2.97E-03	-6.12E-02
0.4	1.027244	0.9792506	9.31E-01	2	2.536859	-0.4916826	-3.52E+00	2.8	1.400039	-0.42304	-2.246119	4	-0.5567574	-6.79E-02	0.4210006	8	7.26E-02	-2.91E-03	-7.84E-02
0.42	1.02608	0.9803938	0.9347077	2.1	2.651222	-0.4629212	-3.577064	2.94	1.680372	-0.4492831	-2.578938	4.2	-0.4417243	-1.00E-01	0.2412963	8.4	8.81E-02	-2.26E-03	-9.26E-02
0.44	1.02485	0.981584	0.9383183	2.2	2.761925	-0.4294315	-3.620788	3.08	1.977278	-0.4727224	-2.922723	4.4	-0.2800746	-1.37E-01	0.005432152	8.8	9.73E-02	-7.37E-04	-9.88E-02
0.46	1.023552	0.9828199	0.9420877	2.3	2.867325	-0.3907692	-3.648863	3.22	2.28915	-0.4924494	-3.274049	4.6	-0.06492908	-1.79E-01	-0.2933681	9.2	9.44E-02	1.95E-03	-9.05E-02
0.48	1.022186	0.9841002	0.9460148	2.4	2.965638	-0.3464808	-3.658599	3.36	2.613796	-0.5074518	-3.6287	4.8	0.2104946	-2.25E-01	-0.6613804	9.6	7.15E-02	6.11E-03	-5.93E-02
0.5	1.020749	0.9854234	0.9500982	2.5	3.054932	-0.2961063	-3.647145	3.5	2.948359	-0.5166103	-3.981579	5	0.5525845	-2.76E-01	-1.103985	10	1.92E-02	1.20E-02	4.76E-03
0.52	1.01924	0.9867884	0.9543367	2.6	3.133124	-0.2391825	-3.611489	3.64	3.289237	-0.5186982	-4.326633	5.2	0.966963	-3.29E-01	-1.625187	10.4	-7.31E-02	1.95E-02	0.1121317
0.54	1.017658	0.9881935	0.9587288	2.7	3.197976	-0.1752467	-3.54847	3.78	3.632005	-0.5123812	-4.656767	5.4	1.458035	-3.85E-01	-2.227052	10.8	-0.2150519	2.86E-02	0.2722115
0.56	1.016001	0.989637	0.963273	2.8	3.247089	-0.103841	-3.454771	3.92	3.971332	-0.4962205	-4.963773	5.6	2.028447	-4.40E-01	-2.909042	11.2	-0.4140377	3.84E-02	0.4907806
0.58	1.014267	0.9911175	0.9679677	2.9	3.277906	-0.02451688	-3.32694	4.06	4.300899	-0.4686789	-5.238256	5.8	2.678461	-0.4944041	-3.667269	11.6	-0.6712334	4.75E-02	0.7662781
0.6	1.012455	0.9926332	0.9728113	3	3.287708	0.06315959	-3.161389	4.2	4.613324	-0.4281282	-5.46958	6	3.40523	-0.5442125	-4.49E+00	12	-0.9773141	5.38E-02	1.084902
0.62	1.010563	0.9941825	0.9778024	3.1	3.273615	0.1596018	-2.954412	4.34	4.900085	-0.3728624	-5.64581	6.2	4.201988	-0.5865092	-5.375007	12.4	-1.306761	5.39E-02	1.414617
0.64	1.008588	0.9957635	0.982939	3.2	3.23259	0.2651963	-2.702198	4.48	5.151461	-0.3011132	-5.753687	6.4	5.057155	-0.6174373	-6.292029	12.8	-1.611355	4.36E-02	1.698476
0.66	1.006529	0.9973744	0.9882194	3.3	3.161443	0.3802958	-2.400851	4.62	5.356469	-0.2110706	-5.77861	6.6	5.953367	-0.6324616	-7.21829	13.2	-1.813493	1.73E-02	1.848088
0.68	1.004385	0.9990131	0.9936417	3.4	3.056837	0.5052112	-2.046414	4.76	5.502823	-0.1009085	-5.70464	6.8	6.866462	-0.6263518	-8.119165	13.6	-1.80055	-3.10E-02	1.738637
0.7	1.002152	1.000678	0.9992042	3.5	2.915298	0.6402039	-1.63489	4.9	5.576911	0.03118399	-5.514544	7	7.764431	-0.5931894	-8.950809	14	-1.422138	-0.107292	1.207554
0.72	0.9998285	1.002367	1.004905	3.6	2.733226	0.7854759	-1.162274	5.04	5.563787	1.87E-01	-5.189855	7.2	8.606379	-0.5264054	-9.659189	14.4	-0.4929101	-0.2165779	5.98E-02
0.74	0.9974129	1.004077	1.010741	3.7	2.50691	0.9411606	-0.6245889	5.18	5.447183	3.68E-01	-4.71098	7.4	9.341547	-0.4188564	-10.17926	14.8	1.195645	-0.360618	-1.916881
0.76	0.9949027	1.005807	1.016712	3.8	2.232545	1.107312	-0.01792095	5.32	5.209565	0.5761069	-4.057351	7.6	9.908447	-0.262948	-10.43434	15.2	3.850281	-0.5355867	-4.921454
0.78	0.9922957	1.007555	1.022814	3.9	1.906255	1.283895	0.6615336	5.46	4.832207	0.8122958	-3.207616	7.8	10.23417	-0.05081466	-10.3358	15.6	7.63344	-0.7287403	-9.09092
0.8	0.9895892	1.009317	1.029046	4	1.524115	1.470769	1.417423	5.6	4.295315	1.077711	-2.139894	8	10.23401	0.2254344	-9.783137	16	12.59882	-0.9145391	-14.4279
0.82	0.9867812	1.011093	1.035405	4.1	1.082182	1.667682	2.253183	5.74	3.57819	1.373051	-0.8320874	8.2	9.811359	0.5733936	-8.664572	16.4	18.60214	-1.050529	-20.7032
0.84	0.9838691	1.012879	1.041888	4.2	0.5765283	1.874253	3.171978	5.88	2.659445	1.698589	0.7377334	8.4	8.858242	0.9999527	-6.858337	16.8	25.18708	-1.073637	-27.33435
0.86	0.9808503	1.014673	1.048495	4.3	0.00327703	2.089957	4.176637	6.02	1.517277	2.054079	2.590881	8.6	7.256321	1.510795	-4.234732	17.2	31.45123	-0.8979306	-33.24709
0.88	0.9777222	1.016472	1.055221	4.4	-0.6413519	2.314114	5.269579	6.16	0.1298059	2.438661	4.747516	8.8	4.87876	2.109792	-0.6591769	17.6	35.90466	-0.4153511	-36.73536
0.9	0.9744825	1.018274	1.062065	4.5	-1.360997	2.545871	6.45274	6.3	-1.524521	2.850749	7.226019	9	1.59297	2.798289	4.003609	18	36.34414	0.4985645	-35.34701
0.92	0.971128	1.020076	1.069023	4.6	-2.159101	2.78419	7.727481	6.44	-3.466443	3.28792	10.04228	9.2	-2.735554	3.574282	9.884117	18.4	29.77993	1.971487	-25.83695
0.94	0.9676566	1.021875	1.076094	4.7	-3.03885	3.027827	9.094502	6.58	-5.715328	3.74679	13.20891	9.4	-8.238123	4.431479	17.10108	18.8	12.46808	4.110861	-4.246357
0.96	0.9640648	1.023669	1.083273	4.8	-4.003114	3.275318	10.55375	6.72	-8.288511	4.222885	16.73428	9.6	-15.03634	5.358265	25.75287	19.2	-19.88116	6.96749	33.81614
0.98	0.9603503	1.025455	1.090559	4.9	-5.054369	3.524966	12.1043	6.86	-11.20056	4.710511	20.62158	9.8	-23.23396	6.336559	35.90708	19.6	-71.63274	10.48407	92.60088
1	0.9565098	1.027229	1.097949	5	-6.19463	3.774817	13.74426	7	-14.4624	5.202614	24.86763	10	-32.90709	7.340617	47.58832	20	-146.3978	14.42809	175.254

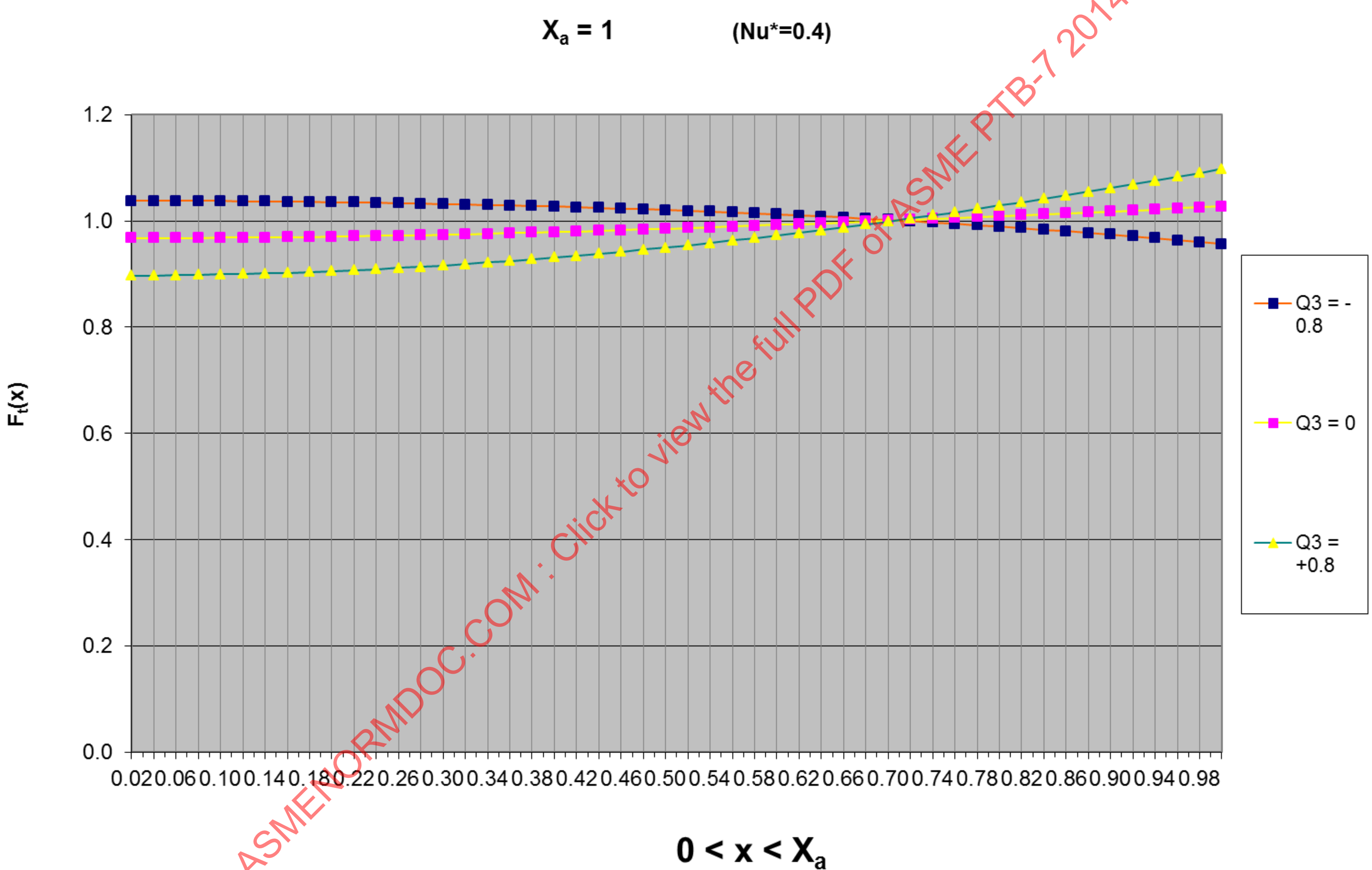
$F_t(X_a)$ $(v^* = 0.4)$

			Fq=Ft(min)			Ft(min)<Fq<Ft(max)						Fq=Ft(max)					
Xa\Q3	**-0.8**	**-0.7**	**-0.6**	**-0.5**	**-0.4**	**-0.3**	**-0.2**	**-0.1**	**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
1	0.9565	0.9653	0.9742	0.9830	0.9919	1.0007	1.0095	1.0184	1.0272	1.0361	1.0449	1.0537	1.0626	1.0714	1.0803	1.0891	1.0979
2	0.3868	0.5103	0.6339	0.7574	0.8809	1.0044	1.1279	1.2514	1.3749	1.4985	1.6220	1.7455	1.8690	1.9925	2.1160	2.2395	2.3631
3	-1.1560	-0.7358	-0.3155	0.1048	0.5250	0.9453	1.3655	1.7858	2.2061	2.6263	3.0466	3.4669	3.8871	4.3074	4.7276	5.1479	5.5682
4	-3.3304	-2.5335	-1.7365	-0.9395	-0.1426	0.6544	1.4514	2.2483	3.0453	3.8423	4.6392	5.4362	6.2332	7.0301	7.8271	8.6241	9.4211
5	-6.1946	-4.9484	-3.7023	-2.4561	-1.2099	0.0363	1.2825	2.5286	3.7748	5.0210	6.2672	7.5134	8.7595	10.0057	11.2519	12.4981	13.7443
6	-9.9120	-8.1122	-6.3123	-4.5124	-2.7126	-0.9127	0.8871	2.6870	4.4868	6.2867	8.0866	9.8864	11.6863	13.4861	15.2860	17.0859	18.8857
7	-14.4624	-12.0043	-9.5461	-7.0880	-4.6299	-2.1718	0.2864	2.7445	5.2026	7.6607	10.1189	12.5770	15.0351	17.4933	19.9514	22.4095	24.8676
8	-19.8122	-16.5960	-13.3799	-10.1637	-6.9475	-3.7313	-0.5151	2.7010	5.9172	9.1334	12.3496	15.5658	18.7820	21.9981	25.2143	28.4305	31.6467
9	-25.9594	-21.8858	-17.8122	-13.7385	-9.6649	-5.5913	-1.5177	2.5560	6.6296	10.7032	14.7768	18.8504	22.9241	26.9977	31.0713	35.1449	39.2186
10	-32.9071	-27.8761	-22.8452	-17.8142	-12.7832	-7.7523	-2.7213	2.3097	7.3406	12.3716	17.4025	22.4335	27.4645	32.4954	37.5264	42.5574	47.5883
11	-40.6555	-34.5672	-28.4789	-22.3906	-16.3023	-10.2140	-4.1257	1.9626	8.0509	14.1392	20.2275	26.3158	32.4041	38.4924	44.5807	50.6690	56.7573
12	-49.2043	-41.9587	-34.7131	-27.4675	-20.2219	-12.9763	-5.7307	1.5149	8.7606	16.0062	23.2518	30.4974	37.7430	44.9886	52.2342	59.4798	66.7254
13	-58.5532	-50.0504	-41.5475	-33.0446	-24.5417	-16.0388	-7.5360	0.9669	9.4698	17.9727	26.4756	34.9784	43.4813	51.9842	60.4871	68.9899	77.4928
14	-68.7023	-58.8422	-48.9820	-39.1219	-29.2618	-19.4017	-9.5415	0.3186	10.1787	20.0388	29.8989	39.7591	49.6192	59.4793	69.3394	79.1996	89.0597
15	-79.6514	-68.3341	-57.0167	-45.6994	-34.3820	-23.0647	-11.7473	-0.4300	10.8874	22.2047	33.5220	44.8394	56.1567	67.4741	78.7914	90.1088	101.4261
16	-91.4006	-78.5261	-65.6515	-52.7770	-39.9024	-27.0279	-14.1533	-1.2788	11.5958	24.4703	37.3449	50.2194	63.0940	75.9685	88.8431	101.7176	114.5922
17	-103.9499	-89.4181	-74.8864	-60.3546	-45.8229	-31.2912	-16.7594	-2.2277	12.3040	26.8358	41.3675	55.8993	70.4310	84.9627	99.4945	114.0262	128.5579
18	-117.2992	-101.0102	-84.7213	-68.4324	-52.1435	-35.8546	-19.5657	-3.2767	13.0122	29.3011	45.5900	61.8789	78.1678	94.4567	110.7457	127.0346	143.3235
19	-131.4485	-113.3024	-95.1563	-77.0102	-58.8642	-40.7181	-22.5720	-4.4259	13.7202	31.8663	50.0123	68.1584	86.3045	104.4506	122.5967	140.7428	158.8888
20	-146.3978	-126.2946	-106.1914	-86.0881	-65.9849	-45.8816	-25.7784	-5.6752	14.4281	34.5313	54.6346	74.7378	94.8410	114.9443	135.0475	155.1508	175.2540

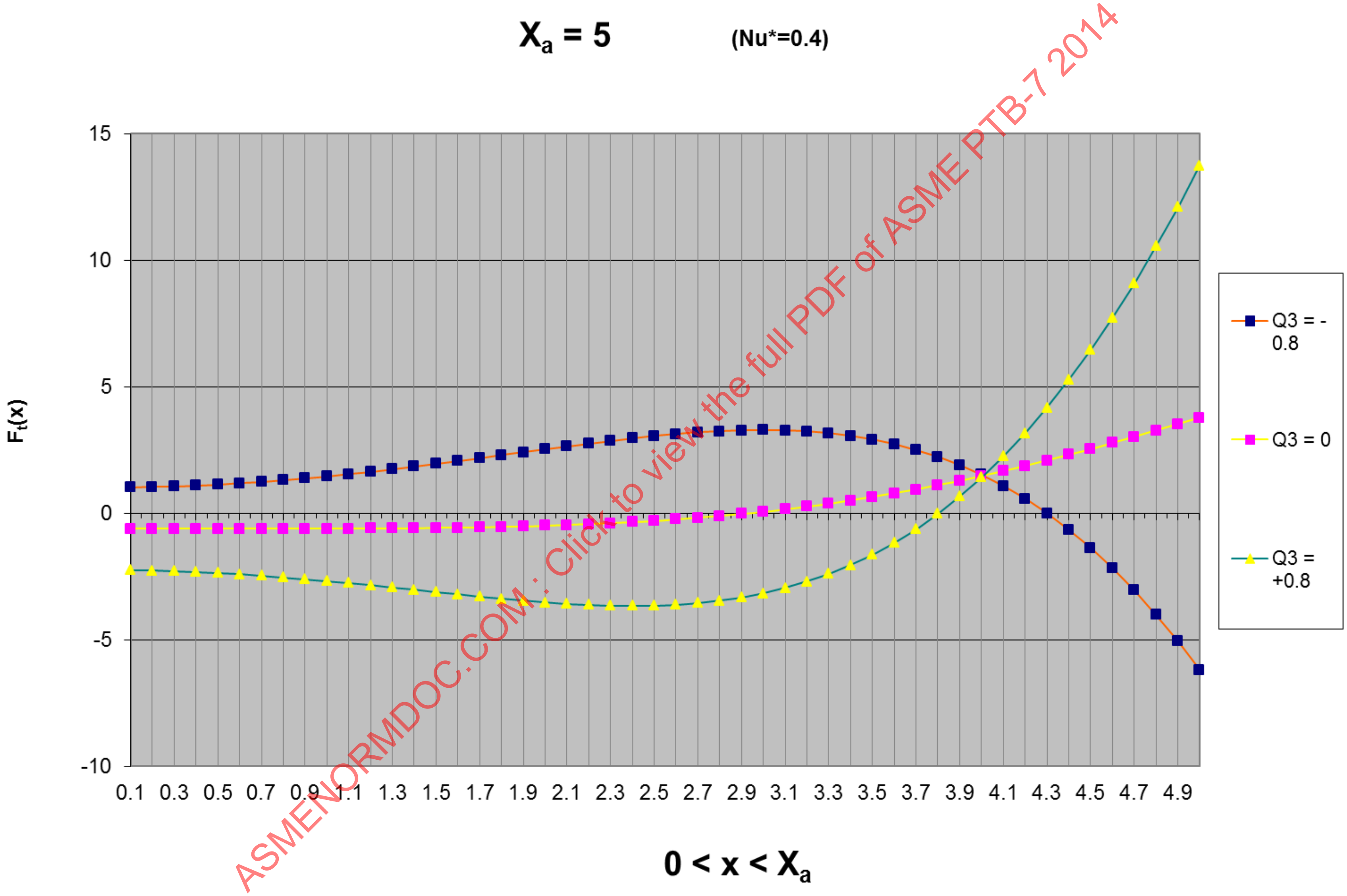
$F_q = (Z_d + Q_3 Z_w) X_a^4 / 2$

							Z(CODAP)>0										
Xa\Q3	**-0.8**	**-0.7**	**-0.6**	**-0.5**	**-0.4**	**-0.3**	**-0.2**	**-0.1**	**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
1	0.9565	0.9653	0.9742	0.9830	0.9919	1.0007	1.0095	1.0184	1.0272	1.0361	1.0449	1.0537	1.0626	1.0714	1.0803	1.0891	1.0979
2	0.3868	0.5103	0.6339	0.7574	0.8809	1.0044	1.1279	1.2514	1.3749	1.4985	1.6220	1.7455	1.8690	1.9925	2.1160	2.2395	2.3631
3	-1.1560	-0.7358	-0.3155	0.1048	0.5250	0.9453	1.3655	1.7858	2.2061	2.6263	3.0466	3.4669	3.8871	4.3074	4.7276	5.1479	5.5682
4	-3.3304	-2.5335	-1.7365	-0.9395	-0.1426	0.6544	1.4514	2.2483	3.0453	3.8423	4.6392	5.4362	6.2332	7.0301	7.8271	8.6241	9.4210
5	-6.1946	-4.9484	-3.7023	-2.4561	-1.2099	0.0363	1.2825	2.5286	3.7748	5.0210	6.2672	7.5134	8.7595	10.0057	11.2519	12.4981	13.7443
6	-9.9120	-8.1122	-6.3123	-4.5124	-2.7126	-0.9127	0.8871	2.6870	4.4868	6.2867	8.0866	9.8864	11.6863	13.4861	15.2860	17.0859	18.8857
7	-14.4624	-12.0043	-9.5461	-7.0880	-4.6299	-2.1718	0.2864	2.7445	5.2026	7.6607	10.1189	12.5770	15.0351	17.4933	19.9514	22.4095	24.8676
8	-19.8122	-16.5960	-13.3799	-10.1637	-6.9475	-3.7313	-0.5151	2.7010	5.9172	9.1334	12.3496	15.5658	18.7820	21.9981	25.2143	28.4305	31.6467
9	-25.9594	-21.8858	-17.8122	-13.7385	-9.6649	-5.5913	-1.5177	2.5560	6.6296	10.7032	14.7768	18.8504	22.9241	26.9977	31.0713	35.1449	39.2186
10	-32.9071	-27.8761	-22.8452	-17.8142	-12.7832	-7.7523	-2.7213	2.3097	7.3406	12.3716	17.4025	22.4335	27.4645	32.4954	37.5264	42.5574	47.5883
11	-40.6555	-34.5672	-28.4789	-22.3906	-16.3023	-10.2140	-4.1257	1.9626	8.0509	14.1392	20.2275	26.3158	32.4041	38.4924	44.5807	50.6690	56.7573
12	-49.2043	-41.9587	-34.7131	-27.4675	-20.2219	-12.9763	-5.7307	1.5149	8.7606	16.0062	23.2518	30.4974	37.7430	44.9886	52.2342	59.4798	66.7254
13	-58.5532	-50.0504	-41.5475	-33.0446	-24.5417	-16.0388	-7.5360	0.9669	9.4698	17.9727	26.4756	34.9784	43.4813	51.9842	60.4871	68.9899	77.4928
14	-68.7023	-58.8421	-48.9820	-39.1219	-29.2618	-19.4017	-9.5415	0.3186	10.1787	20.0388	29.8989	39.7591	49.6192	59.4793	69.3394	79.1996	89.0597
15	-79.6514	-68.3341	-57.0167	-45.6994	-34.3820	-23.0647	-11.7473	-0.4300	10.8874	22.2047	33.5220	44.8394	56.1567	67.4741	78.7914	90.1088	101.4261
16	-91.4006	-78.5261	-65.6515	-52.7770	-39.9024	-27.0279	-14.1533	-1.2788	11.5958	24.4703	37.3449	50.2194	63.0940	75.9685	88.8431	101.7176	114.5922
17	-103.9499	-89.4181	-74.8864	-60.3546	-45.8229	-31.2912	-16.7594	-2.2277	12.3040	26.8358	41.3675	55.8993	70.4310	84.9627	99.4945	114.0262	128.5579
18	-117.2992	-101.0102	-84.7213	-68.4324	-52.1435	-35.8546	-19.5657	-3.2768	13.0122	29.3011	45.5900	61.8789	78.1678	94.4568	110.7457	127.0346	143.3235
19	-131.4485	-113.3024	-95.1563	-77.0102	-58.8642	-40.7181	-22.5720	-4.4259	13.7202	31.8663	50.0123	68.1584	86.3045	104.4506	122.5967	140.7428	158.8888
20	-146.3978	-126.2946	-106.1914	-86.0881	-65.9849	-45.8816	-25.7784	-5.6752	14.4281	34.5313	54.6346	74.7378	94.8410	114.9443	135.0475	155.1508	175.2540

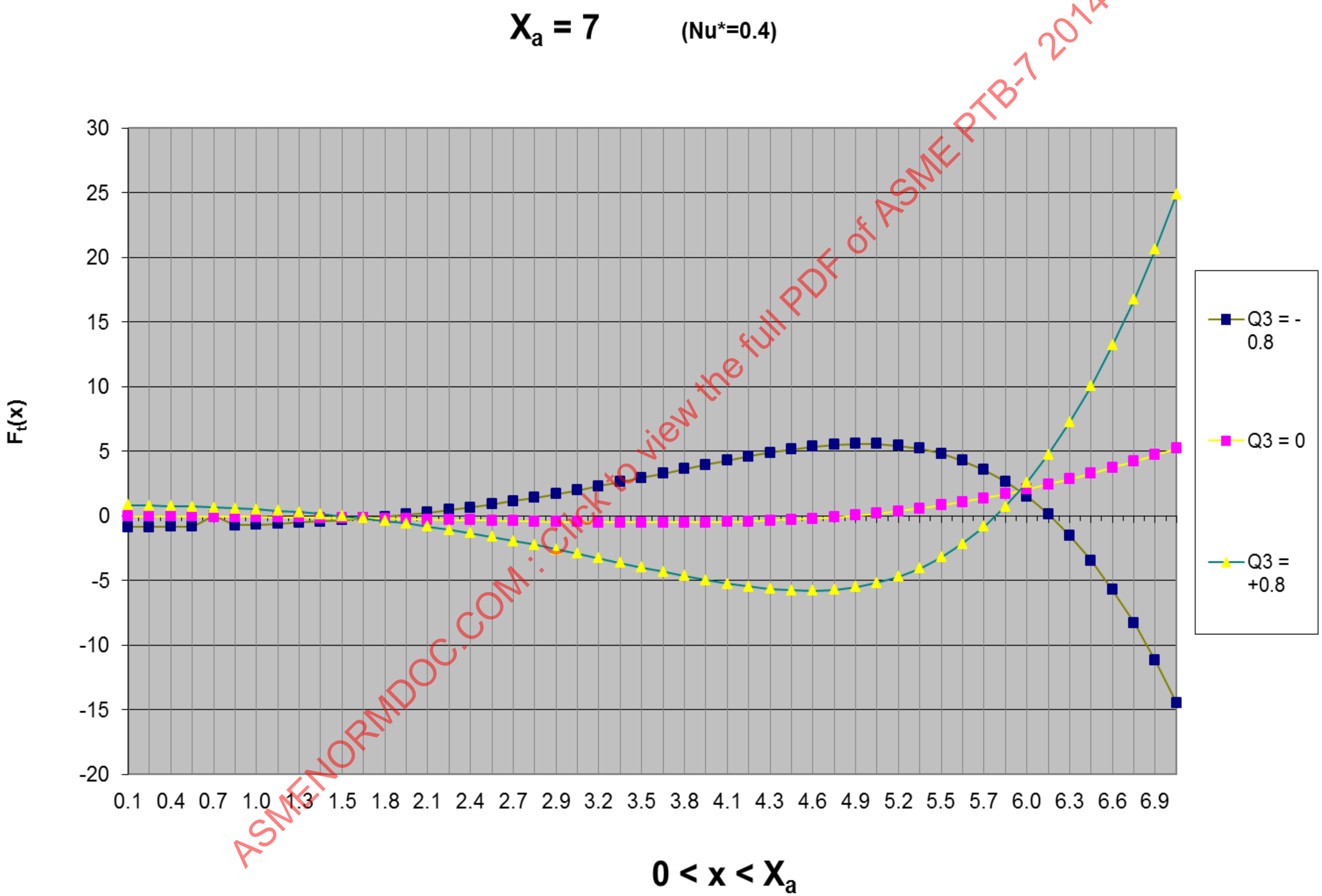
X_a = 1


X_a = 1 (Nu*=0.4)
F_t(x)
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.02 0.06 0.10 0.14 0.18 0.22 0.26 0.30 0.34 0.38 0.42 0.46 0.50 0.54 0.58 0.62 0.66 0.70 0.74 0.78 0.82 0.86 0.90 0.94 0.98
0 < x < X_a
Q3 = -0.8
Q3 = 0
Q3 = +0.8

$X_a = 5$

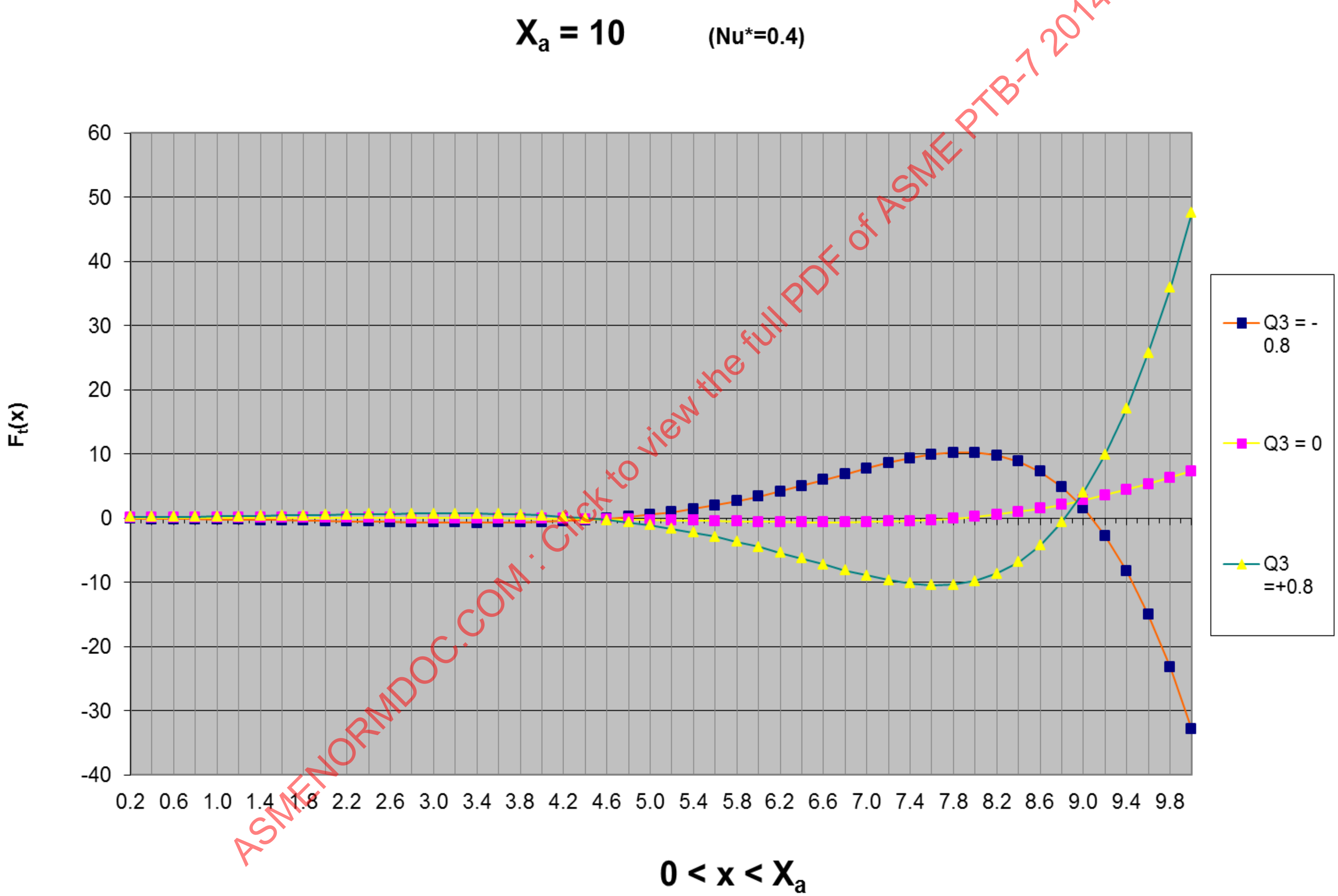

Xa = 5
(Nu*=0.4)
15
10
5
0
-5
-10
Ft(x)
0.1 0.3 0.5 0.7 0.9 1.1 1.3 1.5 1.7 1.9 2.1 2.3 2.5 2.7 2.9 3.1 3.3 3.5 3.7 3.9 4.1 4.3 4.5 4.7 4.9
0 < x < Xa
Q3 = -0.8
Q3 = 0
Q3 = +0.8

$X_a = 7$

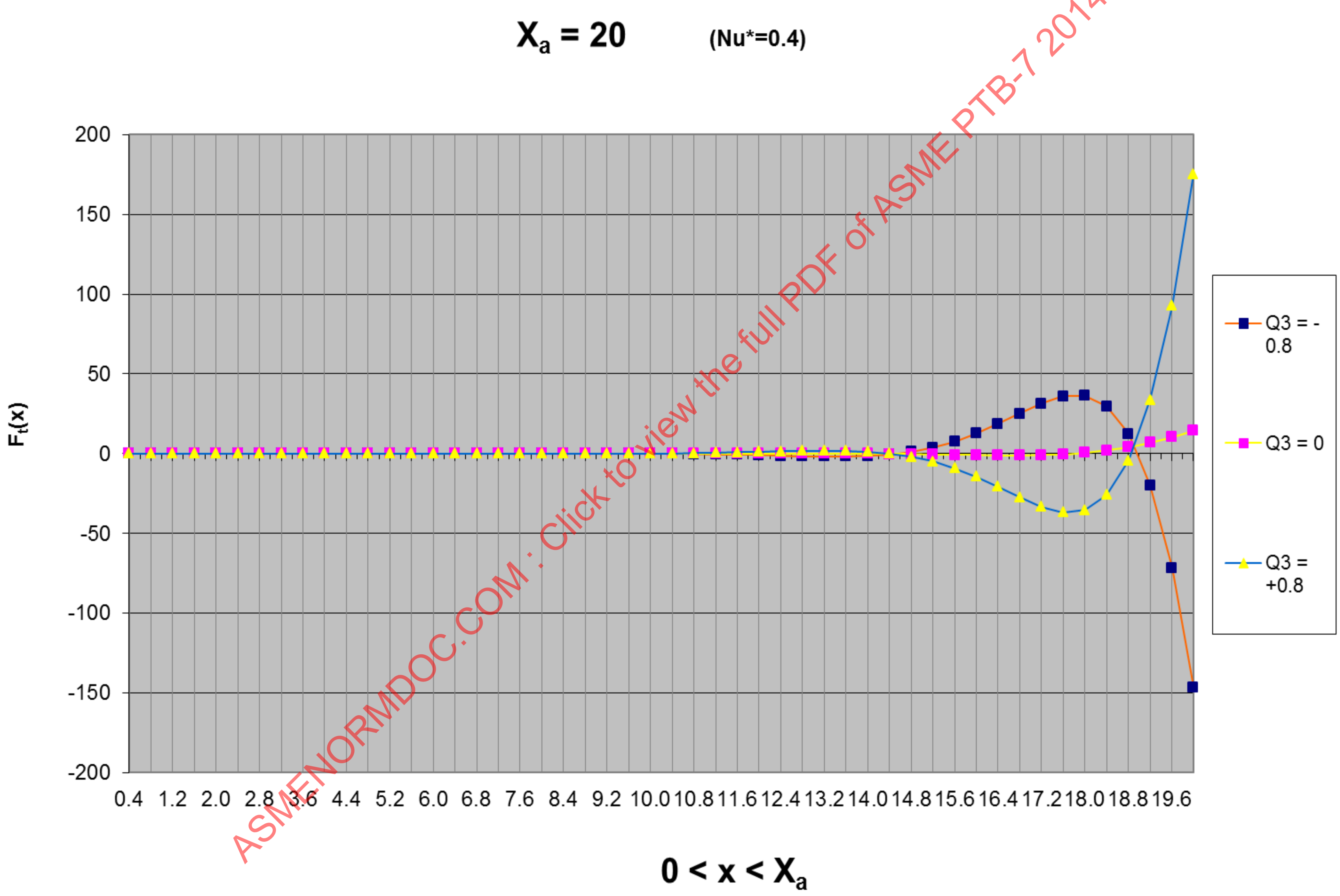


Xa = 7 (Nu*=0.4)
Ft(x)
30
25
20
15
10
5
0
-5
-10
-15
-20
0.1 0.4 0.7 1.0 1.3 1.5 1.8 2.1 2.4 2.7 2.9 3.2 3.5 3.8 4.1 4.3 4.6 4.9 5.2 5.5 5.7 6.0 6.3 6.6 6.9
0 < x < Xa
Q3 = -0.8
Q3 = 0
Q3 = +0.8

X_a = 10


Xa = 10
(Nu*=0.4)
60
50
40
30
20
10
0
-10
-20
-30
-40
Ft(x)
0.2 0.6 1.0 1.4 1.8 2.2 2.6 3.0 3.4 3.8 4.2 4.6 5.0 5.4 5.8 6.2 6.6 7.0 7.4 7.8 8.2 8.6 9.0 9.4 9.8
0 < x < Xa
Q3 = -0.8
Q3 = 0
Q3 =+0.8

$X_a = 20$


Xa = 20 (Nu*=0.4)
Ft(x)
200
150
100
50
0
-50
-100
-150
-200
0.4 1.2 2.0 2.8 3.6 4.4 5.2 6.0 6.8 7.6 8.4 9.2 10.0 10.8 11.6 12.4 13.2 14.0 14.8 15.6 16.4 17.2 18.0 18.8 19.6
0 < x < Xa
Q3 = -0.8
Q3 = 0
Q3 = +0.8

F_t,min vs X_a and Q_3

$F_{t,min}=MinF_t(x)$ (v* = 0.4)

	$\|F_{t,min}\| < F_{t,max}$		$\|F_{t,min}\| > F_{t,max}$			2 values for F_{inside}						$\|F_{t,min}\| < F_{t,max}$					
$X_a \backslash Q_3$	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
1	0.9565	0.9653	0.9742	0.9830	0.9919	0.9941	0.9853	0.9765	0.9677	0.9589	0.9501	0.9413	0.9325	0.9237	0.9149	0.9061	0.8973
2	0.3868	0.5103	0.6339	0.7574	0.8809	0.9159	0.7991	0.6822	0.5654	0.4486	0.3318	0.2150	0.0982	-0.0187	-0.1355	-0.2523	-0.3691
3	-1.1560	-0.7358	-0.3155	0.1048	0.5250	0.6932	0.3721	0.0510	-0.2702	-0.5913	-0.9124	-1.2335	-1.5546	-1.8757	-2.1968	-2.5179	-2.8390
4	-3.3304	-2.5335	-1.7365	-0.9395	-0.1426	0.3679	0.0062	-0.3555	-0.7171	-1.0788	-1.4404	-1.8021	-2.1637	-2.5254	-2.8871	-3.2487	-3.6104
5	-6.1946	-4.9484	-3.7023	-2.4561	-1.2099	0.0081	-0.1968	-0.4018	-0.6088	-0.8977	-1.2503	-1.6307	-2.0251	-2.4271	-2.8343	-3.2446	-3.6586
6	-9.9120	-8.1122	-6.3123	-4.5124	-2.7126	-0.9127	-0.2703	-0.2810	-0.5053	-0.9172	-1.3958	-1.9026	-2.4221	-2.9513	-3.4844	-4.0175	-4.5529
7	-14.4624	-12.0043	-9.5461	-7.0880	-4.6299	-2.1718	-0.2400	-0.2098	-0.5187	-1.0649	-1.6911	-2.3502	-3.0274	-3.7090	-4.3905	-5.0827	-5.7786
8	-19.8122	-16.5960	-13.3799	-10.1637	-6.9475	-3.7313	-0.5151	-0.1858	-0.5545	-1.2450	-2.0372	-2.8708	-3.7126	-4.5685	-5.4348	-6.3010	-7.1672
9	-25.9594	-21.8858	-17.8122	-13.7385	-9.6649	-5.5913	-1.5177	-0.1765	-0.5918	-1.4437	-2.4218	-3.4507	-4.4871	-5.5235	-6.5880	-7.6531	-8.7183
10	-32.9071	-27.8761	-22.8452	-17.8142	-12.7832	-7.7523	-2.7213	-0.1722	-0.6325	-1.6680	-2.8590	-4.0790	-5.3486	-6.6201	-7.8915	-9.1629	-10.4343
11	-40.6555	-34.5672	-28.4789	-22.3906	-16.3023	-10.2140	-4.1257	-0.1715	-0.6747	-1.9039	-3.3266	-4.7998	-6.3010	-7.8021	-9.3032	-10.8043	-12.3054
12	-49.2043	-41.9587	-34.7131	-27.4675	-20.2219	-12.9763	-5.7307	-0.1744	-0.7187	-2.1695	-3.8366	-5.5727	-7.3330	-9.0934	-10.8538	-12.6141	-14.3745
13	-58.5532	-50.0504	-41.5475	-33.0446	-24.5417	-16.0388	-7.5360	-0.1804	-0.7625	-2.4509	-4.4007	-6.3866	-8.4373	-10.4880	-12.5386	-14.5893	-16.6400
14	-68.7023	-58.8422	-48.9820	-39.1219	-29.2618	-19.4017	-9.5415	-0.1883	-0.8077	-2.7544	-5.0134	-7.2927	-9.5720	-11.9125	-14.2749	-16.6373	-18.9996
15	-79.6514	-68.3341	-57.0167	-45.6994	-34.3820	-23.0647	-11.7473	-0.4300	-0.8527	-3.0967	-5.6341	-8.2827	-10.9313	-13.5798	-16.2284	-18.8770	-21.5255
16	-91.4006	-78.5261	-65.6515	-52.7770	-39.9024	-27.0279	-14.1533	-1.2788	-0.8906	-3.4346	-6.3231	-9.2117	-12.2061	-15.2319	-18.2577	-21.2834	-24.3092
17	-103.9499	-89.4181	-74.8864	-60.3546	-45.8229	-31.2912	-16.7594	-2.2277	-0.9428	-3.8003	-7.0446	-10.3976	-13.7506	-17.1037	-20.4567	-23.8098	-27.1628
18	-117.2992	-101.0102	-84.7213	-68.4324	-52.1435	-35.8546	-19.5657	-3.2767	-0.9864	-4.1867	-7.8225	-11.4584	-15.0942	-18.7671	-22.5323	-26.2975	-30.0627
19	-131.4485	-113.3024	-95.1563	-77.0102	-58.8642	-40.7181	-22.5720	-4.4259	-1.0333	-4.6054	-8.5301	-12.7017	-16.8732	-21.0448	-25.2164	-29.3880	-33.5595
20	-146.3978	-126.2946	-106.1914	-86.0881	-65.9849	-45.8816	-25.7784	-5.6752	-1.0736	-4.9554	-9.4954	-14.0354	-18.5754	-23.1154	-27.6554	-32.1954	-36.7354

x(min)

			x(min)=X_a						x(min)=0			0<x(min)<X_a					
$X_a \backslash Q_3$	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
1	1.0000	1.0000	1.0000	1.0000	1.0000	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200
2	2.0000	2.0000	2.0000	2.0000	2.0000	0.0400	0.0400	0.0400	0.0400	0.0400	0.0400	0.0400	0.0400	0.0400	0.0400	0.0400	0.0400
3	3.0000	3.0000	3.0000	3.0000	3.0000	0.0600	0.0600	0.0600	0.0600	0.0600	0.0600	0.0600	0.0600	0.0600	0.0600	0.0600	0.0600
4	4.0000	4.0000	4.0000	4.0000	4.0000	0.0800	0.0800	0.0800	0.0800	0.0800	0.0800	0.0800	0.0800	0.0800	0.0800	0.0800	0.0800
5	5.0000	5.0000	5.0000	5.0000	5.0000	0.1000	0.1000	0.1000	0.7000	1.6000	1.9000	2.1000	2.2000	2.3000	2.3000	2.4000	2.4000
6	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	0.1200	0.9600	2.5200	3.0000	3.2400	3.3600	3.3600	3.4800	3.4800	3.4800	3.6000
7	7.0000	7.0000	7.0000	7.0000	7.0000	7.0000	0.1400	2.3800	3.6400	4.0600	4.3400	4.3400	4.4800	4.4800	4.4800	4.6200	4.6200
8	8.0000	8.0000	8.0000	8.0000	8.0000	8.0000	8.0000	3.2000	4.6400	5.1200	5.2800	5.4400	5.4400	5.6000	5.6000	5.6000	5.6000
9	9.0000	9.0000	9.0000	9.0000	9.0000	9.0000	9.0000	4.1400	5.5800	6.1200	6.3000	6.4800	6.4800	6.4800	6.6600	6.6600	6.6600
10	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	5.0000	6.6000	7.2000	7.4000	7.4000	7.6000	7.6000	7.6000	7.6000	7.6000
11	11.0000	11.0000	11.0000	11.0000	11.0000	11.0000	11.0000	5.7200	7.7000	8.1400	8.3600	8.5800	8.5800	8.5800	8.5800	8.5800	8.5800
12	12.0000	12.0000	12.0000	12.0000	12.0000	12.0000	12.0000	6.7200	8.6400	9.3600	9.3600	9.6000	9.6000	9.6000	9.6000	9.6000	9.6000
13	13.0000	13.0000	13.0000	13.0000	13.0000	13.0000	13.0000	7.5400	9.6200	10.4000	10.4000	10.6600	10.6600	10.6600	10.6600	10.6600	10.6600
14	14.0000	14.0000	14.0000	14.0000	14.0000	14.0000	14.0000	8.4000	10.6400	11.2000	11.4800	11.4800	11.4800	11.7600	11.7600	11.7600	11.7600
15	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	11.7000	12.3000	12.6000	12.6000	12.6000	12.6000	12.6000	12.6000	12.6000
16	16.0000	16.0000	16.0000	16.0000	16.0000	16.0000	16.0000	16.0000	12.8000	13.4400	13.4400	13.4400	13.7600	13.7600	13.7600	13.7600	13.7600
17	17.0000	17.0000	17.0000	17.0000	17.0000	17.0000	17.0000	17.0000	13.6000	14.2800	14.6200	14.6200	14.6200	14.6200	14.6200	14.6200	14.6200
18	18.0000	18.0000	18.0000	18.0000	18.0000	18.0000	18.0000	18.0000	14.7600	15.4800	15.4800	15.4800	15.4800	15.8400	15.8400	15.8400	15.8400
19	19.0000	19.0000	19.0000	19.0000	19.0000	19.0000	19.0000	19.0000	15.5800	16.3400	16.7200	16.7200	16.7200	16.7200	16.7200	16.7200	16.7200
20	20.0000	20.0000	20.0000	20.0000	20.0000	20.0000	20.0000	20.0000	16.8000	17.6000	17.6000	17.6000	17.6000	17.6000	17.6000	17.6000	17.6000

F_t,max vs X_a and Q_3

$F_{t,max}$=Max$F_t(x)$ ($v^* = 0.4$)

	$F_{t,max}$>I $F_{t,min}$ I		$F_{t,max}$<I $F_{t,min}$ I			2 values for $F_{t,inside}$							$F_{t,max}$>I $F_{t,min}$ I				
X_a\Q_3	**-0.8**	**-0.7**	**-0.6**	**-0.5**	**-0.4**	**-0.3**	**-0.2**	**-0.1**	**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
1	1.0381	1.0293	1.0205	1.0117	1.0032	1.0020	1.0095	1.0184	1.0272	1.0361	1.0449	1.0537	1.0626	1.0714	1.0803	1.0891	1.0979
2	1.5000	1.3831	1.2663	1.1495	1.0438	1.0284	1.1279	1.2514	1.3749	1.4985	1.6220	1.7455	1.8690	1.9925	2.1160	2.2395	2.3631
3	2.2987	1.9776	1.6569	1.3670	1.1574	1.1085	1.3655	1.7858	2.2061	2.6263	3.0466	3.4669	3.8871	4.3074	4.7276	5.1479	5.5682
4	2.5990	2.2601	1.9361	1.6366	1.3897	1.2671	1.4891	2.2483	3.0453	3.8423	4.6392	5.4362	6.2332	7.0301	7.8271	8.6241	9.4211
5	3.2877	2.8846	2.4951	2.1198	1.7810	1.5293	1.5752	2.5286	3.7748	5.0210	6.2672	7.5134	8.7595	10.0057	11.2519	12.4981	13.7443
6	4.3381	3.8047	3.2892	2.7797	2.3001	1.8883	1.7178	2.6870	4.4868	6.2867	8.0866	9.8864	11.6863	13.4861	15.2860	17.0859	18.8857
7	5.5769	4.8917	4.2196	3.5475	2.9076	2.3198	1.9388	2.7445	5.2026	7.6607	10.1189	12.5770	15.0351	17.4933	19.9514	22.4095	24.8676
8	6.9813	6.1174	5.2535	4.4218	3.5926	2.8188	2.2248	2.7525	5.9172	9.1334	12.3496	15.5658	18.7820	21.9981	25.2143	28.4305	31.6467
9	8.5253	7.4676	6.4331	5.3985	4.3640	3.3954	2.5710	2.7834	6.6296	10.7032	14.7768	18.8504	22.9241	26.9977	31.0713	35.1449	39.2186
10	10.2342	8.9829	7.7319	6.4808	5.2297	4.0376	2.9645	2.8478	7.3406	12.3716	17.4025	22.4335	27.4645	32.4954	37.5264	42.5574	47.5883
11	12.1389	10.6157	9.1361	7.6613	6.1866	4.7219	3.4132	2.9468	8.0509	14.1392	20.2275	26.3158	32.4041	38.4924	44.5807	50.6690	56.7573
12	14.2426	12.4641	10.6857	8.9072	7.2032	5.5109	3.9181	3.0883	8.7606	16.0062	23.2518	30.4974	37.7430	44.9886	52.2342	59.4798	66.7254
13	16.4752	14.4356	12.3960	10.3564	8.3168	6.3145	4.4432	3.2804	9.4698	17.9727	26.4756	34.9784	43.4813	51.9842	60.4871	68.9899	77.4928
14	18.7983	16.4360	14.0736	11.7904	9.5167	7.2430	5.0294	3.5033	10.1787	20.0388	29.8989	39.7591	49.6192	59.4793	69.3394	79.1996	89.0597
15	21.4523	18.7898	16.1272	13.4647	10.8021	8.1396	5.6805	3.7577	10.8874	22.2047	33.5220	44.8394	56.1567	67.4741	78.7914	90.1088	101.4261
16	24.1031	21.0773	18.0516	15.0258	12.0815	9.2086	6.3356	4.0431	11.5958	24.4703	37.3449	50.2194	63.0940	75.9685	88.8431	101.7176	114.5922
17	26.9523	23.6232	20.2940	16.9649	13.6357	10.3065	6.9774	4.3578	12.3040	26.8358	41.3675	55.8993	70.4310	84.9627	99.4945	114.0262	128.5579
18	30.1805	26.4153	22.6501	18.8849	15.1197	11.3545	7.8050	4.6992	13.0122	29.3011	45.5900	61.8789	78.1678	94.4567	110.7457	127.0346	143.3235
19	33.1856	29.0141	24.8425	20.6709	16.5078	12.5729	8.6379	5.0638	13.7202	31.8663	50.0123	68.1584	86.3045	104.4506	122.5967	140.7428	158.8888
20	36.3441	31.8634	27.3828	22.9021	18.4214	13.9407	9.4600	5.4475	14.4281	34.5313	54.6346	74.7378	94.8410	114.9443	135.0475	155.1508	175.2540

x(max)

	x(max)=0			0<x(max)<X_a								x(max)=X_a					
X_a\Q_3	**-0.8**	**-0.7**	**-0.6**	**-0.5**	**-0.4**	**-0.3**	**-0.2**	**-0.1**	**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**
1	0.0200	0.0200	0.0200	0.0200	0.3800	0.8400	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	0.0400	0.0400	0.0400	0.0400	0.9200	1.6400	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000
3	0.0600	0.0600	0.3600	1.0800	1.7400	2.4000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000
4	1.8400	1.9200	2.0800	2.3200	2.5600	3.0400	3.7600	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000
5	3.0000	3.0000	3.1000	3.2000	3.4000	3.8000	4.4000	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000
6	3.9600	4.0800	4.0800	4.2000	4.3200	4.5600	5.1600	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000
7	4.9000	5.0400	5.0400	5.0400	5.1800	5.4600	5.8800	7.0000	7.0000	7.0000	7.0000	7.0000	7.0000	7.0000	7.0000	7.0000	7.0000
8	5.9200	5.9200	5.9200	6.0800	6.0800	6.2400	6.7200	7.8400	8.0000	8.0000	8.0000	8.0000	8.0000	8.0000	8.0000	8.0000	8.0000
9	6.8400	7.0200	7.0200	7.0200	7.0200	7.2000	7.5600	8.6400	9.0000	9.0000	9.0000	9.0000	9.0000	9.0000	9.0000	9.0000	9.0000
10	7.8000	8.0000	8.0000	8.0000	8.0000	8.2000	8.4000	9.4000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000
11	8.8000	8.8000	9.0200	9.0200	9.0200	9.2400	9.4600	10.3400	11.0000	11.0000	11.0000	11.0000	11.0000	11.0000	11.0000	11.0000	11.0000
12	9.8400	9.8400	9.8400	9.8400	10.0800	10.0800	10.3200	11.0400	12.0000	12.0000	12.0000	12.0000	12.0000	12.0000	12.0000	12.0000	12.0000
13	10.9200	10.9200	10.9200	10.9200	10.9200	11.1800	11.1800	11.9600	13.0000	13.0000	13.0000	13.0000	13.0000	13.0000	13.0000	13.0000	13.0000
14	11.7600	11.7600	11.7600	12.0400	12.0400	12.0400	12.3200	12.8800	14.0000	14.0000	14.0000	14.0000	14.0000	14.0000	14.0000	14.0000	14.0000
15	12.9000	12.9000	12.9000	12.9000	12.9000	12.9000	13.2000	13.8000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000
16	13.7600	13.7600	13.7600	13.7600	14.0800	14.0800	14.0800	14.7200	16.0000	16.0000	16.0000	16.0000	16.0000	16.0000	16.0000	16.0000	16.0000
17	14.9600	14.9600	14.9600	14.9600	14.9600	14.9600	14.9600	15.6400	17.0000	17.0000	17.0000	17.0000	17.0000	17.0000	17.0000	17.0000	17.0000
18	15.8400	15.8400	15.8400	15.8400	15.8400	15.8400	16.2000	16.5600	18.0000	18.0000	18.0000	18.0000	18.0000	18.0000	18.0000	18.0000	18.0000
19	16.7200	16.7200	16.7200	16.7200	17.1000	17.1000	17.1000	17.4800	19.0000	19.0000	19.0000	19.0000	19.0000	19.0000	19.0000	19.0000	19.0000
20	18.0000	18.0000	18.0000	18.0000	18.0000	18.0000	18.0000	18.4000	20.0000	20.0000	20.0000	20.0000	20.0000	20.0000	20.0000	20.0000	20.0000

$F_t(X_a)$ and F_q

$F_t(X_a)$

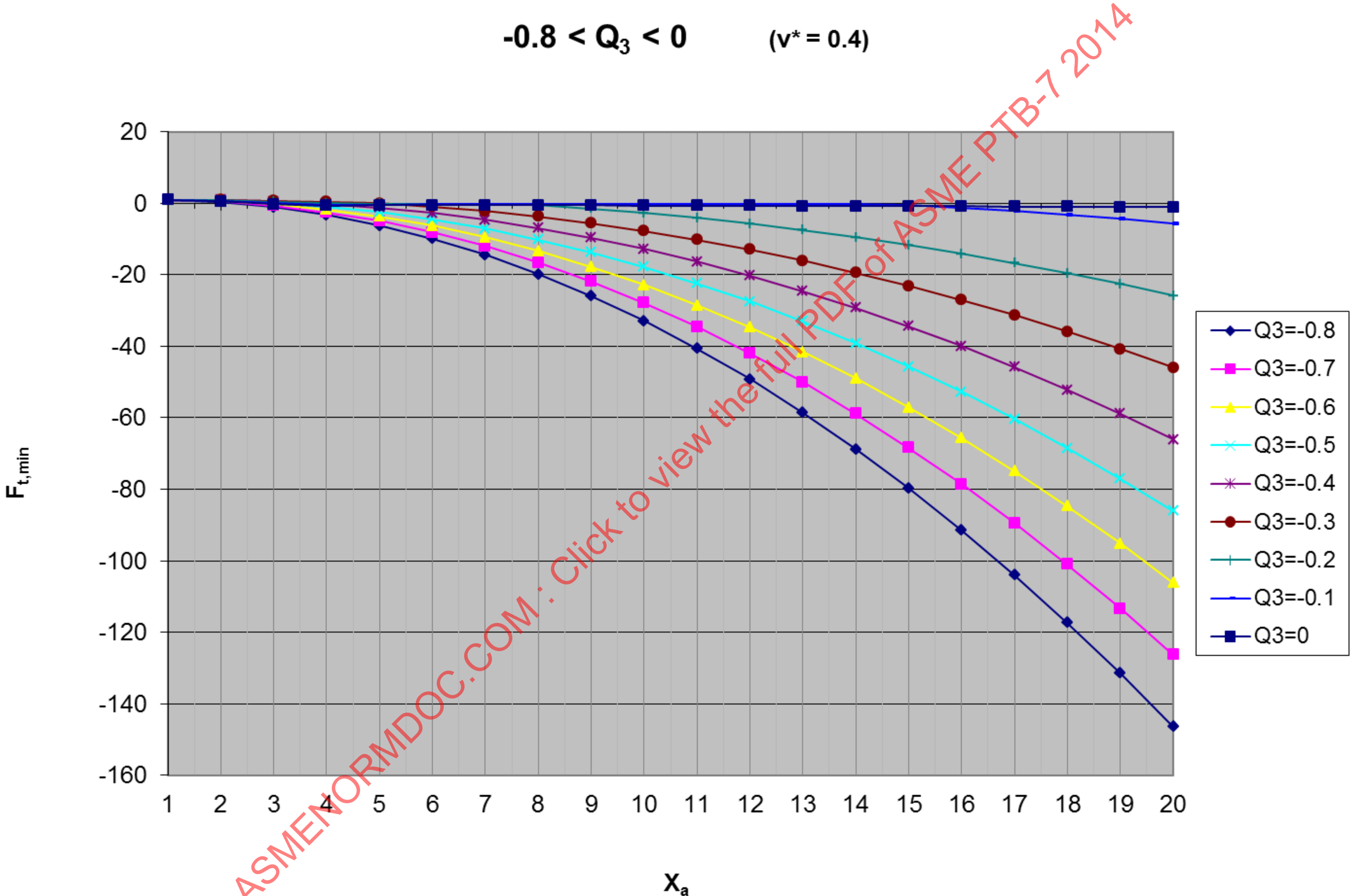
($v^* = 0.4$)

X_a\Q_3			$F_q=F_{t,min}$			$F_{t,min}<F_q<F_{t,max}$						$F_q=F_{t,max}$					
	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
1	0.9565	0.9653	0.9742	0.9830	0.9919	1.0007	1.0095	1.0184	1.0272	1.0361	1.0449	1.0537	1.0626	1.0714	1.0803	1.0891	1.0979
2	0.3868	0.5103	0.6339	0.7574	0.8809	1.0044	1.1279	1.2514	1.3749	1.4985	1.6220	1.7455	1.8690	1.9925	2.1160	2.2395	2.3631
3	-1.1560	-0.7358	-0.3155	0.1048	0.5250	0.9453	1.3655	1.7858	2.2061	2.6263	3.0466	3.4669	3.8871	4.3074	4.7276	5.1479	5.5682
4	-3.3304	-2.5335	-1.7365	-0.9395	-0.1426	0.6544	1.4514	2.2483	3.0453	3.8423	4.6392	5.4362	6.2332	7.0301	7.8271	8.6241	9.4211
5	-6.1946	-4.9484	-3.7023	-2.4561	-1.2099	0.0363	1.2825	2.5286	3.7748	5.0210	6.2672	7.5134	8.7595	10.0057	11.2519	12.4981	13.7443
6	-9.9120	-8.1122	-6.3123	-4.5124	-2.7126	-0.9127	0.8871	2.6870	4.4868	6.2867	8.0866	9.8864	11.6863	13.4861	15.2860	17.0859	18.8857
7	-14.4624	-12.0043	-9.5461	-7.0880	-4.6299	-2.1718	0.2864	2.7445	5.2026	7.6607	10.1189	12.5770	15.0351	17.4933	19.9514	22.4095	24.8676
8	-19.8122	-16.5960	-13.3799	-10.1637	-6.9475	-3.7313	-0.5151	2.7010	5.9172	9.1334	12.3496	15.5658	18.7820	21.9981	25.2143	28.4305	31.6467
9	-25.9594	-21.8858	-17.8122	-13.7385	-9.6649	-5.5913	-1.5177	2.5560	6.6296	10.7032	14.7768	18.8504	22.9241	26.9977	31.0713	35.1449	39.2186
10	-32.9071	-27.8761	-22.8452	-17.8142	-12.7832	-7.7523	-2.7213	2.3097	7.3406	12.3716	17.4025	22.4335	27.4645	32.4954	37.5264	42.5574	47.5883
11	-40.6555	-34.5672	-28.4789	-22.3906	-16.3023	-10.2140	-4.1257	1.9626	8.0509	14.1392	20.2275	26.3158	32.4041	38.4924	44.5807	50.6690	56.7573
12	-49.2043	-41.9587	-34.7131	-27.4675	-20.2219	-12.9763	-5.7307	1.5149	8.7606	16.0062	23.2518	30.4974	37.7430	44.9886	52.2342	59.4798	66.7254
13	-58.5532	-50.0504	-41.5475	-33.0446	-24.5417	-16.0388	-7.5360	0.9669	9.4698	17.9727	26.4756	34.9784	43.4813	51.9842	60.4871	68.9899	77.4928
14	-68.7023	-58.8422	-48.9820	-39.1219	-29.2618	-19.4017	-9.5415	0.3186	10.1787	20.0388	29.8989	39.7591	49.6192	59.4793	69.3394	79.1996	89.0597
15	-79.6514	-68.3341	-57.0167	-45.6994	-34.3820	-23.0647	-11.7473	-0.4300	10.8874	22.2047	33.5220	44.8394	56.1567	67.4741	78.7914	90.1088	101.4261
16	-91.4006	-78.5261	-65.6515	-52.7770	-39.9024	-27.0279	-14.1533	-1.2788	11.5958	24.4703	37.3449	50.2194	63.0940	75.9685	88.8431	101.7176	114.5922
17	-103.9499	-89.4181	-74.8864	-60.3546	-45.8229	-31.2912	-16.7594	-2.2277	12.3040	26.8358	41.3675	55.8993	70.4310	84.9627	99.4945	114.0262	128.5579
18	-117.2992	-101.0102	-84.7213	-68.4324	-52.1435	-35.8546	-19.5657	-3.2767	13.0122	29.3011	45.5900	61.8789	78.1678	94.4567	110.7457	127.0346	143.3235
19	-131.4485	-113.3024	-95.1563	-77.0102	-58.8642	-40.7181	-22.5720	-4.4259	13.7202	31.8663	50.0123	68.1584	86.3045	104.4506	122.5967	140.7428	158.8888
20	-146.3978	-126.2946	-106.1914	-86.0881	-65.9849	-45.8816	-25.7784	-5.6752	14.4281	34.5313	54.6346	74.7378	94.8410	114.9443	135.0475	155.1508	175.2540

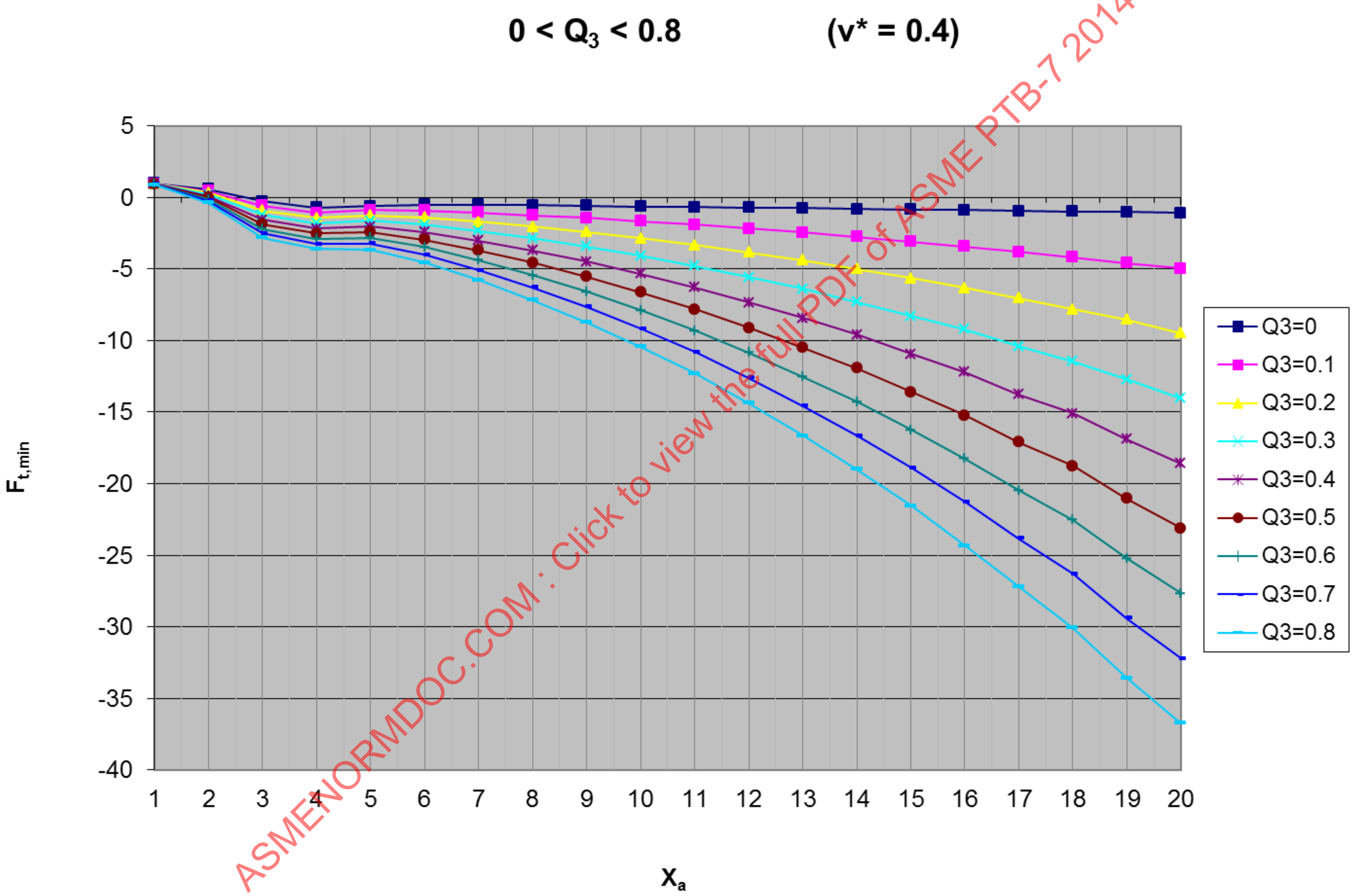
$F_q = (Z_d + Q_3 Z_v) X_a^4/2$

X_a\Q_3							Z(CODAP)>0										
	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
1	0.9565	0.9653	0.9742	0.9830	0.9919	1.0007	1.0095	1.0184	1.0272	1.0361	1.0449	1.0537	1.0626	1.0714	1.0803	1.0891	1.0979
2	0.3868	0.5103	0.6339	0.7574	0.8809	1.0044	1.1279	1.2514	1.3749	1.4985	1.6220	1.7455	1.8690	1.9925	2.1160	2.2395	2.3631
3	-1.1560	-0.7358	-0.3155	0.1048	0.5250	0.9453	1.3655	1.7858	2.2061	2.6263	3.0466	3.4669	3.8871	4.3074	4.7276	5.1479	5.5682
4	-3.3304	-2.5335	-1.7365	-0.9395	-0.1426	0.6544	1.4514	2.2483	3.0453	3.8423	4.6392	5.4362	6.2332	7.0301	7.8271	8.6241	9.4210
5	-6.1946	-4.9484	-3.7023	-2.4561	-1.2099	0.0363	1.2825	2.5286	3.7748	5.0210	6.2672	7.5134	8.7595	10.0057	11.2519	12.4981	13.7443
6	-9.9120	-8.1122	-6.3123	-4.5124	-2.7126	-0.9127	0.8871	2.6870	4.4868	6.2867	8.0866	9.8864	11.6863	13.4861	15.2860	17.0859	18.8857
7	-14.4624	-12.0043	-9.5461	-7.0880	-4.6299	-2.1718	0.2864	2.7445	5.2026	7.6607	10.1189	12.5770	15.0351	17.4933	19.9514	22.4095	24.8676
8	-19.8122	-16.5960	-13.3799	-10.1637	-6.9475	-3.7313	-0.5151	2.7010	5.9172	9.1334	12.3496	15.5658	18.7820	21.9981	25.2143	28.4305	31.6467
9	-25.9594	-21.8858	-17.8122	-13.7385	-9.6649	-5.5913	-1.5177	2.5560	6.6296	10.7032	14.7768	18.8504	22.9241	26.9977	31.0713	35.1449	39.2186
10	-32.9071	-27.8761	-22.8452	-17.8142	-12.7832	-7.7523	-2.7213	2.3097	7.3406	12.3716	17.4025	22.4335	27.4645	32.4954	37.5264	42.5574	47.5883
11	-40.6555	-34.5672	-28.4789	-22.3906	-16.3023	-10.2140	-4.1257	1.9626	8.0509	14.1392	20.2275	26.3158	32.4041	38.4924	44.5807	50.6690	56.7573
12	-49.2043	-41.9587	-34.7131	-27.4675	-20.2219	-12.9763	-5.7307	1.5149	8.7606	16.0062	23.2518	30.4974	37.7430	44.9886	52.2342	59.4798	66.7254
13	-58.5532	-50.0504	-41.5475	-33.0446	-24.5417	-16.0388	-7.5360	0.9669	9.4698	17.9727	26.4756	34.9784	43.4813	51.9842	60.4871	68.9899	77.4928
14	-68.7023	-58.8421	-48.9820	-39.1219	-29.2618	-19.4017	-9.5415	0.3186	10.1787	20.0388	29.8989	39.7591	49.6192	59.4793	69.3394	79.1996	89.0597
15	-79.6514	-68.3341	-57.0167	-45.6994	-34.3820	-23.0647	-11.7473	-0.4300	10.8874	22.2047	33.5220	44.8394	56.1567	67.4741	78.7914	90.1088	101.4261
16	-91.4006	-78.5261	-65.6515	-52.7770	-39.9024	-27.0279	-14.1533	-1.2788	11.5958	24.4703	37.3449	50.2194	63.0940	75.9685	88.8431	101.7176	114.5922
17	-103.9499	-89.4181	-74.8864	-60.3546	-45.8229	-31.2912	-16.7594	-2.2277	12.3040	26.8358	41.3675	55.8993	70.4310	84.9627	99.4945	114.0262	128.5579
18	-117.2992	-101.0102	-84.7213	-68.4324	-52.1435	-35.8546	-19.5657	-3.2768	13.0122	29.3011	45.5900	61.8789	78.1678	94.4568	110.7457	127.0346	143.3235
19	-131.4485	-113.3024	-95.1563	-77.0102	-58.8642	-40.7181	-22.5720	-4.4259	13.7202	31.8663	50.0123	68.1584	86.3045	104.4506	122.5967	140.7428	158.8888
20	-146.3978	-126.2946	-106.1914	-86.0881	-65.9849	-45.8816	-25.7784	-5.6752	14.4281	34.5313	54.6346	74.7378	94.8410	114.9443	135.0475	155.1508	175.2540

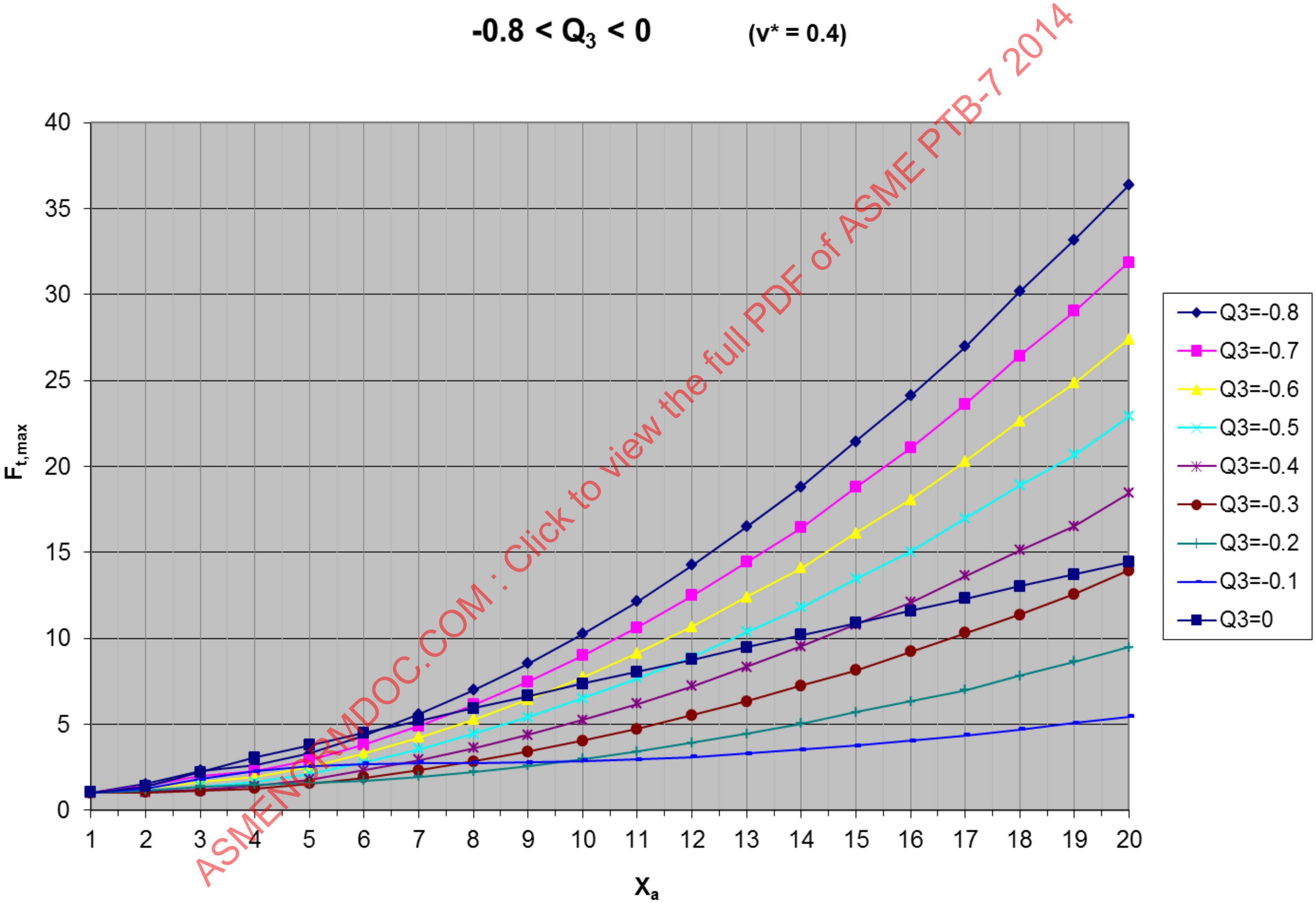
F_t,min for -0.8<Q_3<0.0



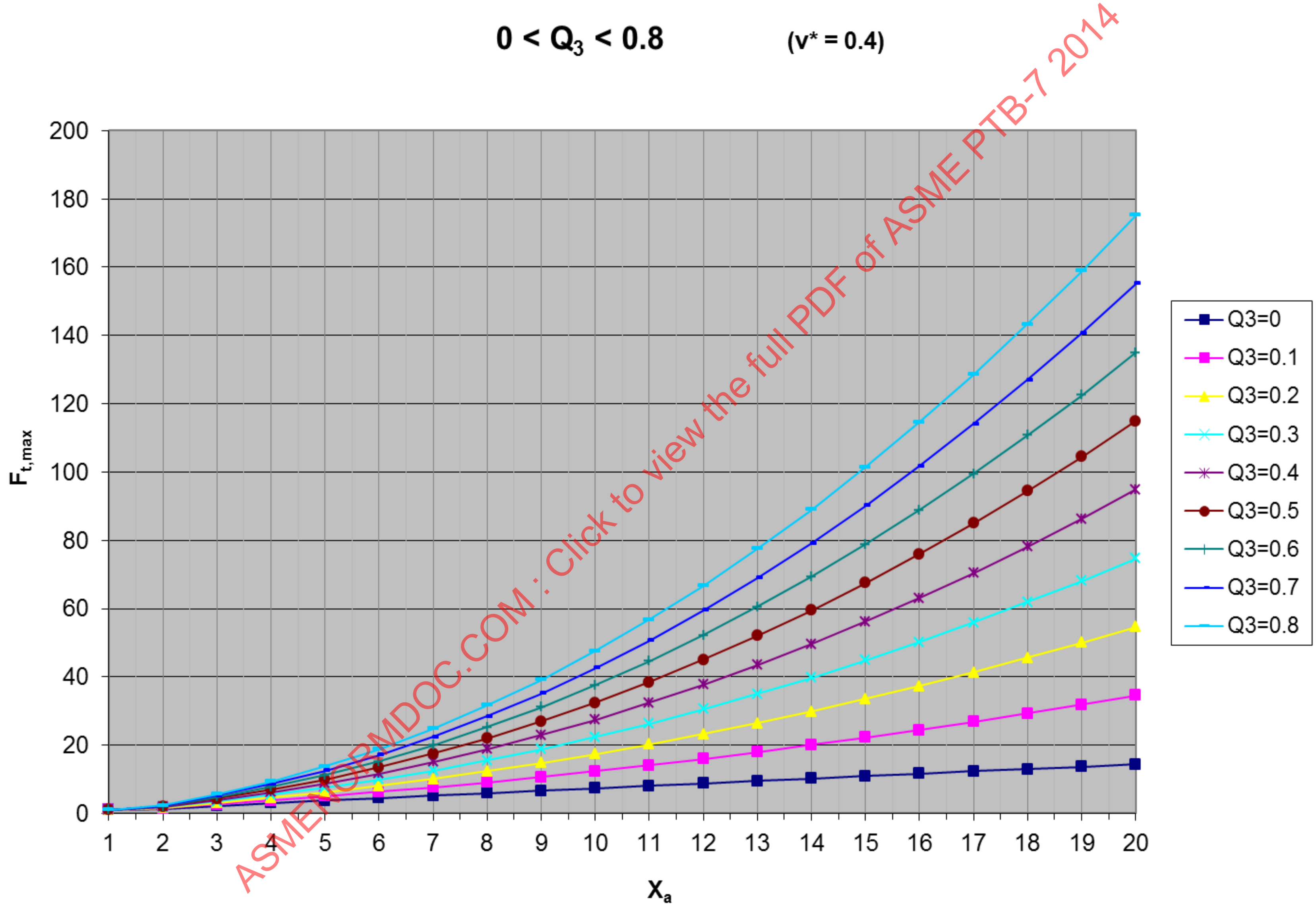
F_t,min for 0<Q_3<0.8



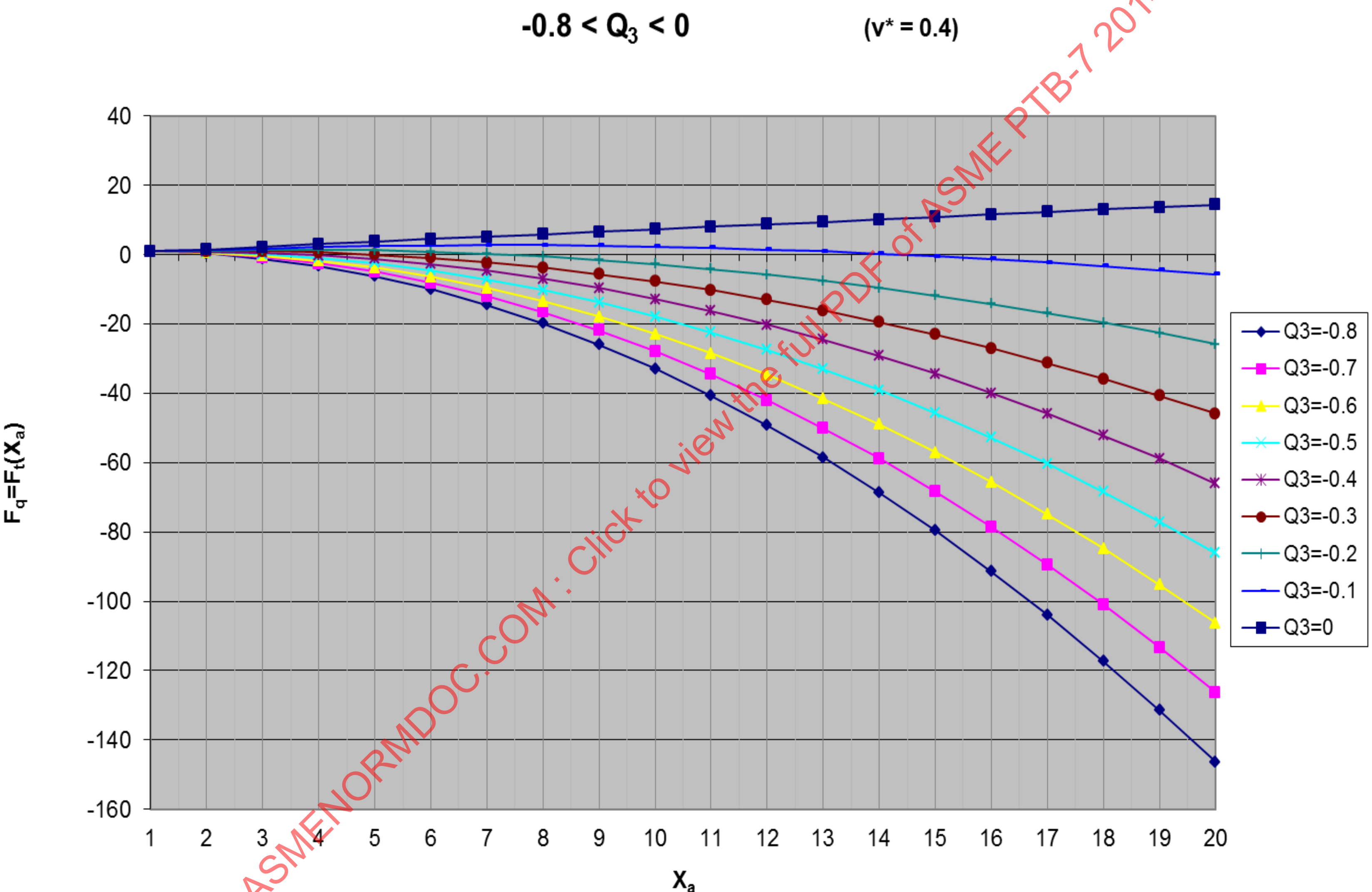
F_t,max for -0.8<Q_3<0.0



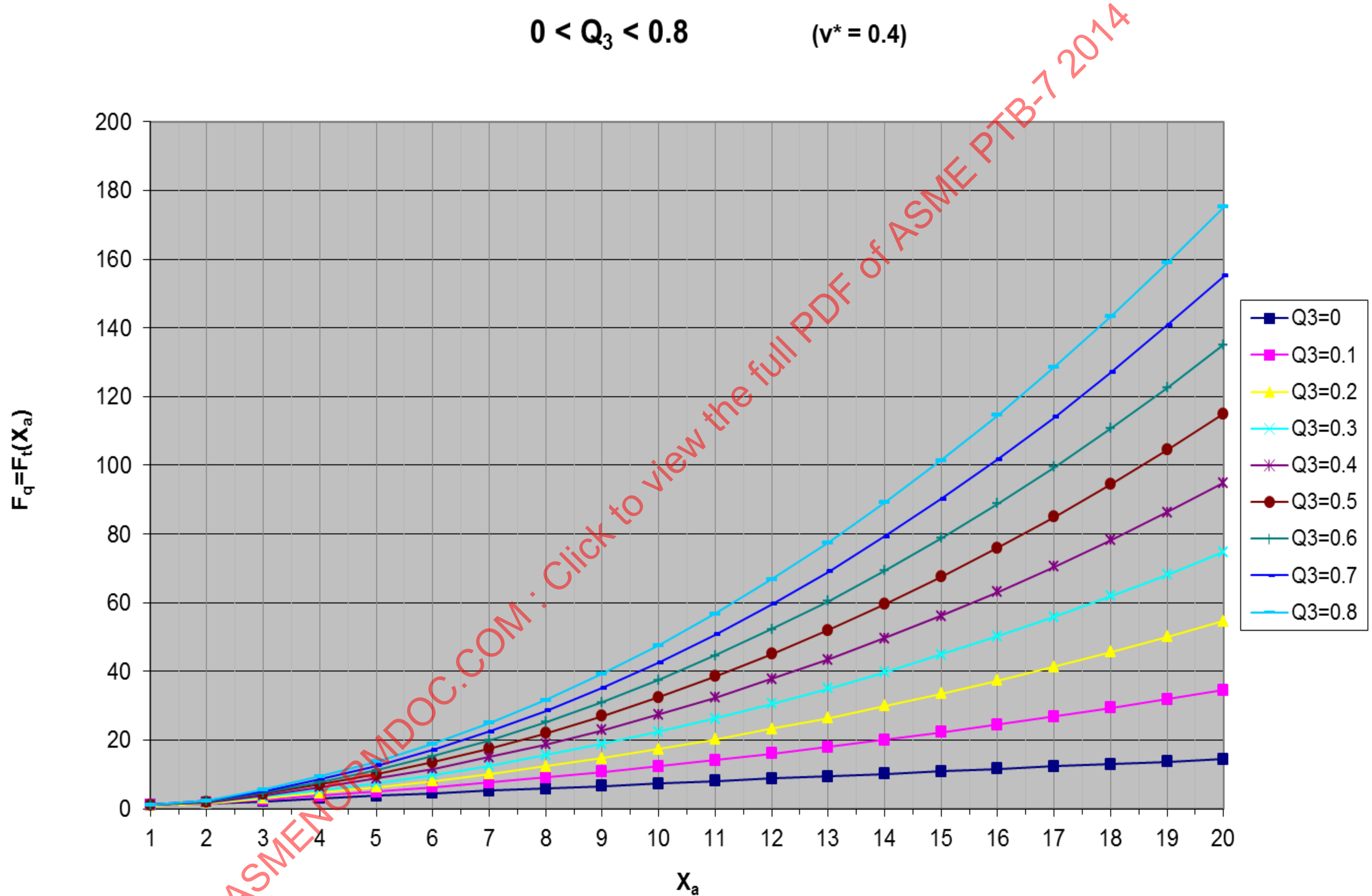
F_t,max for 0<Q_3<0.8



F_q for $-0.8<Q_3<0.0$



F_q for $0<Q_3<0.8$


0 < Q3 < 0.8
(v* = 0.4)
Fq=Ft(Xa)
Xa
0
20
40
60
80
100
120
140
160
180
200
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
Q3=0
Q3=0.1
Q3=0.2
Q3=0.3
Q3=0.4
Q3=0.5
Q3=0.6
Q3=0.7
Q3=0.8

ANNEX P — TABULAR AND GRAPHICAL REPRESENTATION OF COEFFICIENT $F_m(x)$

Annex P provides for $1 \le X_a \le 20$ and $-0.8 \le Q_3 \le +0.8$:

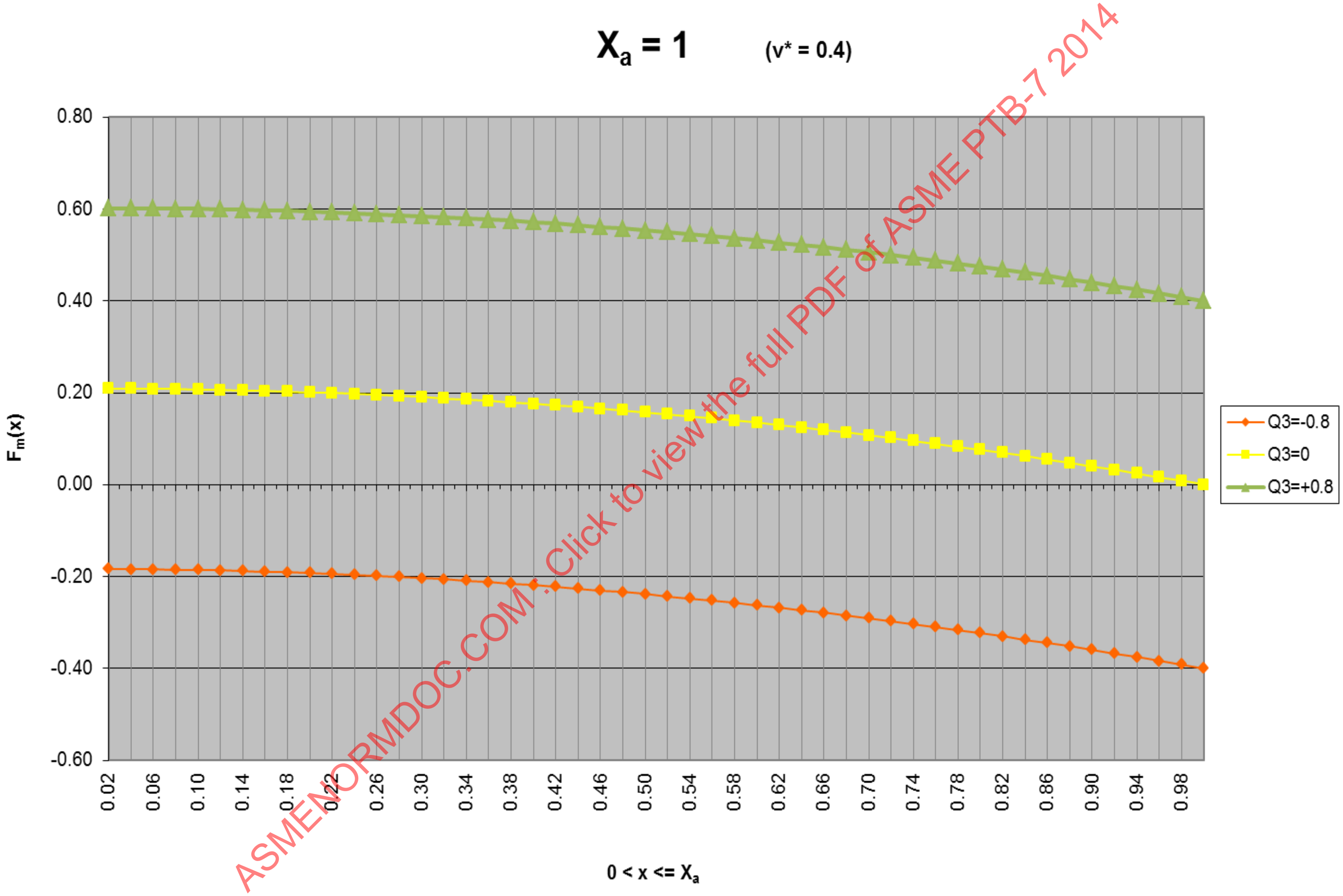
- values and graphs of $F_m(x)$ for $0 \le x \le X_a$
- values and graphs of the maximum of $F_m(x)$: F_m
- location of the maximum of $F_m(x)$: x_{max}

$8F_m(x)$ for X_a=1,3,5,7,10,20

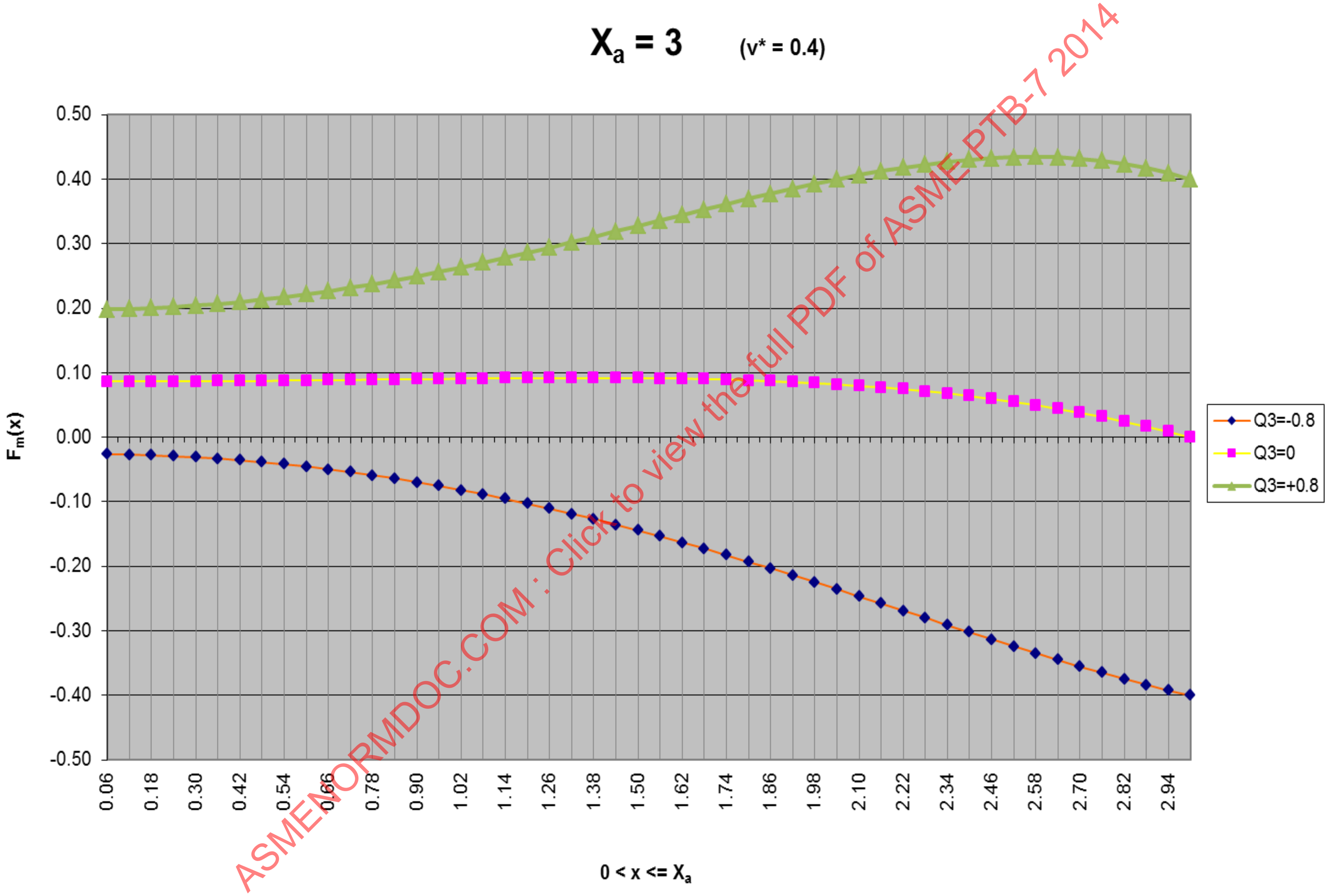
$F_m(x)$ for X_a= 1, 3, 5, 7, 10, 20 and Q_3 = -0.8, 0.0, +0.8 (ν^*=0.4)

X_a=1				X_a=3				X_a=5				X_a=7				X_a=10				X_a = 20			
x	Q_3=-0.8	Q_3=0	Q_3=+0.8	x	Q_3=-0.8	Q_3=0	Q_3=+0.8	x	Q_3=-0.8	Q_3=0	Q_3=+0.8	x	Q_3=-0.8	Q_3=0	Q_3=+0.8	x	Q_3=-0.8	Q_3=0	Q_3=+0.8	x	Q3=-0.8	Q3=0	Q3=+0.8
0.02	-0.18358390	0.20936490	0.60231360	0.06	-0.02616935	0.08616862	0.19850660	0.10	0.05071213	-0.00097628	-0.05266469	0.14	0.01195184	-0.00343116	-0.01881416	0.20	-0.00243681	0.00013826	0.00271334	0.40	-0.00000287	0.00000019	0.00000324
0.04	-0.18384860	0.20911810	0.60208480	0.12	-0.02675549	0.08623743	0.19923040	0.20	0.05044843	-0.00082157	-0.05209158	0.28	0.01217480	-0.00342389	-0.01902258	0.40	-0.00241136	0.00012682	0.00266499	0.80	-0.00000305	0.00000017	0.00000339
0.06	-0.18428980	0.20870680	0.60170350	0.18	-0.02773213	0.08635121	0.20043460	0.30	0.05000487	-0.00056365	-0.05113217	0.42	0.01254269	-0.00341072	-0.01936414	0.60	-0.00236581	0.00010756	0.00258092	1.20	-0.00000329	0.00000015	0.00000359
0.08	-0.18490740	0.20813090	0.60116920	0.24	-0.02909886	0.08650861	0.20211610	0.40	0.04937534	-0.00020240	-0.04978014	0.56	0.01304992	-0.00339006	-0.01983005	0.80	-0.00229549	0.00008024	0.00245598	1.60	-0.00000349	0.00000011	0.00000371
0.10	-0.18570140	0.20739040	0.60048220	0.30	-0.03085506	0.08670776	0.20427060	0.50	0.04855138	0.00026229	-0.04802680	0.70	0.01368848	-0.00335970	-0.02040788	1.00	-0.00219397	0.00004455	0.00228306	2.00	-0.00000353	0.00000004	0.00000362
0.12	-0.18667160	0.20648520	0.59964200	0.36	-0.03299997	0.08694620	0.20689240	0.60	0.04752215	0.00083053	-0.04586110	0.84	0.01444788	-0.00331679	-0.02108145	1.20	-0.00205305	0.00000011	0.00205328	2.40	-0.00000322	-0.00000006	0.00000311
0.14	-0.18781810	0.20541520	0.59864840	0.42	-0.03553258	0.08722100	0.20997460	0.70	0.04627452	0.00150239	-0.04326973	0.98	0.01531492	-0.00325788	-0.02183068	1.40	-0.00186298	-0.00005342	0.00175614	2.80	-0.00000234	-0.00000019	0.00000196
0.16	-0.18914080	0.20418020	0.59750120	0.48	-0.03845168	0.08752862	0.21350890	0.80	0.04479314	0.00227788	-0.04023738	1.12	0.01627357	-0.00317891	-0.02263138	1.60	-0.00161255	-0.00011637	0.00137980	3.20	-0.00000063	-0.00000038	-0.00000012
0.18	-0.19063940	0.20278030	0.59620000	0.54	-0.04175581	0.08786500	0.21748580	0.90	0.04306055	0.00315686	-0.03674683	1.26	0.01730466	-0.00307523	-0.02345513	1.80	-0.00128933	-0.00018895	0.00091142	3.60	0.00000219	-0.00000060	-0.00000340
0.20	-0.19231400	0.20121520	0.59474440	0.60	-0.04544324	0.08822551	0.22189430	1.00	0.04105721	0.00413903	-0.03277915	1.40	0.01838573	-0.00294164	-0.02426902	2.00	-0.00087998	-0.00027119	0.00033759	4.00	0.00000638	-0.00000087	-0.00000811
0.22	-0.19416430	0.19948480	0.59313400	0.66	-0.04951193	0.08860499	0.22672190	1.10	0.03876168	0.00522384	-0.02831400	1.54	0.01949068	-0.00277239	-0.02503546	2.20	-0.00037059	-0.00036287	-0.00035516	4.40	0.00001210	-0.00000115	-0.00001440
0.24	-0.19619030	0.19758900	0.59136810	0.72	-0.05395950	0.08899774	0.23195500	1.20	0.03615077	0.00641046	-0.02332985	1.68	0.02058955	-0.00256123	-0.02571201	2.40	0.00025279	-0.00046343	-0.00117965	4.80	0.00001938	-0.00000140	-0.00002219
0.26	-0.19839170	0.19552740	0.58944650	0.78	-0.05878322	0.08939743	0.23757810	1.30	0.03319965	0.00769765	-0.01780435	1.82	0.02164825	-0.00230147	-0.02625119	2.60	0.00100361	-0.00057184	-0.00214729	5.20	0.00002795	-0.00000158	-0.00003111
0.28	-0.20076830	0.19330010	0.58736840	0.84	-0.06397994	0.08979724	0.24357440	1.40	0.02988207	0.00908375	-0.01171457	1.96	0.02262828	-0.00198603	-0.02660033	2.80	0.00189401	-0.00068655	-0.00326710	5.60	0.00003712	-0.00000159	-0.00004031
0.30	-0.20332000	0.19090670	0.58513340	0.90	-0.06954604	0.09018975	0.24992550	1.50	0.02617054	0.01056653	-0.00503749	2.10	0.02348648	-0.00160751	-0.02670149	3.00	0.00293403	-0.00080530	-0.00454464	6.00	0.00004564	-0.00000133	-0.00004829
0.32	-0.20604660	0.18834690	0.58274040	0.96	-0.07547744	0.09056698	0.25661140	1.60	0.02203660	0.01214312	0.00224964	2.24	0.02417482	-0.00115829	-0.02649139	3.20	0.00413047	-0.00092508	-0.00598062	6.40	0.00005150	-0.00000064	-0.00005278
0.34	-0.20894780	0.18562070	0.58018920	1.02	-0.08176948	0.09092037	0.26361020	1.70	0.01745103	0.01380992	0.01016881	2.38	0.02464022	-0.00063063	-0.02590148	3.40	0.00548573	-0.00104190	-0.00756954	6.80	0.00005182	0.00000062	-0.00005058
0.36	-0.21202330	0.18272770	0.57747860	1.08	-0.08841692	0.09124079	0.27089850	1.80	0.01238420	0.01556249	0.01874078	2.52	0.02482442	-0.00001682	-0.02485806	3.60	0.00699638	-0.00115077	-0.00929793	7.20	0.00004276	0.00000261	-0.00003754
0.38	-0.21527300	0.17966760	0.57460830	1.14	-0.09541386	0.09151853	0.27845090	1.90	0.00680636	0.01739541	0.02798446	2.66	0.02466395	0.00069070	-0.02328254	3.80	0.00865158	-0.00124551	-0.01114261	7.60	0.00001961	0.00000546	-0.00000870
0.40	-0.21869650	0.17644020	0.57157700	1.20	-0.10275370	0.09174329	0.28624030	2.00	0.00068801	0.01930221	0.03791640	2.80	0.02409015	0.00149915	-0.02109184	4.00	0.01043131	-0.00131868	-0.01306868	8.00	-0.00002294	0.00000922	0.00004138
0.42	-0.22229350	0.17304520	0.56838400	1.26	-0.11042900	0.09190422	0.29423750	2.10	-0.00599970	0.02127519	0.04855009	2.94	0.02302931	0.00241520	-0.01819892	4.20	0.01230449	-0.00136149	-0.01502747	8.40	-0.00009023	0.00001381	0.00011785
0.44	-0.22606380	0.16948230	0.56502840	1.32	-0.11843160	0.09198984	0.30241130	2.20	-0.01328455	0.02330535	0.05989525	3.08	0.02140298	0.00344472	-0.01451354	4.40	0.01422692	-0.00136373	-0.01695438	8.80	-0.00018654	0.00001892	0.00022439
0.46	-0.23000680	0.16575110	0.56150910	1.38	-0.12675240	0.09198811	0.31072860	2.30	-0.02119274	0.02538221	0.07195716	3.22	0.01912836	0.00459258	-0.00994320	4.60	0.01613920	-0.00131378	-0.01876677	9.20	-0.00031361	0.00002395	0.00036151
0.48	-0.23412250	0.16185140	0.55782530	1.44	-0.13538120	0.09188642	0.31915400	2.40	-0.02974840	0.02749370	0.08473580	3.36	0.01611890	0.00586228	-0.00439435	4.80	0.01796455	-0.00119868	-0.02036191	9.60	-0.00046844	0.00002786	0.00052415
0.50	-0.23841020	0.15778280	0.55397570	1.50	-0.14430690	0.09167156	0.32765000	2.50	-0.03897302	0.02962602	0.09822506	3.50	0.01228512	0.00725566	0.00222620	5.00	0.01960682	-0.00100418	-0.02161518	10.00	-0.00064056	0.00002911	0.00069879
0.52	-0.24286970	0.15354490	0.54995940	1.56	-0.15351720	0.09132972	0.33617670	2.60	-0.04888477	0.03176349	0.11241170	3.64	0.00753562	0.00877256	0.01000950	5.20	0.02094852	-0.00071502	-0.02237857	10.40	-0.00080874	0.00002562	0.00085998
0.54	-0.24750050	0.14913730	0.54577520	1.62	-0.16299870	0.09084656	0.34469180	2.70	-0.05949798	0.03388843	0.12727480	3.78	0.00177833	0.01041036	0.01904238	5.40	0.02184935	-0.00031520	-0.02247975	10.80	-0.00093753	0.00001471	0.00096695
0.56	-0.25230220	0.14455980	0.54142190	1.68	-0.17273650	0.09020714	0.35315080	2.80	-0.07082227	0.03598100	0.14278430	3.92	-0.00507792	0.01216360	0.02940512	5.60	0.02214497	0.00021158	-0.02172181	11.20	-0.00097407	-0.00000672	0.00096064
0.58	-0.25727440	0.13981190	0.53689830	1.74	-0.18271470	0.08939592	0.36150650	2.90	-0.08286189	0.03801908	0.15890010	4.06	-0.01312157	0.01402351	0.04116859	5.80	0.02164662	0.00088132	-0.01988398	11.60	-0.00084604	-0.00004189	0.00076226
0.60	-0.26241660	0.13489320	0.53220300	1.80	-0.19291550	0.08839688	0.36970930	3.00	-0.09561487	0.03997814	0.17557110	4.20	-0.02243614	0.01597749	0.05439113	6.00	0.02014153	0.00170900	-0.01672352	12.00	-0.00046205	-0.00009364	0.00027478
0.62	-0.26772820	0.12980340	0.52733490	1.86	-0.20331990	0.08719339	0.37770670	3.10	-0.10907220	0.04183109	0.19273430	4.34	-0.03309772	0.01800864	0.06911500	6.20	0.01739438	0.00270762	-0.01197915	12.40	0.00028394	-0.00016356	-0.00061106
0.64	-0.27320870	0.12454200	0.52229270	1.92	-0.21390720	0.08576827	0.38544380	3.20	-0.12321680	0.04354820	0.21031320	4.48	-0.04517204	0.02009514	0.08536233	6.40	0.01315036	0.00388706	-0.00537623	12.80	0.00150222	-0.00025078	-0.00200378
0.66	-0.27885770	0.11910860	0.51707480	1.98	-0.22465500	0.08410390	0.39286280	3.30	-0.13802260	0.04509694	0.22821650	4.62	-0.05871119	0.02220979	0.10313080	6.60	0.00713974	0.00525278	0.00336582	13.20	0.00328846	-0.00035039	-0.00398925
0.68	-0.28467440	0.11350270	0.51167990	2.04	-0.23553890	0.08218204	0.39990300	3.40	-0.15345380	0.04644196	0.24633770	4.76	-0.07374990	0.02431934	0.12238860	6.80	-0.00091539	0.00680417	0.01452373	13.60	0.00569418	-0.00045152	-0.00659722
0.70	-0.29065840	0.10772410	0.50610660	2.10	-0.24653290	0.07998405	0.40650100	3.50	-0.16946340	0.04754493	0.26455320	4.90	-0.09030136	0.02638402	0.14306940	7.00	-0.01129201	0.00853281	0.02835763	14.00	0.00868486	-0.00053518	-0.00975521
0.72	-0.29680910	0.10177210	0.50035330	2.16	-0.25760890	0.07749077	0.41259040	3.60	-0.18599230	0.04836452	0.28272130	5.04	-0.10835280	0.02835690	0.16506660	7.20	-0.02425436	0.01042040	0.04509516	14.40	0.01208474	-0.00057224	-0.01322922
0.74	-0.30312560	0.09564647	0.49441860	2.22	-0.26873650	0.07468259	0.41810170	3.70	-0.20296820	0.04885636	0.30068090	5.18	-0.12786010	0.03018342	0.18822700	7.40	-0.04003901	0.01243653	0.06491207	14.80	0.01551022	-0.00052192	-0.01655405
0.76	-0.30960760	0.08934665	0.48830090	2.28	-0.27988350	0.07153952	0.42296260	3.80	-0.22030430	0.04897301	0.31825030	5.32	-0.14874310	0.03180086	0.21234480	7.60	-0.05883651	0.01453628	0.08790907	15.20	0.01829642	-0.00033153	-0.01895948
0.78	-0.31625400	0.08287223	0.48199850	2.34	-0.29101520	0.06804116	0.42709750	3.90	-0.23789830	0.04866396	0.33522620	5.46	-0.17087890	0.03313792	0.23715470	7.80	-0.08076938	0.01665769	0.11408470	15.60	0.01942719	0.00006158	-0.01930402
0.80	-0.32306440	0.07622273	0.47550990	2.40	-0.30209460	0.06416673	0.43042810	4.00	-0.25563120	0.04787570	0.35138260	5.60	-0.19409630	0.03411439	0.26232500	8.00	-0.10586620	0.01871913	0.14330450	16.00	0.01748464	0.00072468	-0.01603528
0.82	-0.33003790	0.06939774	0.46883340	2.46	-0.31308220	0.05989514	0.43287250	4.10	-0.27336580	0.04655179	0.36646940	5.74	-0.21816920	0.03464086	0.28745090	8.20	-0.13403190	0.02061679	0.17526550	16.40	0.01064304	0.00171949	-0.00720405
0.84	-0.33717370	0.06239678	0.46196730	2.52	-0.32393610	0.05520504	0.43434620	4.20	-0.29094570	0.04463295	0.38021160	5.88	-0.24280950	0.03461862	0.31204670	8.40	-0.16501380	0.02222224	0.20945820	16.80	-0.00325992	0.00308573	0.00943139
0.86	-0.34447110	0.05521942	0.45491000	2.58	-0.33461150	0.05007482	0.43476110	4.30	-0.30819410	0.04205725	0.39230860	6.02	-0.26766090	0.03393968	0.33554020	8.60	-0.19836390	0.02338036	0.24512470	17.20	-0.02653197	0.00481723	0.03616643
0.88	-0.35192910	0.04786516	0.44765940	2.64	-0.34506110	0.04448269	0.43402650	4.40	-0.32491280	0.03876032	0.40243340	6.16	-0.29229090	0.03248701	0.35726490	8.80	-0.23339890	0.02390770	0.28121430	17.60	-0.06126705	0.00682977	0.07492660
0.90	-0.35954690	0.04033359	0.44021410	2.70	-0.35523470	0.03840673	0.43204810	4.50	-0.34088050	0.03467562	0.41023180	6.30	-0.31618470	0.03013497	0.37645470	9.00	-0.26915610	0.02359162	0.31633930	18.00	-0.10876750	0.00892145	0.12661040
0.92	-0.36732360	0.03262423	0.43257210	2.76	-0.36507930	0.03182492	0.42872910	4.60	-0.35585250	0.02973473	0.41532200	6.44	-0.33873830	0.02675002	0.39223830	9.20	-0.30434920	0.02219032	0.34872980	18.40	-0.16870540	0.01072779	0.19016100
0.94	-0.37525830	0.02473663	0.42473160	2.82	-0.37453870	0.02471524	0.42396910	4.70	-0.36955910	0.02386778	0.41729460	6.58	-0.35925150	0.02219172	0.40363500	9.40	-0.33732350	0.01943421	0.37619200	18.80	-0.23800230	0.01167708	0.26135640
0.96	-0.38334990	0.01667034	0.41669060	2.88	-0.38355400	0.01705573	0.41766540	4.80	-0.38170490	0.01700389	0.41571270	6.72	-0.37692320	0.01631409	0.40955140	9.60	-0.36601350	0.01502885	0.39607120	19.20	-0.30944820	0.01095515	0.33135850
0.98	-0.39159750	0.00842497	0.40844740	2.94	-0.39206290	0.00882453	0.40971200	4.90	-0.39196810	0.00907170	0.41011150	6.86	-0.39084540	0.00896733	0.40878010	9.80	-0.38790400	0.00865981	0.40522360	19.60	-0.37015030	0.00749342	0.38513720
1.00	-0.40000000	0.00000000	0.40000000	3.00	-0.40000000	0.00000000	0.40000000	5.00	-0.40000000	0.00000000	0.40000000	7.00	-0.40000000	0.00000000	0.40000000	10.00	-0.40000000	0.00000000	0.40000000	20.00	-0.40000000	0.00000000	0.40000000

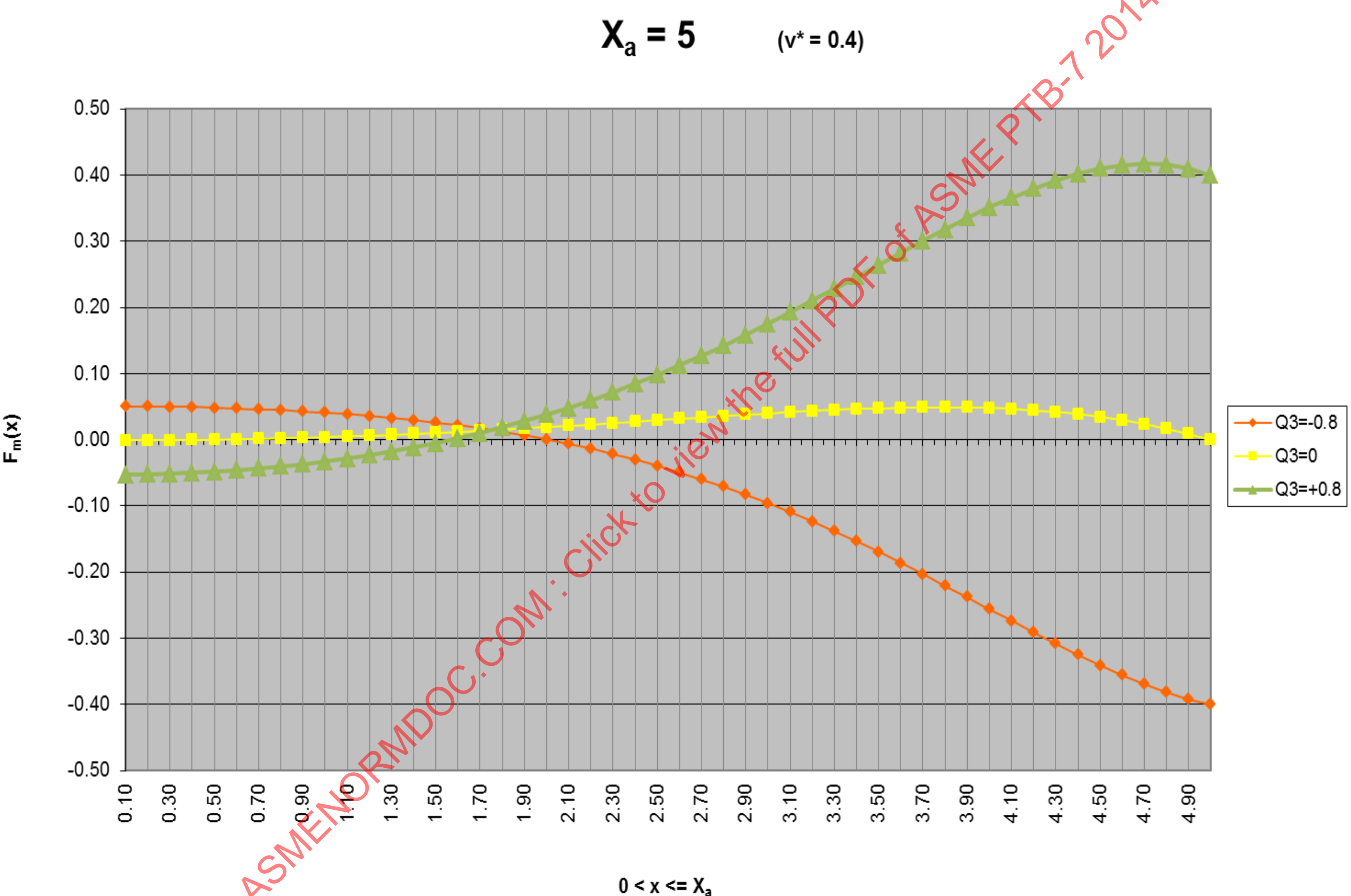
$X_a = 1$


Xa = 1 (v* = 0.4)
0.80
0.60
0.40
0.20
0.00
-0.20
-0.40
-0.60
Fm(x)
0.02 0.06 0.10 0.14 0.18 0.22 0.26 0.30 0.34 0.38 0.42 0.46 0.50 0.54 0.58 0.62 0.66 0.70 0.74 0.78 0.82 0.86 0.90 0.94 0.98
0 < x <= Xa
Q3=-0.8
Q3=0
Q3=+0.8

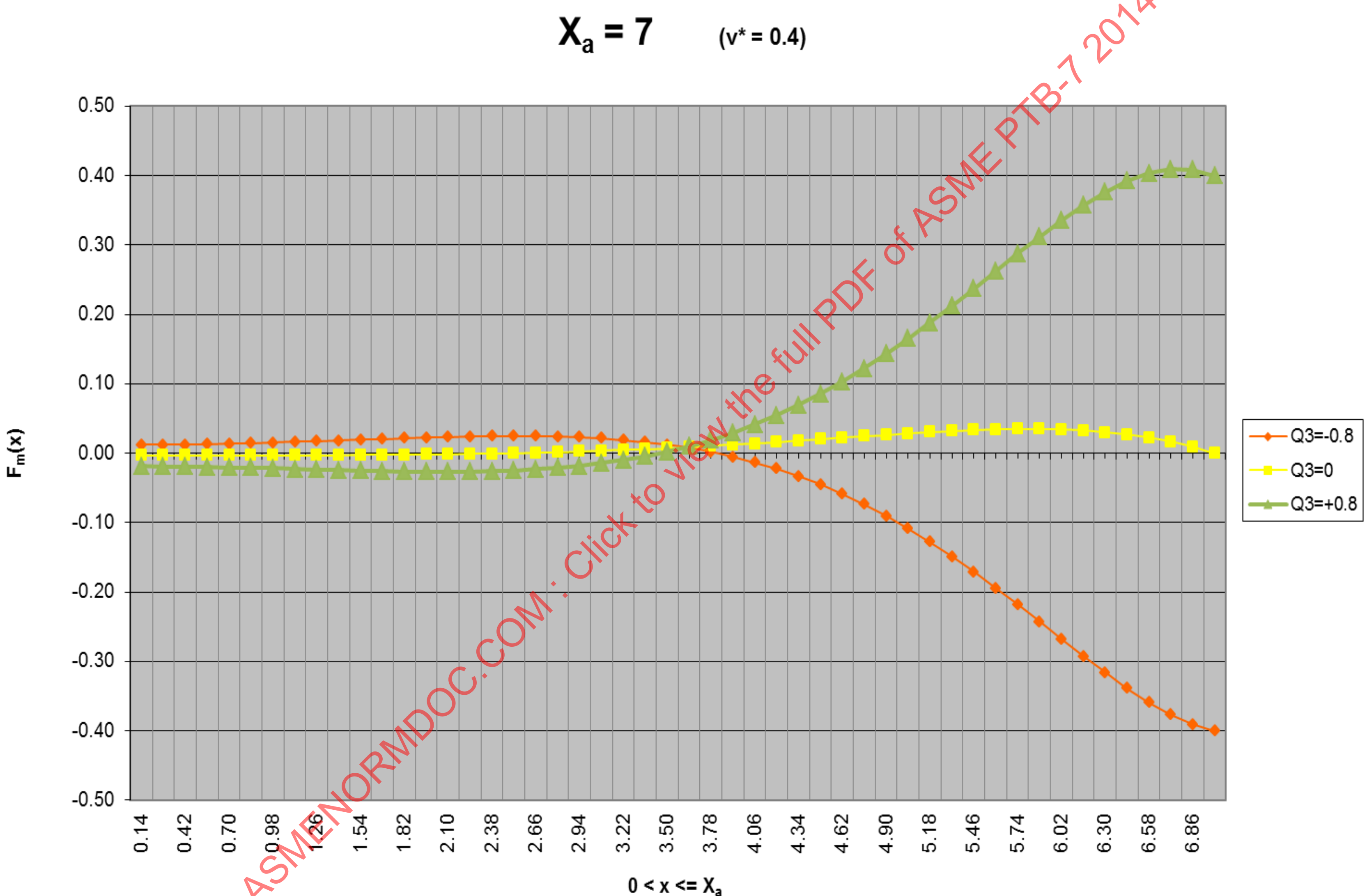
$X_a = 3$


X_a = 3 (v* = 0.4)
$F_m(x)$
0.50
0.40
0.30
0.20
0.10
0.00
-0.10
-0.20
-0.30
-0.40
-0.50
0.06
0.18
0.30
0.42
0.54
0.66
0.78
0.90
1.02
1.14
1.26
1.38
1.50
1.62
1.74
1.86
1.98
2.10
2.22
2.34
2.46
2.58
2.70
2.82
2.94
0 < x <= X_a
Q3=-0.8
Q3=0
Q3=+0.8

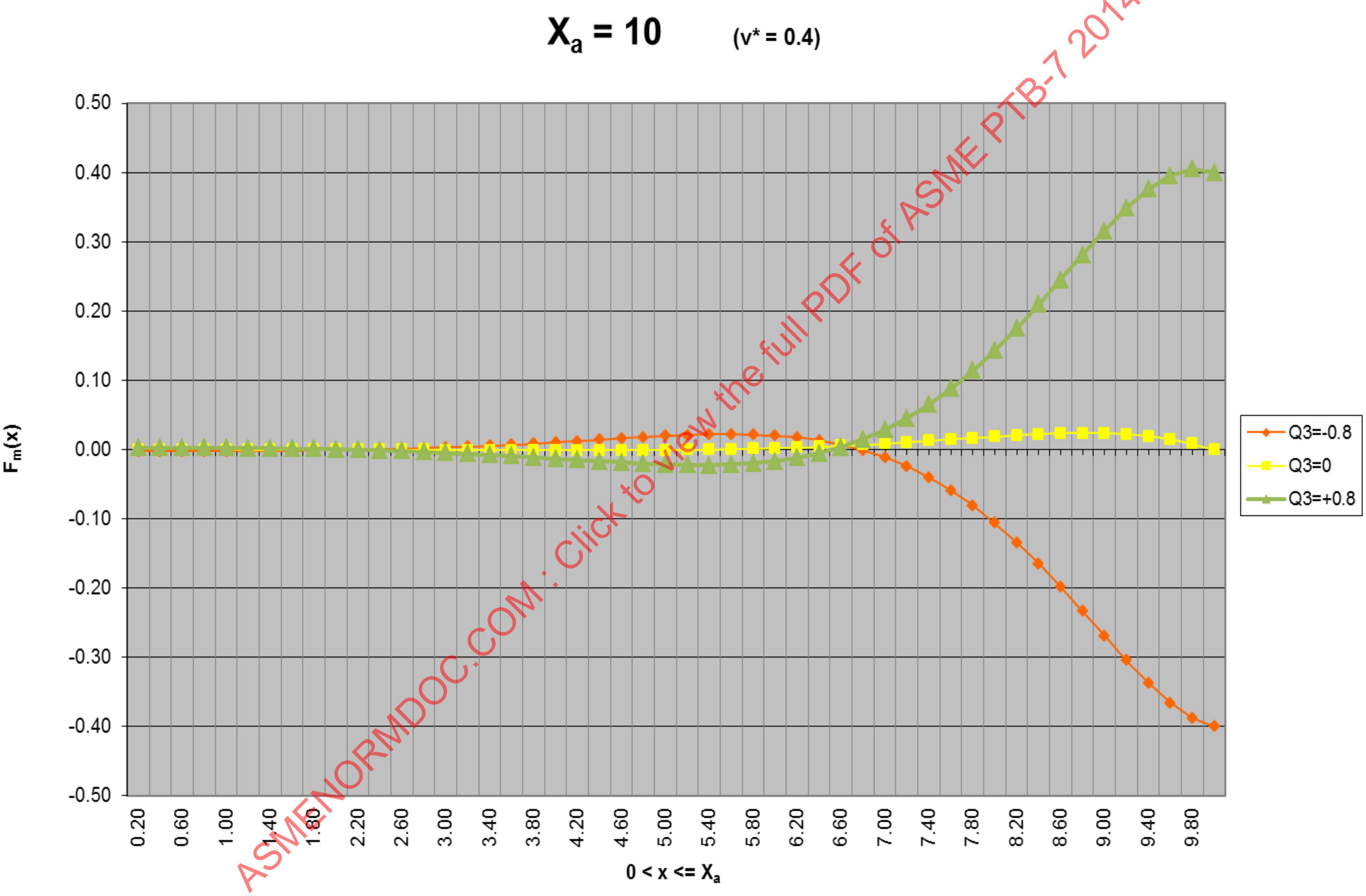
$X_a = 5$


$X_a = 5$ (v* = 0.4)
$F_m(x)$
0.50
0.40
0.30
0.20
0.10
0.00
-0.10
-0.20
-0.30
-0.40
-0.50
0.10 0.30 0.50 0.70 0.90 1.10 1.30 1.50 1.70 1.90 2.10 2.30 2.50 2.70 2.90 3.10 3.30 3.50 3.70 3.90 4.10 4.30 4.50 4.70 4.90
$0 < x <= X_a$
Q3=-0.8
Q3=0
Q3=+0.8

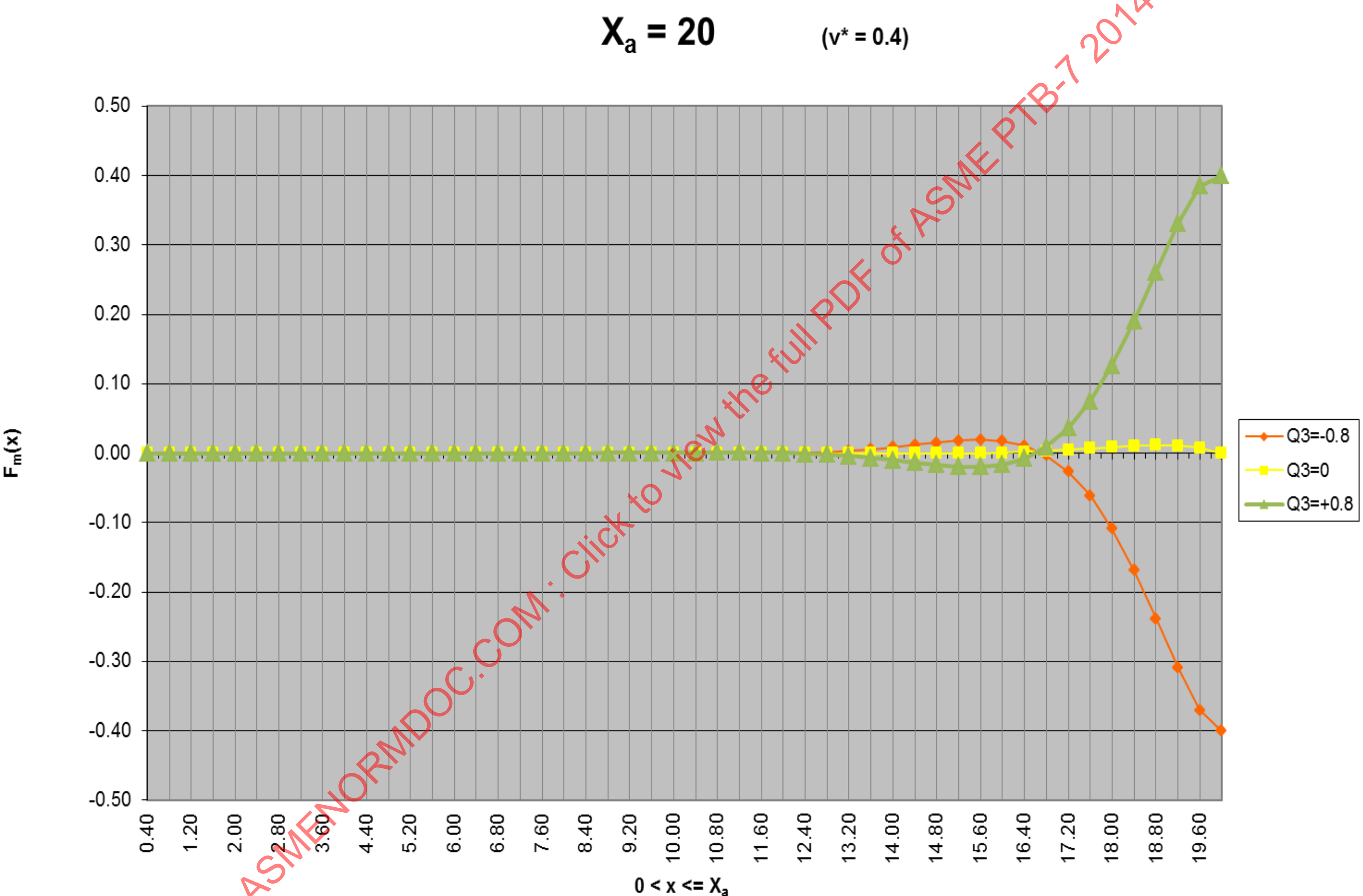
$X_a = 7$


Xa = 7 (v* = 0.4)
Fm(x)
0.50
0.40
0.30
0.20
0.10
0.00
-0.10
-0.20
-0.30
-0.40
-0.50
0.14
0.42
0.70
0.98
1.26
1.54
1.82
2.10
2.38
2.66
2.94
3.22
3.50
3.78
4.06
4.34
4.62
4.90
5.18
5.46
5.74
6.02
6.30
6.58
6.86
0 < x <= Xa
Q3=-0.8
Q3=0
Q3=+0.8

X_a = 10


Xa = 10 (v* = 0.4)
Fm(x)
0.50
0.40
0.30
0.20
0.10
0.00
-0.10
-0.20
-0.30
-0.40
-0.50
0.20
0.60
1.00
1.40
1.80
2.20
2.60
3.00
3.40
3.80
4.20
4.60
5.00
5.40
5.80
6.20
6.60
7.00
7.40
7.80
8.20
8.60
9.00
9.40
9.80
0 < x <= Xa
Q3=-0.8
Q3=0
Q3=+0.8

X_a = 20



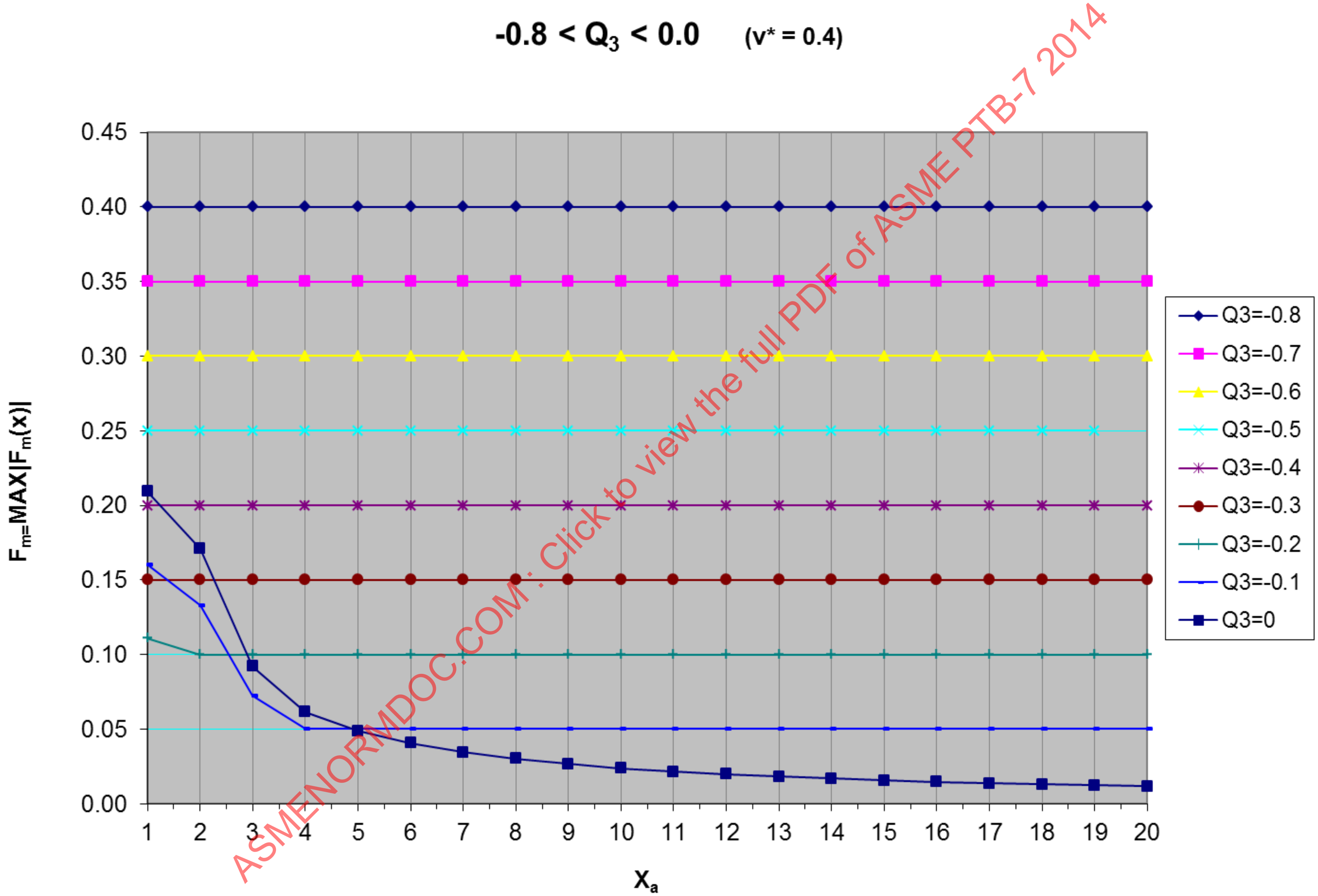
Xa = 20 (v* = 0.4)
Fm(x)
0.50
0.40
0.30
0.20
0.10
0.00
-0.10
-0.20
-0.30
-0.40
-0.50
0.40
1.20
2.00
2.80
3.60
4.40
5.20
6.00
6.80
7.60
8.40
9.20
10.00
10.80
11.60
12.40
13.20
14.00
14.80
15.60
16.40
17.20
18.00
18.80
19.60
0 < x <= Xa
Q3=-0.8
Q3=0
Q3=+0.8

F_m=Max|$F_m(x)$| vs X_a and Q_3

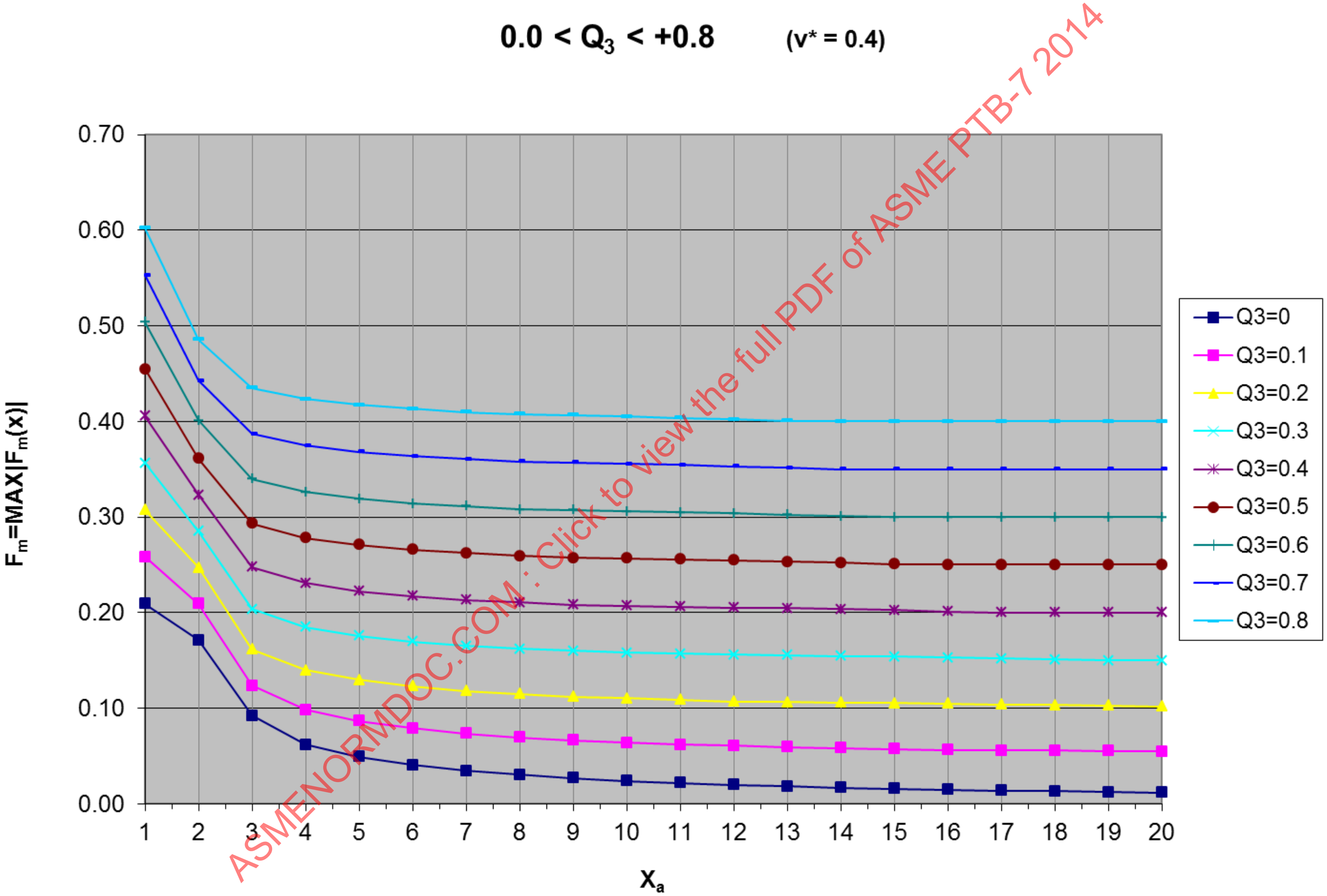
			if $x(max)=X_a$: $F_m(X_a)=\|Q_3/2\|$					F_m=Max|$F_m(x)$|					if $x(max)=X_a$: $F_m(X_a)=\|Q_3/2\|$				
X_a|Q_3	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
1	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1111	0.1602	0.2094	0.2585	0.3076	0.3567	0.4058	0.4550	0.5041	0.5532	0.6023
2	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.1329	0.1710	0.2090	0.2471	0.2851	0.3232	0.3613	0.4009	0.4425	0.4855
3	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0721	0.0920	0.1236	0.1616	0.2035	0.2476	0.2931	0.3396	0.3870	0.4348
4	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0500	0.0616	0.0981	0.1400	0.1846	0.2309	0.2782	0.3263	0.3746	0.4234
5	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0500	0.0490	0.0866	0.1297	0.1755	0.2225	0.2707	0.3189	0.3681	0.4173
6	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0500	0.0407	0.0789	0.1229	0.1694	0.2171	0.2656	0.3141	0.3635	0.4130
7	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0500	0.0346	0.0734	0.1181	0.1652	0.2129	0.2621	0.3112	0.3604	0.4096
8	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0500	0.0301	0.0694	0.1148	0.1620	0.2107	0.2594	0.3081	0.3579	0.4078
9	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0500	0.0266	0.0662	0.1119	0.1600	0.2082	0.2574	0.3071	0.3568	0.4066
10	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0500	0.0239	0.0640	0.1103	0.1579	0.2069	0.2565	0.3061	0.3557	0.4052
11	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0500	0.0216	0.0620	0.1086	0.1567	0.2061	0.2555	0.3049	0.3543	0.4037
12	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0500	0.0198	0.0605	0.1069	0.1561	0.2053	0.2545	0.3037	0.3529	0.4021
13	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0500	0.0181	0.0594	0.1063	0.1553	0.2043	0.2533	0.3023	0.3514	0.4004
14	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0500	0.0169	0.0583	0.1058	0.1546	0.2033	0.2521	0.3009	0.3500	0.4000
15	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0500	0.0157	0.0571	0.1052	0.1537	0.2023	0.2509	0.3000	0.3500	0.4000
16	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0500	0.0146	0.0563	0.1046	0.1529	0.2012	0.2500	0.3000	0.3500	0.4000
17	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0500	0.0137	0.0559	0.1039	0.1520	0.2000	0.2500	0.3000	0.3500	0.4000
18	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0500	0.0130	0.0555	0.1033	0.1511	0.2000	0.2500	0.3000	0.3500	0.4000
19	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0500	0.0123	0.0551	0.1026	0.1501	0.2000	0.2500	0.3000	0.3500	0.4000
20	0.4000	0.3500	0.3000	0.2500	0.2000	0.1500	0.1000	0.0500	0.0117	0.0547	0.1019	0.1500	0.2000	0.2500	0.3000	0.3500	0.4000

				$x(max)=X_a$										$x(max)=X_a$			
X_a|Q_3	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200	0.0200
2	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	0.0400	0.0400	0.0400	0.0400	0.0400	0.0400	0.2800	0.6800	0.8800	1.0000
3	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	0.0600	1.3200	1.8000	2.1000	2.2200	2.3400	2.4000	2.4600	2.5200	2.5800
4	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	2.7200	3.1200	3.2800	3.4400	3.5200	3.5200	3.6000	3.6000	3.6800
5	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000	3.8000	4.2000	4.4000	4.5000	4.6000	4.6000	4.7000	4.7000	4.7000
6	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	6.0000	4.8000	5.1600	5.4000	5.5200	5.6400	5.6400	5.6400	5.7600	5.7600
7	7.0000	7.0000	7.0000	7.0000	7.0000	7.0000	7.0000	7.0000	5.7400	6.3000	6.4400	6.5800	6.7200	6.7200	6.7200	6.7200	6.7200
8	8.0000	8.0000	8.0000	8.0000	8.0000	8.0000	8.0000	8.0000	6.8800	7.3600	7.5200	7.6800	7.6800	7.6800	7.6800	7.8400	7.8400
9	9.0000	9.0000	9.0000	9.0000	9.0000	9.0000	9.0000	9.0000	7.7400	8.2800	8.6400	8.6400	8.6400	8.8200	8.8200	8.8200	8.8200
10	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	10.0000	8.8000	9.4000	9.6000	9.6000	9.8000	9.8000	9.8000	9.8000	9.8000
11	11.0000	11.0000	11.0000	11.0000	11.0000	11.0000	11.0000	11.0000	9.9000	10.3400	10.5600	10.7800	10.7800	10.7800	10.7800	10.7800	10.7800
12	12.0000	12.0000	12.0000	12.0000	12.0000	12.0000	12.0000	12.0000	10.8000	11.5200	11.5200	11.7600	11.7600	11.7600	11.7600	11.7600	11.7600
13	13.0000	13.0000	13.0000	13.0000	13.0000	13.0000	13.0000	13.0000	11.9600	12.4800	12.7400	12.7400	12.7400	12.7400	12.7400	12.7400	12.7400
14	14.0000	14.0000	14.0000	14.0000	14.0000	14.0000	14.0000	14.0000	12.8800	13.4400	13.7200	13.7200	13.7200	13.7200	13.7200	14.0000	14.0000
15	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	15.0000	13.8000	14.4000	14.7000	14.7000	14.7000	14.7000	15.0000	15.0000	15.0000
16	16.0000	16.0000	16.0000	16.0000	16.0000	16.0000	16.0000	16.0000	14.7200	15.6800	15.6800	15.6800	15.6800	16.0000	16.0000	16.0000	16.0000
17	17.0000	17.0000	17.0000	17.0000	17.0000	17.0000	17.0000	17.0000	15.9800	16.6600	16.6600	16.6600	16.6600	17.0000	17.0000	17.0000	17.0000
18	18.0000	18.0000	18.0000	18.0000	18.0000	18.0000	18.0000	18.0000	16.9200	17.6400	17.6400	17.6400	18.0000	18.0000	18.0000	18.0000	18.0000
19	19.0000	19.0000	19.0000	19.0000	19.0000	19.0000	19.0000	19.0000	17.8600	18.6200	18.6200	18.6200	19.0000	19.0000	19.0000	19.0000	19.0000
20	20.0000	20.0000	20.0000	20.0000	20.0000	20.0000	20.0000	20.0000	18.8000	19.6000	19.6000	20.0000	20.0000	20.0000	20.0000	20.0000	20.0000

F_m vs X_a and -0.8<Q_3< 0



F_m vs X_a and $0<Q_3< +0.8$



ANNEX Q—TABULAR AND GRAPHICAL REPRESENTATION OF COEFFICIENT $F_Q(x)$

Annex Q provides for $1 \le X_a \le 20$ and $-0.8 \le Q_3 \le +0.8$:

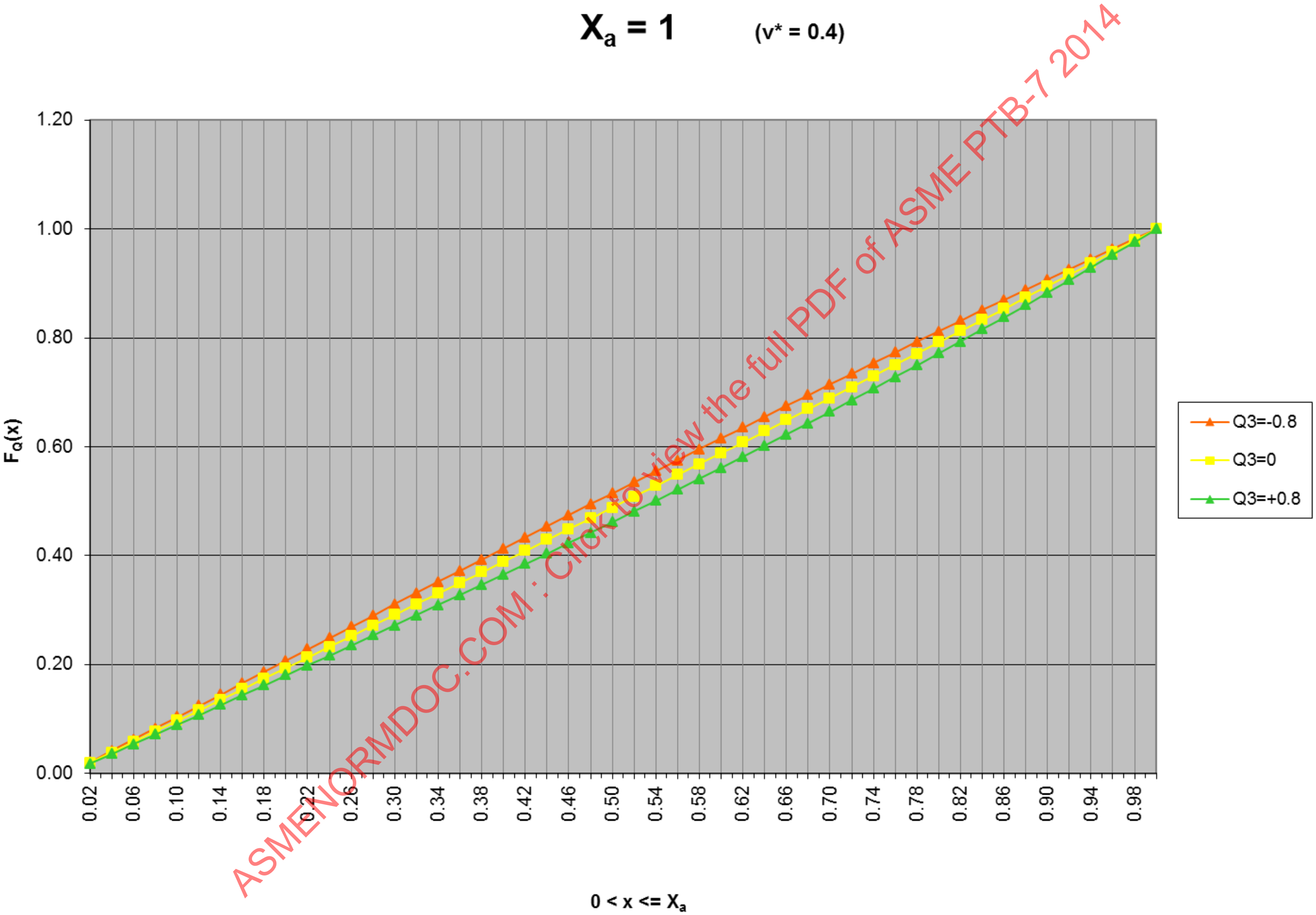
- values and graphs of $F_Q(x)$ for $0 \le x \le X_a$
- values and graphs of the maximum of $F_Q(x)$: F_Q
- location of the maximum of $F_Q(x)$: x_{max}

$F_Q(x)$ for X_a=1,3,5,7,10,20

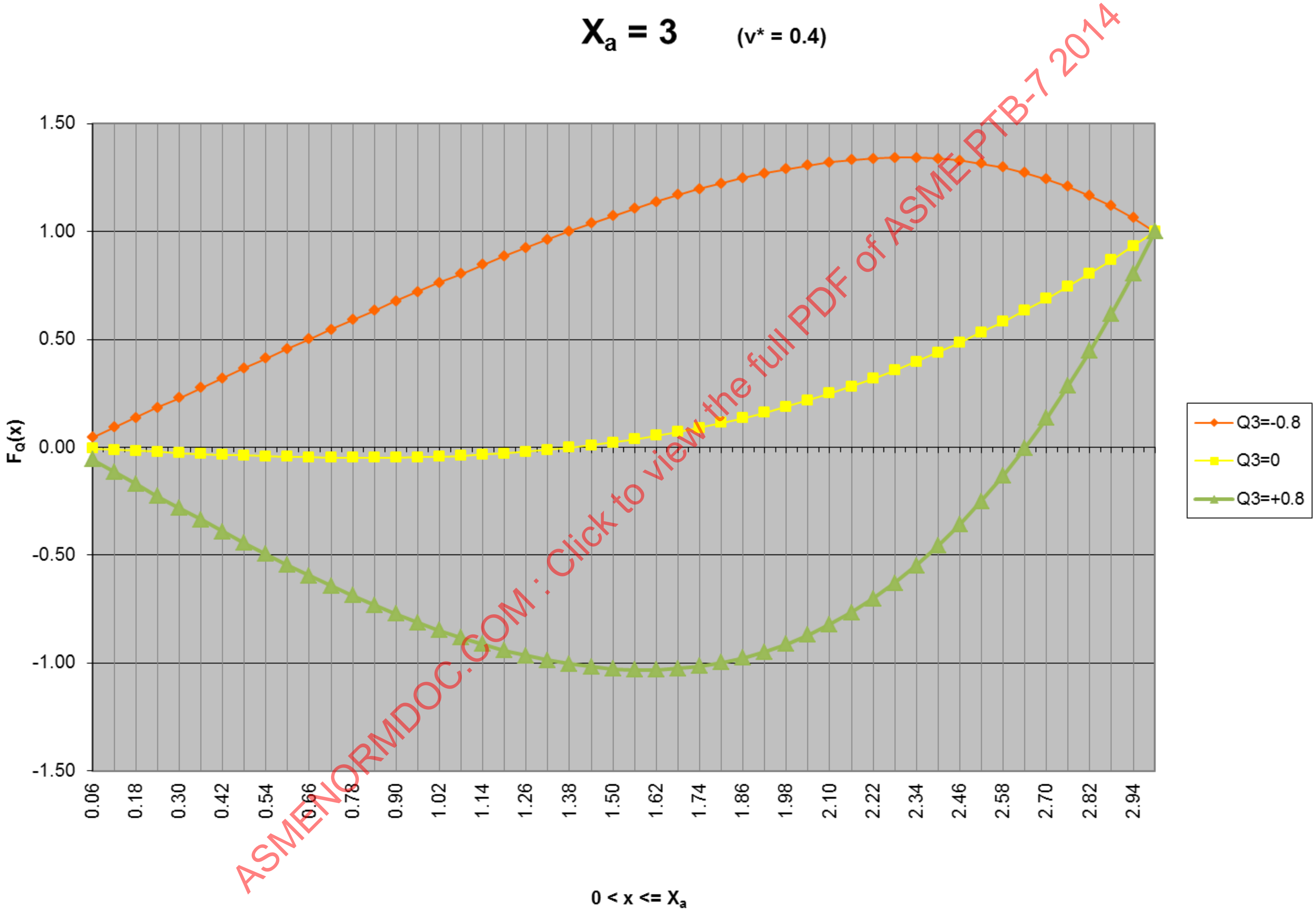
$F_Q(x)$ for X_a = 1, 3, 5, 7, 10, 20 and Q_3 = -0.8, 0.0, +0.8 (ν^*=0.4)

X_a=1				X_a=3				X_a=5				X_a=7				X_a=10				X_a = 20			
x	Q_3=-0.8	Q_3=0	Q_3=+0.8	x	Q_3=-0.8	Q_3=0	Q_3=+0.8	x	Q_3=-0.8	Q_3=0	Q_3=+0.8	x	Q_3=-0.8	Q_3=0	Q_3=+0.8	x	Q_3=-0.8	Q_3=0	Q_3=+0.8	x	Q3=-0.8	Q3=0	Q3=+0.8
0.02	0.02076262	0.01935370	0.01794477	0.06	0.04597750	-0.00541298	-0.05680346	0.10	0.02060735	-0.01213262	-0.04487258	0.14	-0.01755414	-0.00055054	0.01645306	0.20	-0.00193894	0.00089464	0.00372823	0.40	0.00001512	0.00000089	-0.00001335
0.04	0.04152367	0.03870919	0.03589470	0.12	0.09193687	-0.01076614	-0.11346920	0.20	0.04148651	-0.02427055	-0.09002762	0.28	-0.03486280	-0.00117172	0.03251937	0.40	-0.00408697	0.00180124	0.00768945	0.80	0.00002634	0.00000203	-0.00002228
0.06	0.06228157	0.05806826	0.05385495	0.18	0.13785960	-0.01599961	-0.16985880	0.30	0.06290802	-0.03641836	-0.13574480	0.42	-0.05167628	-0.00193401	0.04780826	0.60	-0.00665120	0.00273084	0.01211288	1.20	0.00002952	0.00000367	-0.00002218
0.08	0.08303474	0.07743271	0.07183069	0.24	0.18372650	-0.02105345	-0.22583340	0.40	0.08513980	-0.04857909	-0.18229800	0.56	-0.06773657	-0.00290762	0.06192133	0.80	-0.00983450	0.00369271	0.01721991	1.60	0.00002014	0.00000601	-0.00000811
0.10	0.10378160	0.09680434	0.08982707	0.30	0.22951710	-0.02586758	-0.28125230	0.50	0.10844580	-0.06075350	-0.22995280	0.70	-0.08277331	-0.00416227	0.07444877	1.00	-0.01383283	0.00469333	0.02321949	2.00	-0.00000669	0.00000920	0.00002510
0.12	0.12452060	0.11618490	0.10784920	0.36	0.27520970	-0.03038173	-0.33597320	0.60	0.13308470	-0.07293931	-0.27896330	0.84	-0.09650004	-0.00576692	0.08496620	1.20	-0.01883181	0.00573549	0.03030280	2.40	-0.00005603	0.00001328	0.00008259
0.14	0.14525000	0.13557610	0.12590230	0.42	0.32078050	-0.03453546	-0.38985140	0.70	0.15930820	-0.08513048	-0.32956910	0.98	-0.10861070	-0.00778948	0.09303176	1.40	-0.02500229	0.00681724	0.03863677	2.80	-0.00013251	0.00001811	0.00016873
0.16	0.16596830	0.15497980	0.14399140	0.48	0.36620360	-0.03826807	-0.44273970	0.80	0.18735950	-0.09731634	-0.38199220	1.12	-0.11877650	-0.01029634	0.09818387	1.60	-0.03249485	0.00793082	0.04835649	3.20	-0.00023938	0.00002334	0.00028605
0.18	0.18667380	0.17439770	0.16212160	0.54	0.41145040	-0.04151859	-0.49448750	0.90	0.21747200	-0.10948100	-0.43643390	1.26	-0.12664330	-0.01335183	0.09993967	1.80	-0.04143289	0.00906157	0.05955603	3.60	-0.00037699	0.00002828	0.00043355
0.20	0.20736500	0.19383160	0.18029810	0.60	0.45648950	-0.04422573	-0.54494090	1.00	0.24986680	-0.12160230	-0.49307130	1.40	-0.13182940	-0.01701756	0.09779428	2.00	-0.05190425	0.01018679	0.07227782	4.00	-0.00054077	0.00003186	0.00060450
0.22	0.22804000	0.21328300	0.19852600	0.66	0.50128600	-0.04632790	-0.59394180	1.10	0.28475110	-0.13365150	-0.55205400	1.54	-0.13392410	-0.02135156	0.09122102	2.20	-0.06395109	0.01127458	0.08650026	4.40	-0.00071868	0.00003251	0.00078371
0.24	0.24869730	0.23275380	0.21681020	0.72	0.54580140	-0.04776315	-0.64132770	1.20	0.32231610	-0.14559200	-0.61350010	1.68	-0.13248720	-0.02640728	0.07967261	2.40	-0.07755814	0.01228275	0.10212360	4.80	-0.00088817	0.00002814	0.00094444
0.26	0.26933520	0.25224550	0.23515590	0.78	0.58999320	-0.04846917	-0.68693160	1.30	0.36273460	-0.15737910	-0.67749280	1.82	-0.12704880	-0.03223242	0.06258393	2.60	-0.09263907	0.01315772	0.11895450	5.20	-0.00101313	0.00001614	0.00104542
0.28	0.28995180	0.27175990	0.25356800	0.84	0.63381440	-0.04838331	-0.73058100	1.40	0.40615810	-0.16895880	-0.74407580	1.96	-0.11711080	-0.03886747	0.03937585	2.80	-0.10902130	0.01383357	0.13668840	5.60	-0.00104112	-0.00000642	0.00102829
0.30	0.31054540	0.29129850	0.27205160	0.90	0.67721310	-0.04744254	-0.77209820	1.50	0.45271510	-0.18026740	-0.81325000	2.10	-0.10214920	-0.04634420	0.00946077	3.00	-0.12642890	0.01423129	0.15489150	6.00	-0.00090186	-0.00004256	0.00081673
0.32	0.33111430	0.31086300	0.29061170	0.96	0.72013270	-0.04558348	-0.81129970	1.60	0.50250690	-0.19123070	-0.88496830	2.24	-0.08161756	-0.05468376	-0.02774997	3.20	-0.14446490	0.01425830	0.17298150	6.40	-0.00050796	-0.00009486	0.00031824
0.34	0.35165660	0.33045490	0.30925310	1.02	0.76251060	-0.04274242	-0.84799550	1.70	0.55560510	-0.20176310	-0.95913120	2.38	-0.05495225	-0.06389466	-0.07283707	3.40	-0.16259140	0.01380827	0.19020800	6.80	0.00024051	-0.00016461	-0.00056973
0.36	0.37217050	0.35007560	0.32798080	1.08	0.80427890	-0.03885531	-0.88198940	1.80	0.61204820	-0.21176730	-1.03558300	2.52	-0.02157894	-0.07397044	-0.12636190	3.60	-0.18011100	0.01276156	0.20563420	7.20	0.00144587	-0.00025076	-0.00194738
0.38	0.39265400	0.36972680	0.34679970	1.14	0.84536300	-0.03385787	-0.91307870	1.90	0.67183810	-0.22113350	-1.11410500	2.66	0.01907883	-0.08488717	-0.18885320	3.80	-0.19614680	0.01098615	0.21811910	7.60	0.00319507	-0.00034840	-0.00389187
0.40	0.41310520	0.38940990	0.36571460	1.20	0.88568250	-0.02768556	-0.94105360	2.00	0.73493580	-0.22973880	-1.19441300	2.80	0.06758947	-0.09660070	-0.26079090	4.00	-0.20962510	0.00833940	0.22630390	8.00	0.00553233	-0.00044706	-0.00642645
0.42	0.43352230	0.40912630	0.38473040	1.26	0.92514960	-0.02027364	-0.96569690	2.10	0.80125750	-0.23744640	-1.27615000	2.94	0.12450080	-0.10904370	-0.34258810	4.20	-0.21925940	0.00467078	0.22860100	8.40	0.00842067	-0.00052879	-0.00947825
0.44	0.45390310	0.42887750	0.40385180	1.32	0.96366970	-0.01155732	-0.98678430	2.20	0.87067040	-0.24410590	-1.35888200	3.08	0.19032540	-0.12212230	-0.43456990	4.40	-0.22353960	-0.00017432	0.22319090	8.80	0.01169193	-0.00056633	-0.01282459
0.46	0.47424570	0.44866460	0.42308360	1.38	1.00114100	-0.00147171	-1.00408400	2.30	0.94298810	-0.24955200	-1.44209200	3.22	0.26552360	-0.13571290	-0.53694950	4.60	-0.22072500	-0.00634949	0.20802600	9.20	0.01498676	-0.00052185	-0.01603046
0.48	0.49454780	0.46848920	0.44243060	1.44	1.03745300	0.01004802	-1.01735700	2.40	1.01796600	-0.25360480	-1.52517500	3.36	0.35048380	-0.14965890	-0.64980160	4.80	-0.20884670	-0.01400068	0.18084540	9.60	0.01768924	-0.00034679	-0.01838281
0.50	0.51480750	0.48835230	0.46189720	1.50	1.07248800	0.02306647	-1.02635500	2.50	1.09529500	-0.25606920	-1.60743300	3.50	0.44550030	-0.16376640	-0.77303310	5.00	-0.18571850	-0.02325793	0.13920260	10.00	0.01886495	0.00001631	-0.01883233
0.52	0.53502240	0.50825520	0.48148810	1.56	1.10612000	0.03764805	-1.03082400	2.60	1.17459900	-0.25673520	-1.68806900	3.64	0.55074830	-0.17780120	-0.90635060	5.20	-0.14896020	-0.03422524	0.08050973	10.40	0.01721728	0.00062887	-0.01595954
0.54	0.55519030	0.52819920	0.50120810	1.62	1.13821400	0.05385673	-1.03050100	2.70	1.25542800	-0.25537740	-1.76618300	3.78	0.66625600	-0.19148470	-1.04922500	5.40	-0.09603721	-0.04696843	0.00210035	10.80	0.01108357	0.00154717	-0.00798924
0.56	0.57530900	0.54818520	0.52106140	1.68	1.16862700	0.07175592	-1.02511500	2.80	1.33725200	-0.25175550	-1.84076300	3.92	0.79187370	-0.20449050	-1.20085500	5.60	-0.02431677	-0.06150097	-0.09868518	11.20	-0.00149966	0.00280757	0.00711479
0.58	0.59537610	0.56821440	0.54105270	1.74	1.19720700	0.09140831	-1.01439000	2.90	1.41945800	-0.24561470	-1.91068700	4.06	0.92724110	-0.21644110	-1.36012300	5.80	0.06885336	-0.07776765	-0.22438860	11.60	-0.02262540	0.00440526	0.03143593
0.60	0.61538920	0.58828780	0.56118630	1.80	1.22379000	0.11287580	-0.99803880	3.00	1.50134100	-0.23668590	-1.97471300	4.20	1.07175000	-0.22690450	-1.52555900	6.00	0.18604180	-0.09562604	-0.37729390	12.00	-0.05419668	0.00626580	0.06672829
0.62	0.63534580	0.60840620	0.58146660	1.86	1.24820700	0.13621900	-0.97576930	3.10	1.58210200	-0.22468660	-2.03147500	4.34	1.22450700	-0.23539160	-1.69529100	6.20	0.32960290	-0.11482610	-0.55925520	12.40	-0.09741537	0.00821001	0.11383540
0.64	0.65524330	0.62857060	0.60189800	1.92	1.27027600	0.16149750	-0.94728120	3.20	1.66084200	-0.20932150	-2.07948500	4.48	1.38429100	-0.24135410	-1.86699900	6.40	0.50151430	-0.13498810	-0.77149050	12.80	-0.15204270	0.00991443	0.17187150
0.66	0.67507920	0.64878190	0.62248460	1.98	1.28980700	0.18876910	-0.91226850	3.30	1.73655300	-0.19028430	-2.11712200	4.62	1.54951000	-0.24418280	-2.03787500	6.60	0.70318190	-0.15557860	-1.01433900	13.20	-0.21541390	0.01087201	0.23715790
0.68	0.69485090	0.66904070	0.64323060	2.04	1.30659800	0.21809000	-0.87041780	3.40	1.80812000	-0.16725850	-2.14263700	4.76	1.71815600	-0.24320700	-2.20457000	6.80	0.93520770	-0.17588700	-1.28698200	13.60	-0.28122570	0.01036090	0.30194750
0.70	0.71455550	0.68934780	0.66414010	2.10	1.32043900	0.24951430	-0.82141050	3.50	1.87431000	-0.13991930	-2.15414800	4.90	1.88776300	-0.23769520	-2.36315400	7.00	1.19712200	-0.19500020	-1.58712200	14.00	-0.33817440	0.00743362	0.35304160
0.72	0.73419030	0.70970380	0.68521730	2.16	1.33110900	0.28309390	-0.76492160	3.60	1.93377000	-0.10793560	-2.14964100	5.04	2.05535600	-0.22685640	-2.50906900	7.20	1.48707700	-0.21178020	-1.91063700	14.40	-0.36860900	0.00094339	0.37049570
0.74	0.75375250	0.73010930	0.70646620	2.22	1.33837800	0.31887790	-0.70062230	3.70	1.98502300	-0.07097274	-2.12696900	5.18	2.21740700	-0.20984350	-2.63709400	7.40	1.80150500	-0.22484280	-2.25119000	14.80	-0.34747830	-0.01037070	0.32673690
0.76	0.77323900	0.75056480	0.72789060	2.28	1.34200400	0.35691240	-0.62817870	3.80	2.02646800	-0.02869451	-2.08385700	5.32	2.36979200	-0.18575810	-2.74130800	7.60	2.13474700	-0.23254080	-2.59982900	15.20	-0.24199930	-0.02771412	0.18657110
0.78	0.79264680	0.77107060	0.74949440	2.34	1.34173500	0.39724070	-0.54725330	3.90	2.05637100	0.01923318	-2.01790500	5.46	2.50774900	-0.15365740	-2.81506300	7.80	2.47865200	-0.23295290	-2.94455700	15.60	-0.01263246	-0.05195559	-0.09127872
0.80	0.81197300	0.79162720	0.77128130	2.40	1.33731000	0.43990170	-0.45750610	4.00	2.07287000	0.07313900	-1.92659200	5.60	2.62583700	-0.11256380	-2.85096500	8.00	2.82215600	-0.22388120	-3.26991900	16.00	0.38387920	-0.08320964	-0.55029850
0.82	0.83121420	0.81223470	0.79325520	2.46	1.32845700	0.48493080	-0.35859530	4.10	2.07397000	0.13334220	-1.80728600	5.74	2.71791200	-0.06147613	-2.84086500	8.20	3.15087000	-0.20285870	-3.55658700	16.40	0.98852070	-0.12026780	-1.22905600
0.84	0.85036740	0.83289340	0.81541950	2.52	1.31489500	0.53235890	-0.25017740	4.20	2.05754600	0.20014810	-1.65724900	5.88	2.77709500	0.00061549	-2.77586400	8.40	3.44665700	-0.16717170	-3.78100000	16.80	1.83012500	-0.15988300	-2.14989100
0.86	0.86942890	0.85360340	0.83777790	2.58	1.29633300	0.58221160	-0.13190980	4.30	2.02134200	0.27384340	-1.47365500	6.02	2.79575800	0.07471172	-2.64633400	8.60	3.68726400	-0.11389870	-3.91506100	17.20	2.91076000	-0.19594480	-3.30265000
0.88	0.88839570	0.87436470	0.86033380	2.64	1.27246900	0.63450960	-0.00345021	4.40	1.96297800	0.35469060	-1.25359700	6.16	2.76552400	0.16178290	-2.44195900	8.80	3.84599800	-0.03997031	-3.92593800	17.60	4.18589200	-0.21863160	-4.62315500
0.90	0.90726390	0.89517720	0.88309060	2.70	1.24299500	0.68926760	0.13554060	4.50	1.87995400	0.44292210	-0.99410990	6.30	2.67727600	0.26274380	-2.15178800	9.00	3.89150000	0.05774586	-3.77600800	18.00	5.53950500	-0.21369580	-5.96689700
0.92	0.92603020	0.91604090	0.90605160	2.76	1.20759000	0.74649410	0.28539830	4.60	1.76965500	0.53873340	-0.69218810	6.44	2.52118100	0.37842480	-1.76433100	9.20	3.78764400	0.18233180	-3.42298000	18.40	6.75556100	-0.16212130	-7.07980400
0.94	0.94469080	0.93695540	0.92922000	2.82	1.16592700	0.80619030	0.44645350	4.70	1.62936100	0.64227670	-0.34480800	6.58	2.28673900	0.50953830	-1.26766300	9.40	3.49360700	0.33667290	-2.82026100	18.80	7.48886700	-0.04049700	-7.56986100
0.96	0.96324190	0.95792040	0.95259890	2.88	1.11767100	0.86835070	0.61903010	4.80	1.45626000	0.75365280	0.05104512	6.72	1.96285600	0.65664090	-0.64957460	9.60	2.96417000	0.52327460	-1.91762100	19.20	7.24084700	0.17744890	-6.88594900
0.98	0.98167960	0.97893540	0.97619120	2.94	1.06247900	0.93296140	0.80344380	4.90	1.24745900	0.87290290	0.49834700	6.86	1.53793500	0.82009080	0.10224680	9.80	2.15028600	0.74403930	-0.66220770	19.60	5.34871100	0.51761380	-4.31348300
1.00	0.99999990	0.99999990	0.99999990	3.00	0.99999990	0.99999990	0.99999990	5.00	1.00000000	1.00000000	1.00000000	7.00	1.00000000	1.00000000	1.00000000	10.00	1.00000000	0.99999997	0.99999950	20.00	1.00000200	1.00000000	0.99999840

$X_a = 1$

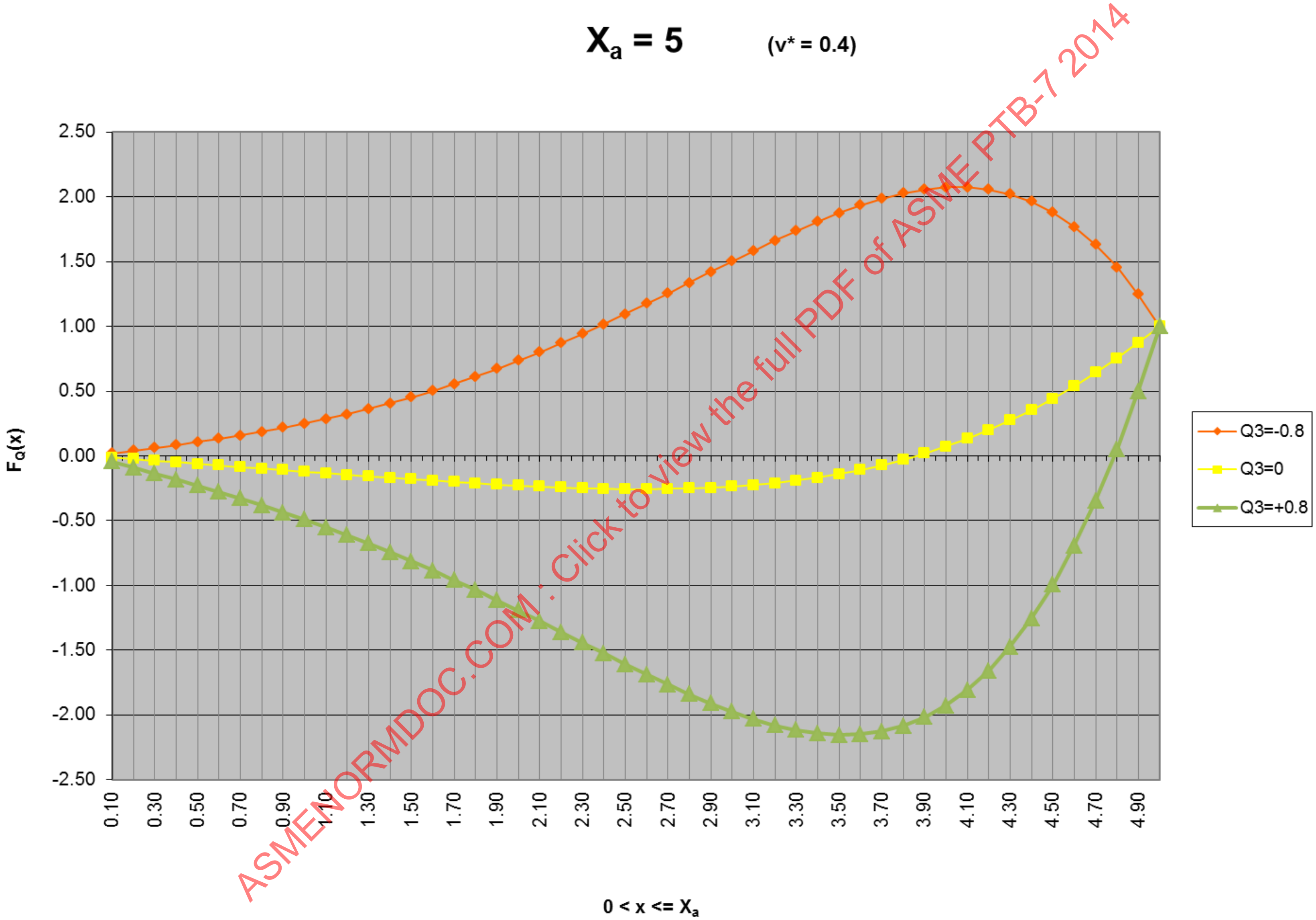

X_a = 1
(v* = 0.4)
1.20
1.00
0.80
0.60
0.40
0.20
0.00
F_Q(x)
0.02
0.06
0.10
0.14
0.18
0.22
0.26
0.30
0.34
0.38
0.42
0.46
0.50
0.54
0.58
0.62
0.66
0.70
0.74
0.78
0.82
0.86
0.90
0.94
0.98
0 < x <= X_a
Q3=-0.8
Q3=0
Q3=+0.8

$X_a = 3$



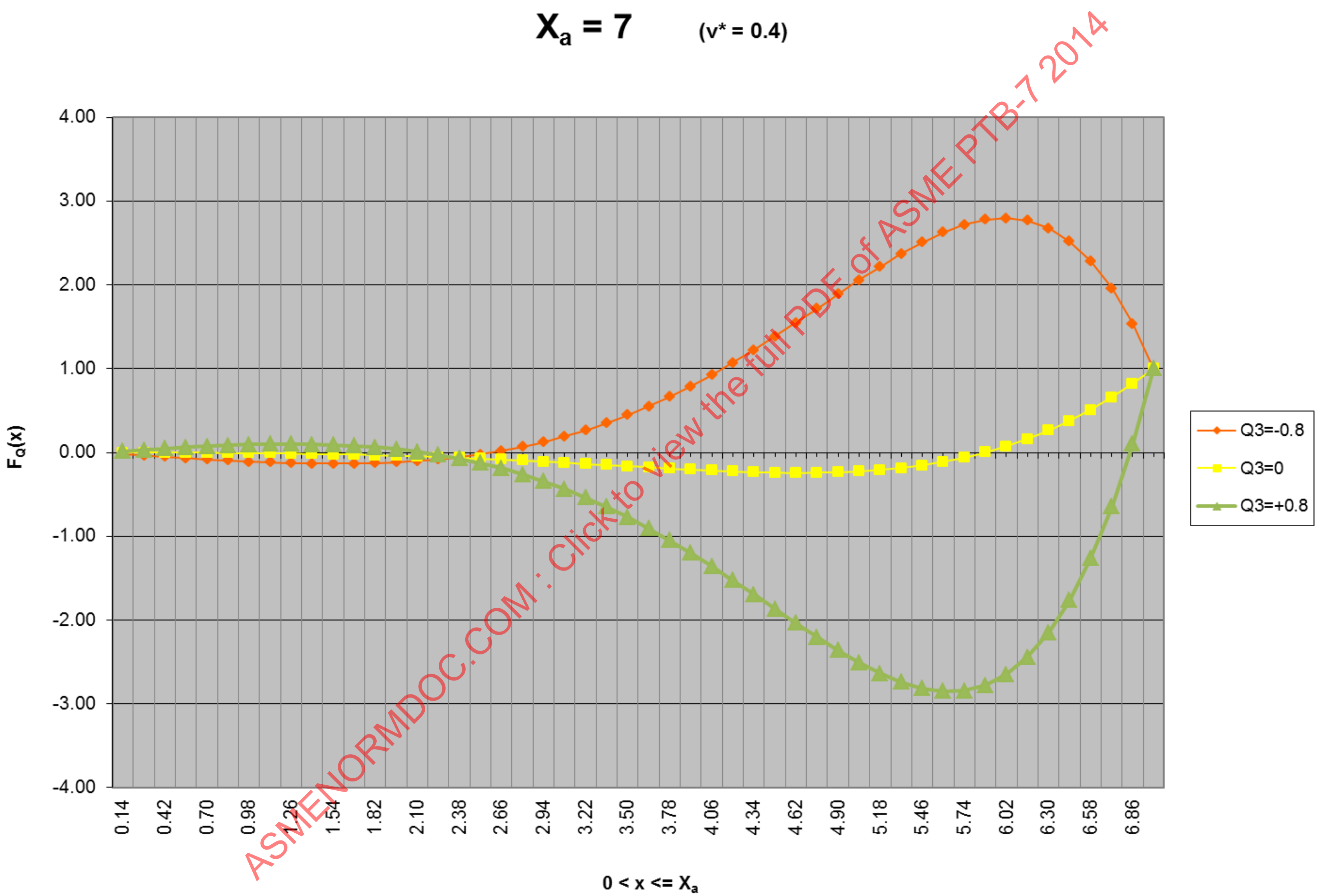
$X_a = 3$ (v* = 0.4)
$F_Q(x)$
1.50
1.00
0.50
0.00
-0.50
-1.00
-1.50
0.06
0.18
0.30
0.42
0.54
0.66
0.78
0.90
1.02
1.14
1.26
1.38
1.50
1.62
1.74
1.86
1.98
2.10
2.22
2.34
2.46
2.58
2.70
2.82
2.94
0 < x <= X_a
Q3=-0.8
Q3=0
Q3=+0.8

X_a = 5

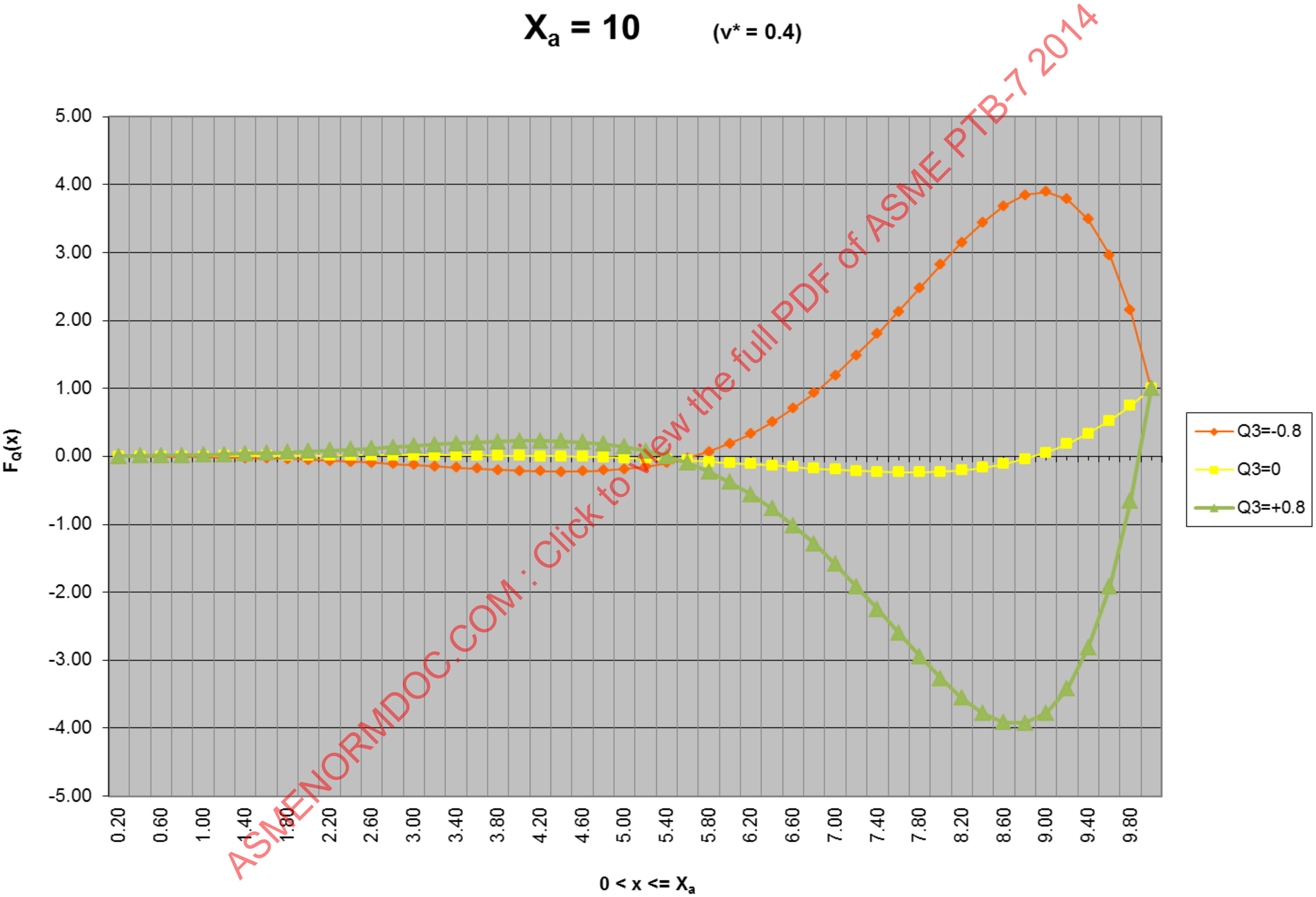


X_a = 5
(v* = 0.4)
$F_Q(x)$
2.50
2.00
1.50
1.00
0.50
0.00
-0.50
-1.00
-1.50
-2.00
-2.50
0.10
0.30
0.50
0.70
0.90
1.10
1.30
1.50
1.70
1.90
2.10
2.30
2.50
2.70
2.90
3.10
3.30
3.50
3.70
3.90
4.10
4.30
4.50
4.70
4.90
0 < x <= X_a
Q3=-0.8
Q3=0
Q3=+0.8

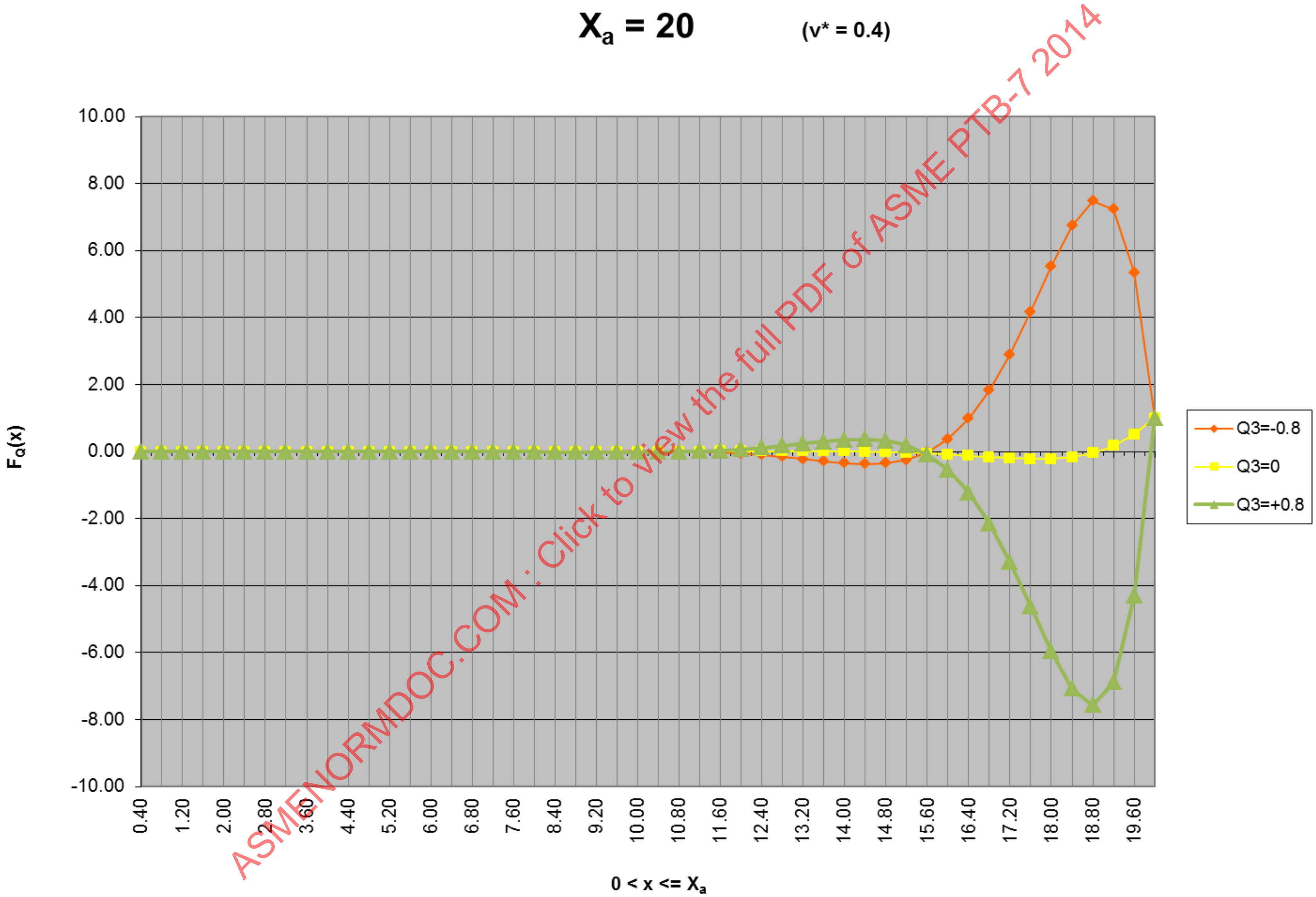
$X_a = 7$


X_a = 7 (v* = 0.4)
4.00
3.00
2.00
1.00
0.00
-1.00
-2.00
-3.00
-4.00
F_Q(x)
0.14 0.42 0.70 0.98 1.26 1.54 1.82 2.10 2.38 2.66 2.94 3.22 3.50 3.78 4.06 4.34 4.62 4.90 5.18 5.46 5.74 6.02 6.30 6.58 6.86
0 < x <= X_a
Q3=-0.8
Q3=0
Q3=+0.8

$X_a = 10$


Xa = 10 (v* = 0.4)
FQ(x)
5.00
4.00
3.00
2.00
1.00
0.00
-1.00
-2.00
-3.00
-4.00
-5.00
0.20 0.60 1.00 1.40 1.80 2.20 2.60 3.00 3.40 3.80 4.20 4.60 5.00 5.40 5.80 6.20 6.60 7.00 7.40 7.80 8.20 8.60 9.00 9.40 9.80
0 < x <= Xa
Q3=-0.8
Q3=0
Q3=+0.8

$X_a = 20$

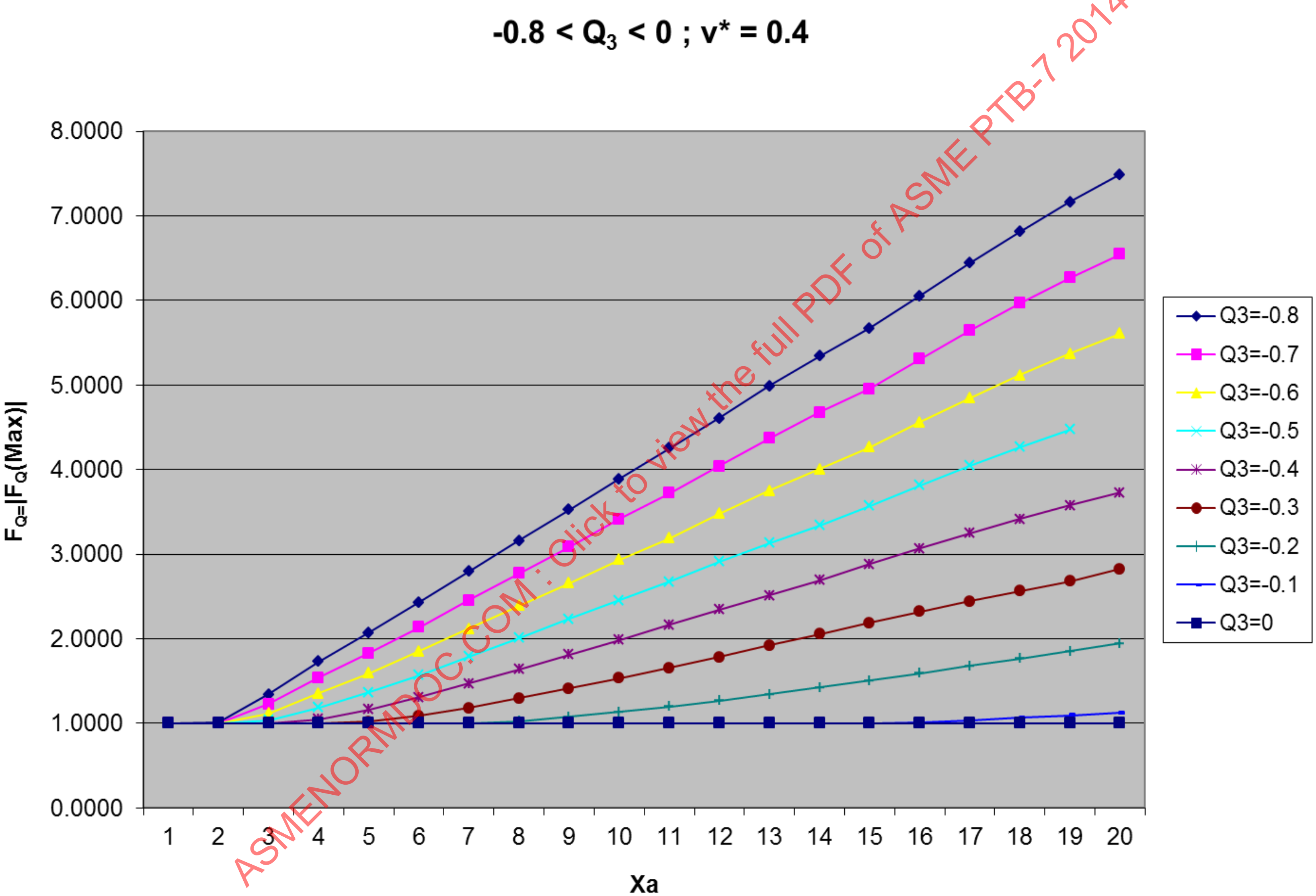

Xa = 20
(v* = 0.4)
FQ(x)
10.00
8.00
6.00
4.00
2.00
0.00
-2.00
-4.00
-6.00
-8.00
-10.00
0.40 1.20 2.00 2.80 3.60 4.40 5.20 6.00 6.80 7.60 8.40 9.20 10.00 10.80 11.60 12.40 13.20 14.00 14.80 15.60 16.40 17.20 18.00 18.80 19.60
0 < x <= Xa
Q3=-0.8
Q3=0
Q3=+0.8

F_Q=Max|$F_Q(x)$| vs X_a and Q_3

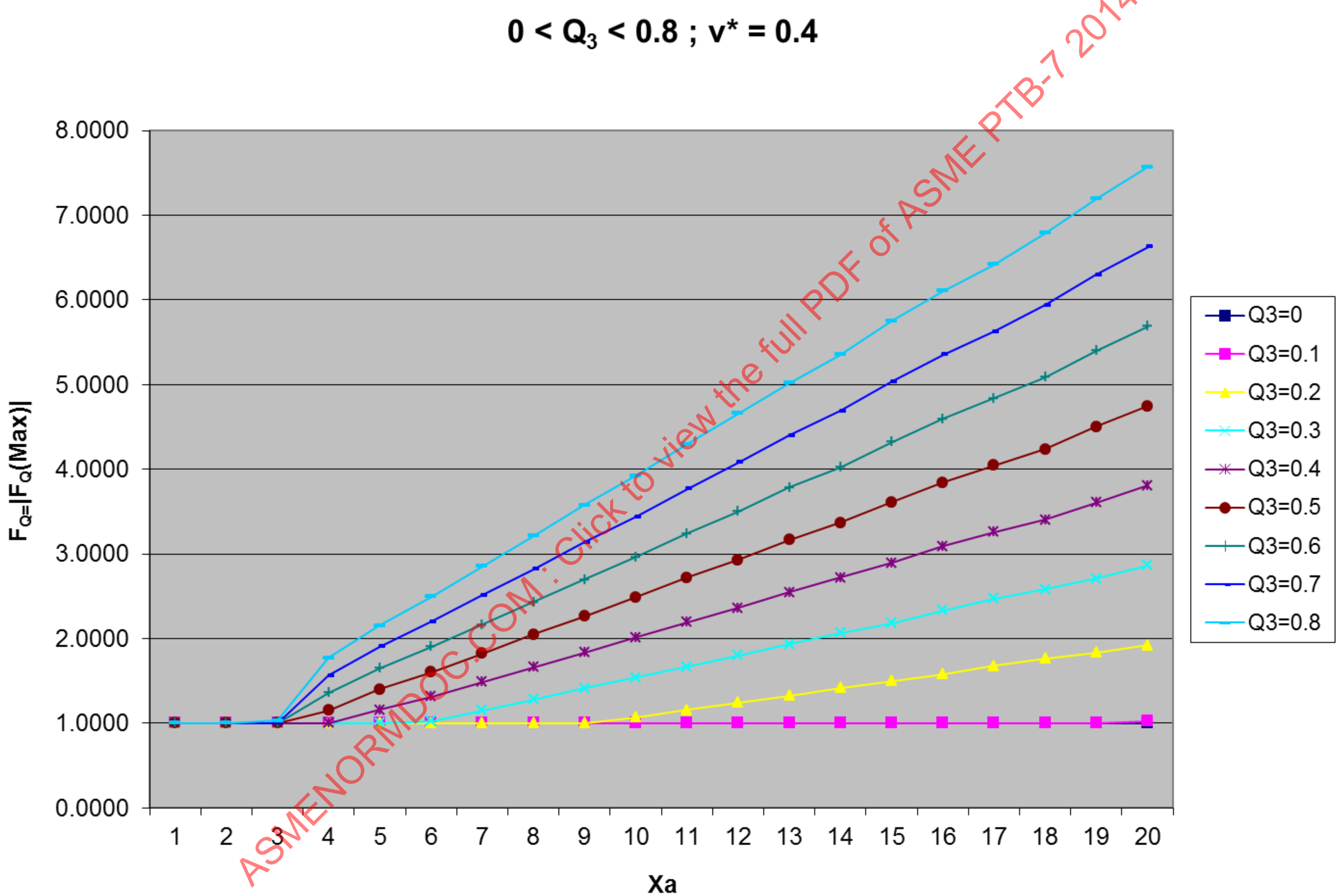
								F_Q=Max|$F_Q(x)$|		x(max)=X_a	$F_Q(X_a)$=1					
X_a\\Q_3	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.0057	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
3	1.3420	1.2251	1.1193	1.0353	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0308
4	1.7329	1.5390	1.3538	1.1841	1.0487	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.1542	1.3577	1.5619	1.7685
5	2.0740	1.8314	1.5932	1.3660	1.1614	1.0171	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.1549	1.4019	1.6506	1.9024	2.1541
6	2.4303	2.1378	1.8505	1.5698	1.3068	1.0852	1.0000	1.0000	1.0000	1.0000	1.0000	1.0261	1.3150	1.6050	1.9012	2.1974	2.4936
7	2.7958	2.4556	2.1155	1.7891	1.4700	1.1820	1.0000	1.0000	1.0000	1.0000	1.0000	1.1517	1.4844	1.8241	2.1664	2.5087	2.8510
8	3.1588	2.7744	2.3900	2.0055	1.6403	1.2933	1.0257	1.0000	1.0000	1.0000	1.0000	1.2779	1.6619	2.0458	2.4297	2.8196	3.2123
9	3.5225	3.0843	2.6566	2.2335	1.8104	1.4120	1.0755	1.0000	1.0000	1.0000	1.0000	1.4094	1.8371	2.2648	2.6966	3.1353	3.5741
10	3.8915	3.4123	2.9331	2.4538	1.9850	1.5343	1.1335	1.0000	1.0000	1.0000	1.0706	1.5393	2.0145	2.4896	2.9648	3.4402	3.9259
11	4.2515	3.7207	3.1899	2.6763	2.1667	1.6570	1.1980	1.0000	1.0000	1.0000	1.1569	1.6651	2.1912	2.7172	3.2433	3.7693	4.2953
12	4.6077	4.0416	3.4755	2.9094	2.3433	1.7842	1.2689	1.0000	1.0000	1.0000	1.2427	1.8012	2.3598	2.9257	3.5027	4.0798	4.6568
13	4.9889	4.3696	3.7504	3.1311	2.5118	1.9196	1.3443	1.0000	1.0000	1.0000	1.3224	1.9318	2.5491	3.1665	3.7838	4.4012	5.0185
14	5.3440	4.6754	4.0067	3.3381	2.6890	2.0548	1.4224	1.0000	1.0000	1.0000	1.4157	2.0670	2.7183	3.3697	4.0210	4.6858	5.3545
15	5.6689	4.9552	4.2629	3.5713	2.8798	2.1882	1.5049	1.0000	1.0000	1.0000	1.4983	2.1818	2.8955	3.6092	4.3229	5.0366	5.7503
16	6.0529	5.3060	4.5591	3.8122	3.0652	2.3183	1.5899	1.0146	1.0000	1.0000	1.5802	2.3343	3.0884	3.8424	4.5965	5.3506	6.1046
17	6.4433	5.6434	4.8435	4.0437	3.2438	2.4439	1.6768	1.0385	1.0000	1.0000	1.6787	2.4682	3.2577	4.0471	4.8366	5.6261	6.4156
18	6.8142	5.9642	5.1141	4.2640	3.4139	2.5639	1.7651	1.0644	1.0000	1.0000	1.7633	2.5831	3.4029	4.2368	5.0868	5.9369	6.7870
19	7.1635	6.2662	5.3689	4.4717	3.5744	2.6783	1.8541	1.0921	1.0000	1.0000	1.8342	2.7064	3.6036	4.5009	5.3982	6.2954	7.1927
20	7.4889	6.5477	5.6065	4.6654	3.7242	2.8262	1.9433	1.1215	1.0000	1.0268	1.9228	2.8640	3.8052	4.7463	5.6875	6.6287	7.5699

			x(max)=X_a	$F_Q(X_a)$=1					x(max)			x(max)=X_a	$F_Q(X_a)$=1				
X_a\\Q_3	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.8800	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000	2.0000
3	2.2800	2.4000	2.5200	2.7000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	3.0000	1.5600
4	3.1200	3.2000	3.2800	3.4400	3.6800	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	4.0000	2.4000	2.4000	2.4800	2.4800
5	4.1000	4.1000	4.2000	4.3000	4.5000	4.8000	5.0000	5.0000	5.0000	5.0000	5.0000	5.0000	3.4000	3.4000	3.5000	3.5000	3.5000
6	5.0400	5.0400	5.1600	5.1600	5.4000	5.6400	6.0000	6.0000	6.0000	6.0000	6.0000	4.3200	4.4400	4.5600	4.5600	4.5600	4.5600
7	6.0200	6.0200	6.0200	6.1600	6.3000	6.4400	7.0000	7.0000	7.0000	7.0000	7.0000	5.4600	5.4600	5.6000	5.6000	5.6000	5.6000
8	7.0400	7.0400	7.0400	7.0400	7.2000	7.3600	7.8400	8.0000	8.0000	8.0000	8.0000	6.5600	6.5600	6.5600	6.5600	6.7200	6.7200
9	7.9200	7.9200	8.1000	8.1000	8.1000	8.2800	8.6400	9.0000	9.0000	9.0000	9.0000	7.5600	7.5600	7.5600	7.7400	7.7400	7.7400
10	9.0000	9.0000	9.0000	9.0000	9.2000	9.2000	9.6000	10.0000	10.0000	10.0000	8.4000	8.6000	8.6000	8.6000	8.6000	8.8000	8.8000
11	9.9000	9.9000	9.9000	10.1200	10.1200	10.1200	10.5600	11.0000	11.0000	11.0000	9.4600	9.6800	9.6800	9.6800	9.6800	9.6800	9.6800
12	11.0400	11.0400	11.0400	11.0400	11.0400	11.2800	11.2800	12.0000	12.0000	12.0000	10.5600	10.5600	10.5600	10.8000	10.8000	10.8000	10.8000
13	11.9600	11.9600	11.9600	11.9600	11.9600	12.2200	12.2200	13.0000	13.0000	13.0000	11.4400	11.7000	11.7000	11.7000	11.7000	11.7000	11.7000
14	12.8800	12.8800	12.8800	12.8800	13.1600	13.1600	13.4400	14.0000	14.0000	14.0000	12.6000	12.6000	12.6000	12.6000	12.6000	12.8800	12.8800
15	13.8000	13.8000	14.1000	14.1000	14.1000	14.1000	14.4000	15.0000	15.0000	15.0000	13.5000	13.8000	13.8000	13.8000	13.8000	13.8000	13.8000
16	15.0400	15.0400	15.0400	15.0400	15.0400	15.0400	15.3600	15.6800	16.0000	16.0000	14.7200	14.7200	14.7200	14.7200	14.7200	14.7200	14.7200
17	15.9800	15.9800	15.9800	15.9800	15.9800	15.9800	16.3200	16.6600	17.0000	17.0000	15.6400	15.6400	15.6400	15.6400	15.6400	15.6400	15.6400
18	16.9200	16.9200	16.9200	16.9200	16.9200	16.9200	17.2800	17.6400	18.0000	18.0000	16.5600	16.5600	16.5600	16.9200	16.9200	16.9200	16.9200
19	17.8600	17.8600	17.8600	17.8600	17.8600	18.2400	18.2400	18.6200	19.0000	19.0000	17.4800	17.8600	17.8600	17.8600	17.8600	17.8600	17.8600
20	18.8000	18.8000	18.8000	18.8000	18.8000	19.2000	19.2000	19.6000	20.0000	18.4000	18.8000	18.8000	18.8000	18.8000	18.8000	18.8000	18.8000

F_Q vs X_a and -0.8<Q_3<0



Graph F_Q vs X_a and $0<Q_3<+0.8$



F_{Q}/F_{m}

$X_a \backslash Q_3$	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8
1	2.5	2.857143	3.333333	4	5	6.666667	8.998657	6.240394	4.77635	3.868719	3.250954	2.803314	2.464029	2.198006	1.983826	1.807681	1.660265
2	2.51432	2.857143	3.333333	4	5	6.666667	10	7.523566	5.848846	4.783954	4.047103	3.506946	3.093996	2.767785	2.494105	2.259883	2.05977
3	3.35501	3.500383	3.73087	4.141388	5	6.666666	10	13.86455	10.87076	8.093174	6.187609	4.914857	4.039271	3.412036	2.944396	2.58429	2.371012
4	4.332228	4.397226	4.512783	4.736272	5.24374	6.666666	9.999999	20	16.22084	10.19497	7.142984	5.417763	4.330965	4.148806	4.161247	4.170054	4.176638
5	5.184925	5.232549	5.310653	5.46412	5.80719	6.780873	10	20	20.41941	11.54998	7.711372	5.697706	5.190115	5.178172	5.175274	5.16785	5.162176
6	6.075725	6.108031	6.168247	6.2791	6.534085	7.23438	10	20	24.54934	12.67999	8.133738	6.055543	6.057201	6.043075	6.052838	6.045625	6.037598
7	6.989395	7.016077	7.051653	7.156484	7.35005	7.87972	10	20	28.86764	13.61935	8.465816	6.970071	6.971029	6.959758	6.960386	6.960843	6.96119
8	7.89696	7.926769	7.96651	8.022152	8.20139	8.622153	10.25692	20	33.19143	14.41635	8.714232	7.888013	7.886964	7.886306	7.885861	7.878157	7.877766
9	8.80632	8.812226	8.855463	8.934016	9.051845	9.413327	10.75545	20	37.58715	15.10373	8.940473	8.806101	8.822027	8.798765	8.780189	8.786229	8.790792
10	9.72875	9.749374	9.77687	9.815368	9.92494	10.22883	11.33498	20	41.82752	15.61794	9.707451	9.747581	9.734529	9.705679	9.686176	9.672886	9.688325
11	10.62868	10.63051	10.63296	10.70518	10.83327	11.04675	11.98035	20	46.23938	16.14145	10.64935	10.62359	10.62964	10.63336	10.63587	10.63769	10.63906
12	11.51917	11.54736	11.58494	11.63755	11.71648	11.89494	12.6891	20	50.55819	16.52118	11.62482	11.54198	11.49611	11.49684	11.53397	11.56074	11.58096
13	12.47236	12.48471	12.50117	12.52422	12.5588	12.79733	13.44304	20	55.22265	16.83206	12.438	12.43668	12.47522	12.49884	12.51481	12.52633	12.53502
14	13.36011	13.35825	13.35577	13.3523	13.44501	13.6986	14.22442	20	59.26641	17.16312	13.38468	13.37383	13.3682	13.36474	13.3624	13.38801	13.38615
15	14.17231	14.15778	14.20962	14.28528	14.39875	14.58788	15.04863	20	63.69394	17.51477	14.24479	14.19137	14.31318	14.38781	14.40964	14.39027	14.37574
16	15.13227	15.15997	15.19691	15.24862	15.32618	15.45545	15.89881	20.29252	68.54565	17.7724	15.11083	15.26828	15.3501	15.36974	15.32167	15.28733	15.26158
17	16.10813	16.12396	16.14506	16.1746	16.21892	16.29277	16.76836	20.77014	72.90249	17.8918	16.15014	16.23864	16.28462	16.18856	16.12208	16.07459	16.03898
18	17.0356	17.04047	17.04695	17.05604	17.06966	17.09236	17.65105	21.28718	76.86542	18.01652	17.07245	17.09924	17.01448	16.94706	16.95614	16.96263	16.96749
19	17.90864	17.90342	17.89646	17.88672	17.87211	17.85515	18.54106	21.84126	81.1034	18.14656	17.87644	18.03008	18.01824	18.00362	17.99388	17.98693	17.98171
20	18.72217	18.7077	18.68842	18.66142	18.62093	18.84149	19.43298	22.43002	85.63786	18.77245	18.86903	19.09339	19.02589	18.98539	18.9584	18.93911	18.92465

ANNEX R — DETERMINATION OF THE ALLOWABLE BUCKLING STRESS LIMITS

If the tubes are under compression, tube buckling may restrict the tube's load carrying ability. If a substantial number of tubes are above their buckling limit, it is possible that the bundle cannot sustain the required loading. This is true for either pressure or thermal load conditions. For this reason, no distinction is made between primary and secondary allowable compressive loads in the tubes.

The maximum permissible buckling stress limit in UHX parallels that as given in TEMA [12] .

(a) **When tubes are under compression, the axial tube stress $\sigma_t(x)$ is negative and must be limited to:**

- The critical Euler's stress $\sigma_{cr} = \dfrac{\pi^2 E_t}{\lambda_t^2}$ where: λ_t is the slenderness ratio of the tube: $\lambda_t = \dfrac{l_t}{r_t}$

r_t is the tube radius of gyration: $r_t = \dfrac{\sqrt{d_t^2 + (d_t - 2\,t_t)^2}}{4}$

$l_t = k\,l$ is the largest unsupported buckling length of the tube, obtained from the unsupported tube spans, l, and their corresponding method of support k, where:

k= 0.6 for unsupported spans between two tubesheets,

k= 0.8 for unsupported spans between a tubesheet and a tube support

k= 1.0 for unsupported spans between two tube supports.

- The yield strength,$S_{y,t}$ of the tube material to prevent entering in the plastic range, which might lead to the failing of the tube bundle.

Figure 69 shows the forbidden range for σ_t represented by the hatched areas.

So as to ensure a smooth transition, TEMA has drawn a line from point A_o (λ_o ; $S_{y,t}/2$) to point A (0 ; $S_{y,t}$) which equation is written:

$$\sigma_{t,bk} = S_{y,t}\left(1 - \frac{\lambda_t}{2\lambda_o}\right) \qquad \text{with: } \lambda_o = \sqrt{\frac{2\pi^2 E_t}{S_{y,t}}}$$

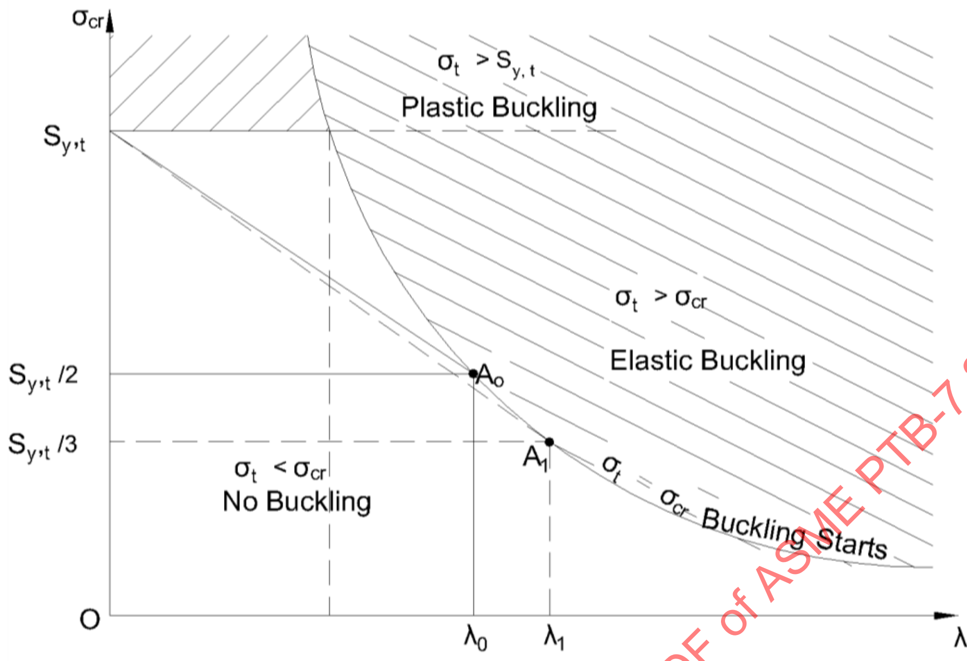


Figure 69 — Tube Buckling

(b) **Accordingly, the buckling stress limit $\sigma_{t,bk}$** is given by:

- $\sigma_{t,bk} = S_{y,t}\left(1 - \dfrac{\lambda_t}{2\lambda_o}\right)$ if $\lambda_t < \lambda_o$. The tubes have a low slenderness ratio and plastic buckling prevails.
- $\sigma_{t,bk} = \dfrac{\pi^2 E_t}{\lambda_t^2}$ if $\lambda_t \geq \lambda_o$. The tubes have a high slenderness ratio and elastic buckling prevails.

Note: The slope of line AA$_o$ is written: $-\dfrac{S_{y,t}}{2\lambda_o}$ *The slope of Euler's curve at point A$_o$ is written:*

$$\left[\frac{d\sigma}{d\lambda}\right]_{A_o} = -\frac{2\pi^2 E_t}{\lambda_o^3} = -\frac{S_{y,t}}{\lambda_o}$$ *which is half of the slope of line AAo*

A smoother transition would be obtained if the line was tangent to the Euler's curve, at point A$_1$ on Figure 69,

with a slope: $\left[\dfrac{d\sigma_{cr}}{d\lambda_t}\right]_{A_1} = -\dfrac{2\pi^2 E_t}{\lambda_1^3}$. *Equation of line AA$_1$ is written:* $\sigma_{t,bk} = S_{y,t} - \dfrac{2\pi^2 E_t}{\lambda_1^3}\lambda$

The abscissa λ_1 of point λ_1 A$_1$ is obtained from: $S_{y,t} - \dfrac{2\pi^2 E_t}{\lambda_1^3}\lambda_1 = \dfrac{\pi^2 E_t}{\lambda_1^2}$,

which leads to: $\lambda_1 = \sqrt{1.5}\lambda_o$

The ordinate of point A$_1$ is obtained from: $[\sigma_{cr}]_{A_1} = \dfrac{\pi^2 E_t}{\lambda_1^2} = \dfrac{\pi^2 E_t}{1.5\lambda_o^2} = \dfrac{S_{y,t}}{3}$

Line AA$_1$, starting at $S_{y,t}/3$ instead of $S_{y,t}/2$, would ensure a better transition between elastic and plastic buckling.

(c) **The safety factor** applied to $\sigma_{t,bk}$ used in UHX is based on TEMA approach as follows.
The tube stress distribution throughout the tube bundle is given in Section 8.5 of PART 3:

$$\sigma_t(x)=\frac{1}{x_t-x_s}\left[\left(x_sP_s-x_tP_t\right)-F_t(x)\,P_e\right] \quad \text{with} \quad F_t(x)=\frac{X_a^4}{2}\left[Q_3\,Z_w(x)+Z_d(x)\right] \qquad \text{[VIII-5]}$$

x=kr

At periphery ($x=ka_o=Xa$): $F_t(X_a)=F_q=\frac{X_a^4}{2}\left[Q_3\,Z_w+Z_d\right]$

The following simplifications are made in TEMA.

(1) No unperforated rim: $\left.\begin{array}{l} a_o=a_s\Rightarrow\rho_s=1\\ a_o=a_c\Rightarrow\rho_c=1\end{array}\right\} a_o=a_s=a_c=a\Rightarrow Q_1=-\frac{\Phi\,Z_v}{1+\Phi\,Z_m}$

(2) No flange is considered: $W=0,\ A=D_o\ \Rightarrow K=1\Rightarrow\Phi=\frac{1-\upsilon^{*2}}{E^*}[\lambda_s+\lambda_c]$

(3) Radial displacement due to pressures P_s and P_t acting at TS-shell-channel connection is ignored:

$$\left.\begin{array}{l} w_s=0\Rightarrow\delta_s=0\Rightarrow\omega_s=0\Rightarrow\omega_s^*=0\\ w_c=0\Rightarrow\delta_c=0\Rightarrow\omega_c=0\Rightarrow\omega_c^*=0\end{array}\right\}\Rightarrow Q_2=0\Rightarrow Q_3=Q_1=-\frac{\Phi\,Z_v}{1+\Phi\,Z_m}$$

$$F_t(x)=\frac{X_a^4}{2}\left[-\frac{\Phi\,Z_v}{1+\Phi\,Z_m}Z_w(x)+Z_d(x)\right]$$

(4) The maximum of $F_t(x)$ appears either inside of the tube-bundle (r<R) or at its periphery (r=R). TEMA considers only the outermost tube row.

At periphery $x=X_a=kR$: $F_t(X_a)=F_q=\frac{X_a^4}{2}\left[-\frac{\Phi\,Z_v}{1+\Phi\,Z_m}Z_w+Z_d\right]$

(5) The TS is considered

- either simply supported: $\lambda_s=0,\lambda_c=0\Rightarrow\Phi=0\Rightarrow Q_1=0$
- or clamped: $\lambda_s=\infty,\lambda_c=\infty\Rightarrow\Phi=\infty\Rightarrow Q_1=-\frac{Z_v}{Z_m}$

Curves $F_t(x)$ as a function of x=r/R for various values of X_a=kR (X_a=1, 2, 3, …., 20) represent the tube stress distribution $\sigma_t(x)$ throughout the tube bundle given by [VIII-5] above. Examination of these curves shows that:

The stress distribution varies smoothly throughout the tube bundle when X_a is low, i.e. when the tube bundle rigidity is significantly lower than the TS rigidity:

$$\left.\begin{array}{ll} X_a=3 \text{ when the TS is simply supported, which leads to} & F_q=2.25\\ X_a=7 \text{ when the TS is clamped, which leads to} & F_q=2.7\end{array}\right\}\text{in both cases } F_q \text{ is about 2.5.}$$

In that case, a large amount of adjacent tube rows may not take over the extra compressive load, which would lead to a general buckling of the tube bundle.
Accordingly, a higher safety factor F_S=2.0 is used.

The stress distribution varies significantly throughout the tube bundle when X_a is high, i.e. when the tube bundle rigidity is significantly higher than the TS rigidity:

$$\left.\begin{array}{l} X_a = 5 \text{ when the TS is simply supported, which leads to } F_q = 3.79 \\ X_a = 10 \text{ when the TS is clamped, which leads to } F_q = 3.83 \end{array}\right\} \text{in both cases } F_q \text{ is about 4.}$$

In that case, if the outermost tube row buckles, the adjacent tube rows can take over the extra compressive load. Accordingly, a lower safety factor F_S=1.25 is used.

In the intermediate range of F_q ($2.5 \le F_q \le 4$), a linear interpolation is used (see Figure 70):

$$F_S = 3.25 - 0.5F_q$$

Thus, the safety factor is written: $F_S = MAX\left[\left(3.25 - 0.5F_q\right), (1.25)\right]$

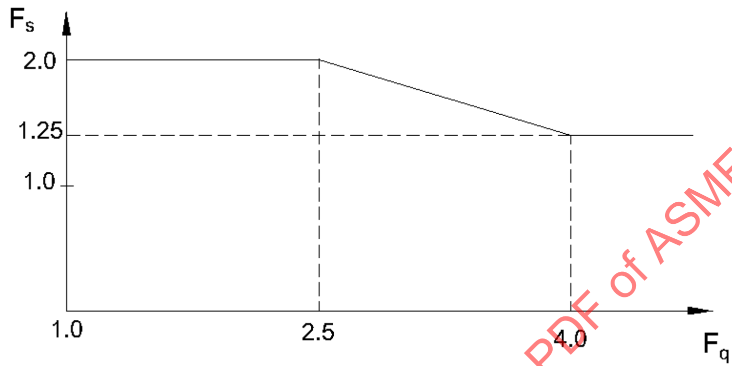


Figure 70 — Determination of Buckling Safety Factor, FS

(d) **Moving to UHX/TEMA notations** (F_t=λ_t ; C_t=λ_o), the maximum permissible buckling stress limit is written, having in mind that it cannot exceed the allowable stress of the tube material:

$$S_{tb} = MIN\left\{\left[\frac{1}{F_S}\frac{\pi^2 E_t}{F_t^2}\right], [S_t]\right\} \quad \text{when } C_t \le F_t$$

$$S_{tb} = MIN\left\{\left[\frac{S_{y,t}}{F_S}\left(1 - \frac{F_t}{2C_t}\right)\right], [S_t]\right\} \quad \text{when } C_t > F_t$$

with: $F_t = \dfrac{l_t}{r_t}$ $\qquad r_t = \dfrac{\sqrt{d_t^2 + (d_t - 2t_t)^2}}{4}$ $\qquad C_t = \dfrac{2\pi^2 E_t}{S_{y,t}}$

$$F_S = MAX\left[\left(3.25 - 0.5F_q\right), (1.25)\right] \qquad F_q = \frac{X_a^4}{2}\left[Q_3 Z_w + Z_d\right]$$

The minimum value of $\sigma_t(x)$, given by: $\sigma_{t,\min}$ =MIN$\left[\left(\sigma_{t,1}\right), \left(\sigma_{t,2}\right)\right]$ **must not exceed the buckling stress limit:** $\left|\sigma_{t,\min}\right| \le S_{tb}$

ANNEX S — COMMON INTERSECTION OF CURVES $\Sigma_t(x)$

1 General

Numerical calculations of H.E. according to UHX-13 show that $\sigma_t(x)$ curves ($0<x<X_a$) intersect at the same point $x=x_o$ for the 7 loading cases (ASME 2013) , whatever the Q_3 value is for each of these loading cases (see Figure 71).

$$\sigma_t(x) = \frac{1}{x_t - x_s}\left[\left(P_s \cdot x_s - P_t \cdot x_t\right) - P_e \cdot F_t(x)\right]$$

$\sigma_t(x)$ is calculated from $F_t(x)$: $$F_t(x) = \left[Z_d(x) + Q_3 \cdot Z_w(x)\right] \cdot \frac{X_a^4}{2}$$

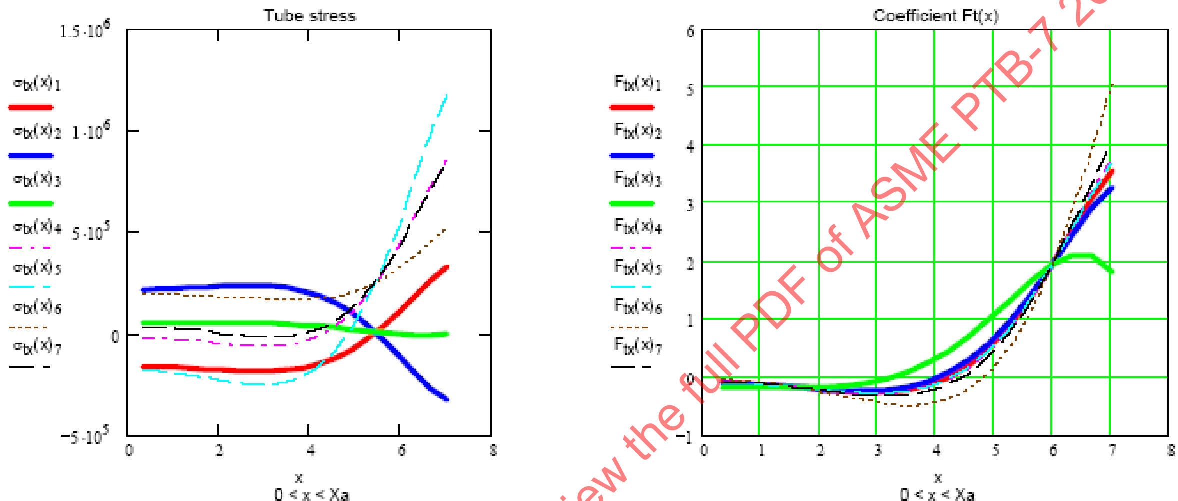


Figure 71 — Graphs Giving $\sigma_t(x)$ and $F_t(x)$ for the 7 Loading Cases (ASME 2013)

2 Determination of Common Intersection x_o for $\sigma_t(x)$

Curves $F_t(x)$ will intersect at the same point of abscissa x_o, whatever Q_3 value is, if:

$$\frac{2}{X_a^4} F_t(x_o) = Z_d(x_o) + Q_3 \cdot Z_w(x_o) = K$$

Where K is a constant independent of Q_3 . This is only possible if $Z_w(x_o) = 0$, which means that the value x_o of common intersection is obtained for value(s) of x_o which make $Z_w(x)$ equal to 0.
Parametric calculations on X_a show that there is always at least 1 value of x_o which makes $Z_w(x_o) = 0$. For high values of X_a ($X_a>6$), there are 2 values of x_o. For Example E4.18.7 ($X_a=7$): $Z_w(x) = 0$ for $x_o=1.68$ and $x_o=5.97$.

The curves $\sigma_t(x)$ for the 3 pressure loading cases (ASME 2013) intersect at the same point of abscissa x_1.This point is different from the intersection point x_o of $F_t(x)$, due to the presence of P_e.
The curves $\sigma_t(x)$ for the 4 pressure and thermal loading cases (ASME 2013) intersect at the same point of abscissa x_1, but their values are generally higher than for the 3 pressure loading cases due to higher values of P_e for thermal loading cases.

For Example E4.18.7 (2013):

- curves relative to pressure loading cases 1, 2 and 3 intersect at the same point ($x_1 = 5.5$; $\sigma_t=0$)

- curves relative to pressure and thermal loading cases 4, 5, 6 and 7 intersect at the same point (x_1 =5.5; σ_t= 1700 psi)

3 Generalization to Other Stresses

The same principle applies also to:

the tubesheet bending stress σ(x), calculated from: $F_m(x) = [Q_v(x) + Q_3 \cdot Q_m(x)] \cdot \frac{1}{2}$

Value of intersection is obtained for $Q_m(x_o)$=0. There is no value of x_o for X_a<5, and one value of x_o for $X_a \geq 5$

the tubesheet shear stress τ(x), calculated from: $F_Q(x) = |Q_\beta(x) + Q_3 \cdot Q_\alpha(x)|$

Value of intersection is obtained for $Q_\alpha(x_o)$=0. There is always one solution for x_o=X_a (as $Q_\alpha(X_a)$=0), and a 2nd value of x_o for $X_a \geq 7$

the tubesheet slope θ(x), calculated from: $F_\theta(x) = [Z_v(x) + Q_3 \cdot Z_m(x)] \cdot \frac{X_a^3}{2}$

ANNEX T — DETERMINATION OF STRESSES IN U-TUBE TS HES USING THE FIXED TS RULES

UHX -12 rules can be obtained using the fixed TS analysis developed in PART 3, provided that the tubes have no axial rigidity, which implies that they do not play the role of an elastic foundation.

Accordingly: $k_t = 0 \quad \Rightarrow \quad k_w = \dfrac{N_t\, k_t}{\pi\, a_o^2} = 0 \quad \Rightarrow X_a = 0$

Modifications of PART 3 analysis are as follows.

(a) **Equilibrium of the Unperforated Rim** (see Sections 6.2 and 6.3 of PART 3)

(1) **due to axial loads** axial equilibrium of the shell: $2\pi a_s^{'} V_s = \pi a_s^2 P_s$ leads to: $a_o V_a = \dfrac{a_o^2}{2} P_e$

(2) **due to applied moments:** explicating V_a,
equation [VI.2b'] of PART 3 becomes:

$$[R\, M_R] = -a_o\, M_a + \frac{P_s}{4} a_o^3 \left[(\rho_s - 1)(\rho_s^2 + 1) \right] - \frac{P_c}{4} a_o^3 \left[(\rho_c - 1)(\rho_c^2 + 1) \right]$$
$$-\left[a_s^{'}\, k_s \left(1 + t_s^{'} + \frac{t_s^{'2}}{2} \right) + a_c^{'}\, k_c \left(1 + t_c^{'} + \frac{t_c^{'2}}{2} \right) \right] \theta_a + a_o \left(\omega_c\, P_c - \omega_s\, P_s \right)$$

equation [VI.3a'] of PART 3 becomes:

$$\Phi \left[M_a\, Z_m + (a_o\, V_a)\, Z_v \right] = -M_a + \left(\omega_s^*\, P_s - \omega_c^*\, P_c \right) + \frac{1}{2\pi} \left[W_c\, \gamma_{bc} - W_s\, \gamma_{bs} \right]$$

with: $\Phi = (1 + \upsilon^*) F$ $\quad \boxed{\omega_s^* = a_o^2 \left[\dfrac{(\rho_s - 1)(\rho_s^2 + 1)}{4} \right] - \omega_s} \quad \boxed{\omega_c^* = a_o^2 \left[\dfrac{(\rho_c - 1)(\rho_c^2 + 1)}{4} \right] - \omega_c}$

(b) **Stresses in the Tubesheet** (see Section 6.3 of PART 3)

(1) **moment at TS periphery**
Equation [VI.3a] of PART 3 becomes:

$$M_a = \left(\frac{a_o^2}{2} P_e \right) \underbrace{\frac{-\Phi\, Z_v}{1 + \Phi\, Z_m}}_{Q_1} + \underbrace{\frac{\left(\omega_s^*\, P_s - \omega_c^*\, P_c \right) + \left(W_c \dfrac{\gamma_{bc}}{2\pi} - W_s \dfrac{\gamma_{bs}}{2\pi} \right)}{1 + \Phi\, Z_m}}_{Q_2}$$

$$M_a = (a_o V_a)\, Q_1 + Q_2 \qquad Q_1 = \frac{-\Phi\, Z_v}{1 + \Phi\, Z_m} \qquad Q_2 = \frac{\left(\omega_s^*\, P_s - \omega_c^*\, P_c \right) + (W_c - W_s) \dfrac{\gamma_b}{2\pi}}{1 + \Phi\, Z_m}$$

From Annex F, for X_a=0:

$$Z_m = \frac{1}{1 + \upsilon^*} \qquad \Phi Z_m = F \qquad Z_v = \frac{1}{4(1 + \upsilon^*)} \qquad \Phi Z_v = \frac{F}{4}$$

$$\boxed{Q_1 = \frac{\dfrac{F}{4}}{1 + F}} \qquad \boxed{Q_2 = \frac{\left(\omega_s^*\, P_s - \omega_c^*\, P_c \right) + (W_c - W_s) \dfrac{\gamma_b}{2\pi}}{1 + F}}$$

Explicating ω_s^* *and* ω_c^*, Q_2 is written:

$$Q_2 = \frac{P_s \frac{a_o^2}{4}\left[(\rho_s - 1)(\rho_s^2 + 1)\right] - P_c \frac{a_o^2}{4}\left[(\rho_c - 1)(\rho_c^2 + 1)\right] + (\omega_c P_c - \omega_s P_s) + (W_c - W_s)\frac{\gamma_b}{2\pi}}{1 + F}$$

$$M_a = \frac{\overbrace{\overbrace{P_s \frac{a_o^2}{4}\left[(\rho_s - 1)(\rho_s^2 + 1)\right] - P_c \frac{a_o^2}{4}\left[(\rho_c - 1)(\rho_c^2 + 1)\right]}^{M_{TS}} + (\omega_c P_c - \omega_s P_s) + \frac{\gamma_b}{2\pi}(W_c - W_s)}^{M^*} - F\frac{a_o^2}{8} P_e}{1 + F} = \frac{M^* - F\left(\frac{a_o^2}{8} P_e\right)}{1 + F}$$

W_s and W_c values to be used in UHX-12 are given in Section 5.2(d) of PART 5.

(2) **moment at TS center** is given by Annex F when x=0

$$M_0 = M(0) = a_o^2 \, P_e \, F_m(0) \quad \text{with:} \quad F_m(0) = \frac{Q_3}{2} + \frac{3 + \upsilon^*}{16} \qquad Q_3 = Q_1 + \frac{2}{a_0^2 \, P_e} Q_2$$

Using Q_1 and Q_2 above: $Q_3 = \frac{-\frac{F}{4}}{1 + F} + \frac{2}{(a_o^2 P_e)} \frac{(\omega_s^* \, P_s - \omega_c^* \, P_c) + (W_c - W_s)\frac{\gamma_b}{2\pi}}{1 + F}$

$$M_o = \frac{-\frac{F}{4} a_o^2 P_e + (\omega_s^* \, P_s - \omega_c^* \, P_c) + (W_c - W_s)\frac{\gamma_b}{2\pi}}{1 + F} + \frac{a_o^2}{16}(3 + \upsilon^*) P_e = M_a + \frac{a_o^2}{16}(3 + \upsilon^*) P_e$$

(3) **maximum moment in TS**

Annex P shows that for low values of X_a (X_a<1), the maximum of the TS moment M(x) appears either at the center or at TS periphery: $M = MAX\left[|M_P|, |M_o|\right]$

(c) **Stresses in the Shell and Channel** (see Section 8.6 an 8.7 of PART 3)

(1) axial membrane stress

From [VIII.6a] of PART 3:

$$\sigma_{s,m} = \frac{a_o^2}{(D_s + t_s)t_s}\left[(P_s - P_t) + (\rho_s^2 - 1)(P_s - P_t)\right] + \frac{a_s^2}{(D_s + t_s)t_s} P_t = \frac{a_o^2}{(D_s + t_s)t_s}\left[\rho_s^2 (P_s - P_t)\right] + \frac{a_s^2}{(D_s + t_s)t_s}$$

$$\sigma_{s,m} = \frac{a_o^2}{(D_s + t_s)t_s}\left[\rho_s^2 (P_s - P_t)\right] + \frac{a_s^2}{(D_s + t_s)t_s} P_t$$

$$\sigma_{s,m} = \frac{a_s^2}{(D_s + t_s)t_s} P_s$$ which is the classical formula for cylinders

Axial membrane stress in the channel is written in the same way:

$$\sigma_{c,m} = \frac{a_c^2}{(D_c + t_c)t_c} P_t$$

(2) **bending stress**

From [VIII.6c] of PART 3:

$$\sigma_{s,b} = \frac{6}{t_s^2} k_s \left\{ \beta_s \, \delta_s \, P_s + \frac{6\left(1-\nu^{*2}\right)}{E^*} \left(\frac{a_o}{h}\right)^3 \left(1+\frac{h\,\beta_s}{2}\right) \left[P_e \left(Z_v + Q_1 Z_m\right) + \frac{2}{a_o^2}\left(Q_2 Z_m\right) \right] \right\}$$

From Annex F, for X_a=0: $Z_m = \frac{1}{1+\upsilon^*}$ $\qquad Z_v = \frac{1}{4\left(1+\upsilon^*\right)}$

$$\sigma_{s,b} = \frac{6}{t_s^2} k_s \left\{ \beta_s \, \delta_s \, P_s + \frac{6\left(1-\nu^{*2}\right)}{E^*} \left(\frac{a_o}{h}\right)^3 \left(1+\frac{h\,\beta_s}{2}\right) \left[P_e \left(\frac{1}{4(1+\upsilon^*)} + Q_1 \frac{1}{(1+\upsilon^*)} \right) + \frac{2}{a_o^2} \left(Q_2 \frac{1}{(1+\upsilon^*)} \right) \right] \right\}$$

$$\sigma_{s,b} = \frac{6}{t_s^2} k_s \left\{ \beta_s \, \delta_s \, P_s + 6 \frac{1-\nu^{*2}}{E^*} \left(\frac{a_o}{h}\right)^3 \left(1+\frac{h\,\beta_s}{2}\right) \frac{1}{1+\upsilon^*} \frac{2}{a_o^2} \left[P_e \frac{a_o^2}{8} + \underbrace{P_e Q_1 \frac{a_o^2}{2} + Q_2}_{M_a} \right] \right\}$$

$$\boxed{\sigma_{s,b} = \frac{6}{t_s^2} k_s \left[\beta_s \delta_s P_s + 6 \frac{1-\nu^{*2}}{E^*} \frac{D_o}{h^3} \left(1+\frac{h\beta_s}{2}\right) \left(M_a + \frac{D_o^2}{32} P_e \right) \right]}$$

Axial bending stress in the channel is written in the same way:

$$\boxed{\sigma_{c,b} = \frac{6}{t_c^2} k_c \left[\beta_c \delta_c P_c - 6 \frac{1-\nu^*}{E^*} \frac{D_o}{h^3} \left(1+\frac{h\,\beta_c}{2}\right) \left(M_a + \frac{D_o^2}{32} P_e \right) \right]}$$

(d) **In conclusion** the general formulas of PART 3, applied to U-Tube TS HE, match the UHX-12 formulas obtained in Section 6 of PART 5. This implies that UHX-12 rules could be written in the same way as UHX-13 rules, using for (ω_s^*, ω_c^*) and (Q1, Q2) the formulas given here above.

ANNEX U — CALCULATION OF A U-TUBE TS USING FLOATING OR FIXED TS HE SOFTWARE

A U-Tube HE can be calculated using a floating TS HE software, such as the Mathcad software used in Section 11.3 of PART 4 as follows:

- use an immersed floating head TS HE as explained in Section 3.5.1 of PART 4.
- use N_t=0 so that the tubes have no axial rigidity.

Calculation could be also performed using a fixed TS HE software, such as the Mathcad software used in Annex V as follows:

- use a bellows of rigidity close to 0 to simulate an immersed floating TS
- use L_t=infinity ($Lt=10^{20}$), so that the tubes have no axial rigidity.
- use $P_e=P_s-P_t$

ANNEX V — UHX-13 – EXAMPLE E4.18.7 (PTB-4 2013 EDITION) WITH GENERAL EQUATIONS

This Annex provides a Mathcad calculation sheet for the fixed TS HE defined in Example E4.18.7 (PTB-4 2013 Edition).The TS is integral with shell and channel (configuration a). The data are shown in the calculation sheet, which is divided in 2 parts.

Part 1 follows strictly the steps 1 to 11 of UHX-13.5 calculation procedure. It includes the use of the elastic-plastic procedure at the TS-shell-channel connection.
Part 2 provides the equations which enable to calculate at any radius of the perforated tubesheet:

- **the net effective pressure q(r),**
- **the deflection w(r),**
- **the rotation θ(r) ,**
- **the bending moment M_r(r), and the bending stress σ(r),**
- **the shear force Q_r(x) and the shear stress τ(x),**
- **the tube axial stress σ_t(r)**

These quantities are also given in graphical format. Their maximum values are determined and they match the maximum stress values obtained in Part 1. The positive directions of these quantities are shown on Figure 45.

The equations are taken from Section 8 of PART 3 and therefore depend on axial load V_a and bending moment M_a acting at the periphery of the tubesheet (see Figure 40) which are determined from the equivalent pressure P_e:

$$V_a = \frac{a_o}{2} \cdot P_e \qquad M_a = (a_o V_a) \cdot Q_1 + Q_2$$

These equations are general and do not depend on coefficient Q_3.Thus, they apply whether $P_e \neq 0$ or $P_e = 0$. The calculation sheet provides also the determination of the moment M_R acting on the unperforated rim and the edge loads Q_s, Q_c, M_s, M_c and axial force V_s and elastic stretch Δ_s of the shell. The positive directions of these quantities are shown on Figure 40. See Annex Y for UHX-13- Example E4.18.7 (PTB-4 2013 Edition) with General Equations.

A fixed tubesheet heat exchanger with the tubesheet construction in accordance with Configuration a as shown in VIII-1, Figure UHX-13.1, Configuration a.

- For the Design Condition, the shell side design pressure is 325 psig at 400°F, and the tube side design pressure is 200 psig at 300°F.
- There is one operating condition. For Operating Condition 1, the shell side design pressure is 325 psig at 400°F, the tube side design pressure is 200 psig at 300°F, the shell mean metal temperature is 151°F, and the tube mean metal temperature is 113°F. For this example, the operating pressures and operating metal temperatures are assumed to be the same as the design values.
- The tube material is SA-249, Type 304L (S30403). The tubes are 1 in. outside diameter and are 0.049 in. thick.
- The tubesheet material is SA-240, Type 304L (S30403). The tubesheet outside diameter is 43.125 in. There are 955 tube holes on a 1.25 in. triangular pattern. There is no pass partition lane and the outermost tube radius from the tubesheet center is 20.125 in. The distance between the outer tubesheet faces is 240 in. The option for the effect of differential radial expansion is not required. There is no corrosion allowance on the tubesheet.

- The shell material is SA-240, Type 304L (S30403). The shell inside diameter is 42 in. and the thickness is 0.5625 in. There is no corrosion allowance on the shell and no expansion joint in the shell. The efficiency of shell circumferential welded joint (Category B) is 0.85.
- The channel material is SA-516, Grade 70 (K02700). The inside diameter of the channel is 42.125 in. and the channel is 0.375 in. thick. There is no corrosion allowance on the channel.

FIXED TUBESHEET RULES according to UHX-13 (July 2013 Edition)

Example E4.18.7 (PTB- 4 2013 edition) Fixed Tubesheet configuration "a"

1 - GEOMETRIC Data (from Fig.UHX-13.1)

Types of Operating Conditions

$x := 1$

x=1 NORMAL operating condition
x=2 STARTUP operating condition
x=3 SHUTDOWN operating condition
x=4 UPSET operating condition
x=5 CLEANING operating condition
x=6 OTHER operating condition

Configuration types: a, b, c, d

Config := "a"

"**a**" for shell/channel integral both sides
"**b**" for shell integral channel gask - TS extended
"**c**" for shell integral channel gask - TS not extended
"**d**" for gasketed both sides

YELLOW :most important data and results

Tubesheet Data (from Fig.UHX-13.1)

$h := 1.375 \cdot in$	**Tubesheet thickness**
$Layout := 0$	**For triangular pitch : "Layout"=0** **For square pitch : "Layout"=1**
$r_o := 20.125 \cdot in$	**Radius to outer tube**
$A := 43.125 \cdot in$	**Outside Diameter of Tubesheet**
$C := 0 \cdot in$	**Bolt Circle Diameter**
$C_p := 126.4 \cdot in$	**Perimeter of the tube layout**
$A_p := 1272.4 \cdot in^2$	**Total area enclosed by C_p**
$A_L := 0 \cdot in^2$	**Total Untubed Lanes Area**
$c_t := 0 \cdot in$	**Tubesheet Corr. Allow. (Tubeside)**
$c_s := 0 \cdot in$	**Tubesheet Corr. Allow. (Shellside)**
$h_g := 0 \cdot in$	**Groove depth**

SS := "NO" **SS="YES" for TS calculated as Simply Supported acc. to UHX-13.9**

Tube Data (from Fig.UHX-11.1)

$p := 1.25 \cdot in$	**Tube Pitch**
$N_t := 955$	**Number of Tubes**
$d_t := 1 \cdot in$	**Tube OutsideDiameter**
$t_t := 0.049 \cdot in$	**Tube Thickness**
$L_t := 240 \cdot in$	**Tube Length**
$L := L_t - 2 \cdot h$	$L = 237.250\, in$ Effective length of tubes
$\rho := 0.909$	**Tube expansion depth ratio**
$l_{tx} := 1.25 \cdot in$	**Length of Expanded Portion of Tube**
$kl := 48 \cdot in$	**k=0.6 for spans between Tubesheets** **k=0.8 for spans between TS/support plate** **k=1.0 for spans between support plates** **l unsupported tube span**

Shell Data (from Fig.UHX-13.1)

$D_s := 42 \cdot in$	**Shell ID**
$t_s := 0.5625 \cdot in$	**Shell Thickness away from TS**
$G_s := 0 \cdot in$	**Shell Gasket Diameter**
$t_{s1} := 0.5625 \cdot in$	**Shell Thickness near TS**
$C_s := 0 \cdot in$	**Shell Corrosion Allowance**
$E_{sw} := 0.85$	**Shell joint efficiency**
$l_1 := 0 \cdot in$	**Thick Shell Length @ one end**
$l'_1 := 0 \cdot in$	**Thick Shell Length @ other end**

Channel Data (from Fig.UHX-13.1)

$D_c := 42.125 \cdot in$	**Channel ID**
$t_c := 0.375 \cdot in$	**Channel Thickness**
$G_c := 0.1 \cdot in$	**Channel Gasket Diameter**
$G_1 := 0.1 \cdot in$	**Channel Contact mid-point TS/Flange**
$C_c := 0 \cdot in$	**Channel Corrosion Allowance**
CHAN := "CYL"	**"CYL" for Cylindrical Channel** **"HEMI" for Hemisherical Channel**

Corroded thicknesses:

$h := h - c_s - c_t$	**Tubesheet thickness**	$h = 1.375\, in$
$t_s := t_s - C_s$	**Shell Thickness away from TS**	$t_s = 0.563\, in$
$t_{s1} := t_{s1} - C_s$	**Shell Thickness near TS**	$t_{s1} = 0.563\, in$
$t_c := t_c - C_c$	**Channel Thickness**	$t_c = 0.375\, in$

Corroded length:

$L_t := L_t - 2c_t$ $L_t = 240.000\, in$

Corroded diameters:

$D_s := D_s + 2 \cdot C_s$	**Shell ID**	$D_s = 42.000\, in$
$D_c := D_c + 2 \cdot C_c$	**Channel ID**	$D_c = 42.125\, in$

Expansion Joint data $D_J := 0 \cdot in$ **Inside Diameter of Bellows** $K_J := 10^{20} \cdot \frac{lb}{in}$ **Bellows Axial Rigidity**

2 - Design (D) and Operating (O) PRESSURES data (from UHX-13.3)

Maximum and Minimum DESIGN PRESSURES (D)

$P_{sD\_max} := 325 \cdot \frac{lb}{in^2}$ maximum Shellside Design Pressure

$P_{sD\_min} := 0 \cdot \frac{lb}{in^2}$ minimum Shellside Design Pressure

$P_{tD\_max} := 200 \cdot \frac{lb}{in^2}$ maximumTubeside Design Pressure

$P_{tD\_min} := 0 \cdot \frac{lb}{in^2}$ minimum Tubeside Design Pressure

OPERATING PRESSURES (O) **for Operating Condition** x

$P_{sO\_x} := 325 \cdot \frac{lb}{in^2}$ Shellside Operating Pressure

$P_{tO\_x} := 200 \cdot \frac{lb}{in^2}$ Tubeside Operating Pressure

DESIGN PRESSURES P_{sD} and P_{tD} (from Table UHX-13.4-1)

$$P_{sD} := \begin{pmatrix} P_{sD\_min} \\ P_{sD\_max} \\ P_{sD\_max} \\ P_{sD\_min} \end{pmatrix} \qquad P_{sD} = \begin{pmatrix} 0.000 \\ 325.000 \\ 325.000 \\ 0.000 \end{pmatrix} \frac{lb}{in^2}$$

$$P_{tD} := \begin{pmatrix} P_{tD\_max} \\ P_{tD\_min} \\ P_{tD\_max} \\ P_{tD\_min} \end{pmatrix} \qquad P_{tD} = \begin{pmatrix} 200.000 \\ 0.000 \\ 200.000 \\ 0.000 \end{pmatrix} \frac{lb}{in^2}$$

OPERATING PRESSURES $P_{so,x}$ and $P_{to,x}$ for oper.cond. x (from Table UHX-13.4-2)

$$P_{sO} := \begin{pmatrix} 0 \cdot \frac{lb}{in^2} \\ P_{sO\_x} \\ P_{sO\_x} \\ 0 \cdot \frac{lb}{in^2} \end{pmatrix} \qquad P_{sO} = \begin{pmatrix} 0.000 \\ 325.000 \\ 325.000 \\ 0.000 \end{pmatrix} \frac{lb}{in^2}$$

$$P_{tO} := \begin{pmatrix} P_{tO\_x} \\ 0 \cdot \frac{lb}{in^2} \\ P_{tO\_x} \\ 0 \cdot \frac{lb}{in^2} \end{pmatrix} \qquad P_{tO} = \begin{pmatrix} 200.000 \\ 0.000 \\ 200.000 \\ 0.000 \end{pmatrix} \frac{lb}{in^2}$$

Determination of DESIGN and OPERATING PRESSURES P_s and P_t

2013 Design LC1= 2010 Design LC1
2013 Design LC2= 2010 Design LC2
2013 Design LC3= 2010 Design LC3
2013 Design LC4= not explicited in 2010

2013 Operating LC5= 2010 Operating LC5
2013 Operating LC6= 2010 Operating LC6
2013 Operating LC7= 2010 Operating LC7
2013 Operating LC8= 2010 Operating LC4

$$P_s := \begin{pmatrix} P_{sD\_min} \\ P_{sD\_max} \\ P_{sD\_max} \\ P_{sD\_min} \\ 0 \cdot \frac{lb}{in^2} \\ P_{sO\_x} \\ P_{sO\_x} \\ 0 \cdot \frac{lb}{in^2} \end{pmatrix} \qquad P_s = \begin{pmatrix} 0.0 \\ 325.0 \\ 325.0 \\ 0.0 \\ 0.0 \\ 325.0 \\ 325.0 \\ 0.0 \end{pmatrix} \frac{lb}{in^2}$$

$$P_t := \begin{pmatrix} P_{tD\_max} \\ P_{tD\_min} \\ P_{tD\_max} \\ P_{tD\_min} \\ P_{tO\_x} \\ 0 \cdot \frac{lb}{in^2} \\ P_{tO\_x} \\ 0 \cdot \frac{lb}{in^2} \end{pmatrix} \qquad P_t = \begin{pmatrix} 200.0 \\ 0.0 \\ 200.0 \\ 0.0 \\ 200.0 \\ 0.0 \\ 200.0 \\ 0.0 \end{pmatrix} \frac{lb}{in^2}$$

3 - TEMPERATURE Data for Design (D) and Operating (O) conditions

set Farenheit temp. degF := R

	Design conditions		Operating conditions	
Tubesheet	$T_D := 400 \cdot degF$	**Tubesheet Design Temp.**	$T_{O\_x} := 400 \cdot degF$	**TS Oper.Temp.** for Oper. Cond. x
Tubes	$T_{tD} := 300 \cdot degF$	**Tube Design Temp.**	$T_{tO\_x} := 300 \cdot degF$	**Tube Oper. Temp.** for Oper. Cond. x
Shell	$T_{sD} := 400 \cdot degF$	**Shell Design Temp.**	$T_{sO\_x} := 400 \cdot degF$	**Shell Oper.Temp.** for Oper. Cond. x
Channel	$T_{cD} := 300 \cdot degF$	**Channel Design Temp.**	$T_{cO\_x} := 300 \cdot degF$	**Channel Oper.Temp.** for Oper. Cond. x

$T_a := 70 \cdot degF$ **Ambient temperature**

$T_{tm\_x} := 113 \cdot degF$ **Mean Tube temp. along L** $T_{tm.} := T_{tm\_x}$ $T_{tm.} = 113.0\,degF$

$T_{sm\_x} := 151 \cdot degF$ **Mean Shell temp. along L** $T_{sm.} := T_{sm\_x}$ $T_{sm.} = 151.0\,degF$

Additional Temperature Data for Radial Thermal Expansion from UHX-13.8.4 (if required)

Tubesheet	$T'_x := 70 \cdot degF$	**TS temp.@ rim**	$T' := T'_x$	$T' = 70.0\,degF$
Shell	$T'_{sx} := 70 \cdot degF$	**Shell temp. @ Tubesheet**	$T'_s := T'_{sx}$	$T'_s = 70.0\,degF$
Channel	$T'_{cx} := 70 \cdot degF$	**Channel temp.@ tubesheet**	$T'_c := T'_{cx}$	$T'_c = 70.0\,degF$

4 - MATERIAL Data

TUBESHEET Material is SA-240/304L

$S_D := 15800 \cdot \frac{lb}{in^2}$ **TS allowable stress @ T_D**

$S_{PS} := 47400 \cdot \frac{lb}{in^2}$ **TS allowable P+S stress @ T_{O_x}**

$S_a := 20000 \cdot \frac{lb}{in^2}$ **TS allowable stress @ Ta**

$E_D := 26.4 \cdot 10^6 \cdot \frac{lb}{in^2}$ **TS elastic modulus @ T_D**

$E_O := 26.4 \cdot 10^6 \cdot \frac{lb}{in^2}$ **TS elastic modulus @ T_{O_x}**

$\nu := 0.3$ **TS Poisson's ratio**

$\alpha' := 8.984 \cdot 10^{-6} \cdot \frac{in}{in \cdot degF}$ **TS coeff. expansion@ rim**

TUBE Material is SA-249/304L

$S_{tD} := 16706 \cdot \frac{lb}{in^2}$ **Tube allowable stress @ T_{tD}**

$S_{tO} := 16706 \cdot \frac{lb}{in^2}$ **Tube allowable stress @ T_{tO_x}**

$E_{tD} := 27.0 \cdot 10^6 \cdot \frac{lb}{in^2}$ **Tube elastic modulus @T_{tD}**

$E_{tO} := 27.0 \cdot 10^6 \cdot \frac{lb}{in^2}$ **Tube elastic modulus@ T_{tO_x}**

$S_{ytD} := 19200 \cdot \frac{lb}{in^2}$ **Tube yield stress @T_{tD}**

$S_{ytO} := 19200 \cdot \frac{lb}{in^2}$ **Tube yield stress @ T_{tO_x}**

$S_{tT} := 15765 \cdot \frac{lb}{in^2}$ **Tube allowable stress @ T_D**

$E_{tT} := 26.4 \cdot 10^6 \cdot \frac{lb}{in^2}$ **Tube elastic modulus @ T_D**

$\nu_t := 0.3$ **Tube Poisson's ratio**

$\alpha_{tm} := 8.652 \cdot 10^{-6} \cdot \frac{in}{in \cdot degF}$ **Tube coeff. expansion@$T_{t,m}$**

SHELL Material is SA-240/304L

$S_{sD} := 15800 \cdot \frac{lb}{in^2}$ **Shell allow. stress @ T_{sD}**

$S_{PSs} := 47400 \cdot \frac{lb}{in^2}$ **Shell allowable P+S stress @ T_{sO_x}**

$E_{sD} := 26.4 \cdot 10^6 \cdot \frac{lb}{in^2}$ **Thin Shell elast mod. @ T_{sD}**

$E_{sO} := 26.4 \cdot 10^6 \cdot \frac{lb}{in^2}$ **Thin Shell elast mod. @ T_{sO_x}**

$S_{ysD} := 17500 \cdot \frac{lb}{in^2}$ **Shell yield stress @ T_{sD}**

$S_{ysO} := 17500 \cdot \frac{lb}{in^2}$ **Shell yield stress @ T_{sO_x}**

$\nu_s := 0.3$ **Shell Poisson' ratio**

$\alpha_{sm} := 8.802 \cdot 10^{-6} \cdot \frac{in}{in \cdot degF}$ **Shell coeff expansion @ $T_{s,m}$**

SHELL Band Material is SA-240/304L

$S_{s1} := 15800 \cdot \frac{lb}{in^2}$ **Shell allow. stress @ T_s**

$S_{ys1} := 17500 \cdot \frac{lb}{in^2}$ **Shell yield stress @ T_s**

$E_{s1} := 26.4 \cdot 10^6 \cdot \frac{lb}{in^2}$ **Thick Shell elast mod. @T_s.**

$\alpha_{sm1} := 8.802 \cdot 10^{-6} \cdot \frac{in}{in \cdot degF}$ **Thick Shell coeff. expansion@T_s**

$\nu_{s1} := 0.3$ **Thick Shell Poisson' ratio**

$\alpha'_s := 8.984 \cdot 10^{-6} \cdot \frac{in}{in \cdot degF}$ **Shell coeff. expansion @ rim**

CHANNEL Material is SA-516/grade70

$S_{cD} := 20000 \cdot \frac{lb}{in^2}$ **Channel allow. stress @ T_{cD}**

$S_{PSc} := 67200 \cdot \frac{lb}{in^2}$ **Channel allowable P+S stress @T_{cO_x}**

$E_{cD} := 28.3 \cdot 10^6 \cdot \frac{lb}{in^2}$ **Channel elast. modulus @ T_{cD}**

$E_{cO} := 28.3 \cdot 10^6 \cdot \frac{lb}{in^2}$ **Channel elast. modulus @ T_{cO_x}**

$S_{ycD} := 33600 \cdot \frac{lb}{in^2}$ **Channel yield stress @ T_{cD}**

$\nu_c := 0.3$ **Channel Poisson's ratio**

$\alpha'_c := 6.666 \cdot 10^{-6} \cdot \frac{in}{in \cdot degF}$ **Channel coeff. expansion @T_c**

5 - Flange Design and Operating BOLT LOADS data (from UHX-13.3)

Maximum and Minimum Flange DESIGN BOLT LOADS

$W_{m1s} := 0.0 \cdot lb$ **Shell flange Design bolt load**

$W_{m1c} := 0.0 \cdot lb$ **Channel flange Design bolt load**

$W_{m1max} := \max(W_{m1s}, W_{m1c})$ $W_{m1max} = 0.000\,lb$

Flange BOLT LOADS for GASKET SEATING Condition

$W_s := 0.0 \cdot lb$ **Shell flange bolt load for Gasket Seating**

$W_c := 0.0 \cdot lb$ **Channel flange bolt load for Gasket Seating**

$W_{max} := \max(W_s, W_c)$ $W_{max} = 0.000\,lb$

Determination of EFFECTIVE BOLT LOAD W* for each Configuration a , b , c , d

$$W^*_a := \begin{pmatrix} 0.0 \cdot lb \\ 0.0 \cdot lb \\ 0.0 \cdot lb \\ 0.0 \cdot lb \\ 0.0 \cdot lb \\ 0.0 \cdot lb \\ 0.0 \cdot lb \\ 0.0 \cdot lb \end{pmatrix} \quad W^*_b := \begin{pmatrix} W_{m1c} \\ 0.0 \cdot lb \\ W_{m1c} \\ 0.0 \cdot lb \\ W_c \\ W_c \\ W_c \\ W_c \end{pmatrix} \quad W^*_c := \begin{pmatrix} W_{m1c} \\ 0.0 \cdot lb \\ W_{m1c} \\ 0.0 \cdot lb \\ W_c \\ W_c \\ W_c \\ W_c \end{pmatrix} \quad W^*_d := \begin{pmatrix} W_{m1c} \\ W_{m1s} \\ W_{m1max} \\ 0.0 \cdot lb \\ W_{max} \\ W_{max} \\ W_{max} \\ W_{max} \end{pmatrix}$$

$$W^* := \begin{vmatrix} W^*_a & \text{if Config = "a"} \\ W^*_b & \text{if Config = "b"} \\ W^*_c & \text{if Config = "c"} \\ W^*_d & \text{if Config = "d"} \end{vmatrix} \qquad W^* = \begin{pmatrix} 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \end{pmatrix} lb$$

Minimum required thickness h_r of the TS flanged extension (from UHX-9)

For flanged Config. **b , d** (extended as a flange) from UHX-9.5a

For unflanged Config. **c** See UHX-9.5b

For unflanged Config. **d** See UHX-9.5c

$h_G := \frac{C - G_c}{2}$ **Gasket moment arm** $h_G = -0.050\,in$

$h_{rG} := \sqrt{\frac{1.9 W_c}{S_a \cdot G_c} \cdot h_G}$ $h_{rG} = 0.000\,in$

$h_{rO} := \sqrt{\frac{1.9 W_{m1c}}{S_D \cdot G_c} \cdot h_G}$ $h_{rO} = 0.000\,in$

$h_r := \max(h_{rG}, h_{rO})$ $h_r = 0.000\,in$

Start of Calculations

L = effective length of shell/tubes $L := L_t - 2\cdot h$ $L = 237.250\,in$ $l_{s1} := L - l_1 - l'_1$ $l_{s1} = 237.250\,in$

D_o = equivalent diameter of outer tubes $D_o := 2\cdot r_o + d_t$ $D_o = 41.250\,in$ $a_o := \frac{D_o}{2}$ $a_o = 20.625\,in$

UHX-13.5.1 Step 1 Determine D_0, μ, μ* and h'_g from UHX-11.5.1 :

$$\rho := \frac{l_{tx}}{h} \quad \rho = 0.909$$

$$d^* := \max\left[d_t - 2\cdot t_t\cdot\left(\frac{E_{tT}}{E_D}\right)\cdot\left(\frac{S_{tT}}{S_D}\right)\cdot\rho, \left(d_t - 2t_t\right)\right] \quad d^* = 0.9111\,in$$

$$p^* := \frac{p}{\sqrt{1 - \frac{4\cdot\min\left(A_L, 4D_o\cdot p\right)}{\pi\cdot D_o^2}}} \quad p^* = 1.250\,in$$

$$\mu := \frac{p - d_t}{p} \quad \mu = 0.200 \qquad \mu^* := \frac{p^* - d^*}{p^*} \quad \mu^* = 0.271$$

Shell Radial Dim.

$$a_s := \begin{cases} \frac{G_s}{2} & \text{if Config = "d"} \\ \frac{D_s}{2} & \text{otherwise} \end{cases} \quad a_s = 21.000\,in$$

Chan Rad. Dim.

$$a_c := \begin{cases} \frac{D_c}{2} & \text{if Config = "a"} \\ \frac{G_c}{2} & \text{otherwise} \end{cases} \quad a_c = 21.063\,in$$

$$\rho_s := \frac{a_s}{a_o} \quad \rho_s = 1.018 \qquad x_s := 1 - N_t\cdot\left(\frac{d_t}{2\cdot a_o}\right)^2 \quad x_s = 0.439$$

$$\rho_c := \frac{a_c}{a_o} \quad \rho_c = 1.021 \qquad x_t := 1 - N_t\cdot\left(\frac{d_t - 2\cdot t_t}{2\cdot a_o}\right)^2 \quad x_t = 0.543$$

UHX-13.5.2 Step 2 Calculate the shell axial stiffness K_s, tube axial stiffness K_t, and stiffness factors K_{st} and J

$$K^*_s := \frac{\pi \cdot (D_s + t_s)}{\frac{L - (l_1 + l'_1)}{E_{sD} \cdot t_s} + \frac{l_1 + l'_1}{E_{s1} \cdot t_{s1}}} \qquad K^*_s = 8.3695 \times 10^6 \frac{lb}{in}$$

$$K_t := \frac{\pi \cdot t_t \cdot (d_t - t_t) \cdot E_{tD}}{L} \qquad K_t = 1.666 \times 10^4 \frac{lb}{in}$$

$$K_{st} := \frac{K^*_s}{N_t \cdot K_t} \qquad K_{st} = 0.526$$

$$J := \frac{1}{1 + \frac{K^*_s}{K_J}} \qquad J = 1$$

Calculate shell and channel parameters:

SS = "NO" Use SS=YES for Simply Supported calculation in a 2nd step (see UHX-13.9)

$$\beta_s := \begin{cases} \frac{\left[12 \cdot (1 - \nu_s^2)\right]^{0.25}}{\left[(D_s + t_{s1}) \cdot t_{s1}\right]^{0.5}} & \text{if SS = "NO"} \wedge (\text{Config = "a"} \vee \text{Config = "b"} \vee \text{Config = "c"}) \\ 0 \cdot \frac{1}{in} & \text{otherwise} \end{cases} \qquad \beta_s = 0.372 \frac{1}{in}$$

$$\beta_c := \begin{cases} \frac{\left[12 \cdot (1 - \nu_c^2)\right]^{0.25}}{\left[(D_c + t_c) \cdot t_c\right]^{0.5}} & \text{if SS = "NO"} \wedge \text{Config = "a"} \\ 0 \cdot \frac{1}{in} & \text{otherwise} \end{cases} \qquad \beta_c = 0.455 \frac{1}{in}$$

$$k_s := \beta_s \cdot \frac{E_{s1} \cdot t_{s1}^3}{6 \cdot (1 - \nu_s^2)} \qquad k_s = 3.1971 \times 10^5 \, lb$$

$$k_c := \beta_c \cdot \frac{E_{cD} \cdot t_c^3}{6 \cdot (1 - \nu_c^2)} \qquad k_c = 1.245 \times 10^5 \, lb$$

$$h'_s := h \cdot \beta_s \qquad h'_s = 0.511$$

$$h'_c := h \cdot \beta_c \qquad h'_c = 0.626$$

$$\lambda_s := \frac{6 \cdot D_s}{h^3} \cdot k_s \cdot \left(1 + h'_s + \frac{h'^2_s}{2}\right) \qquad \lambda_s = 5.0868 \times 10^7 \frac{lb}{in^2}$$

$$\lambda_c := \frac{6 \cdot D_c}{h^3} \cdot k_c \cdot \left(1 + h'_c + \frac{h'^2_c}{2}\right) \qquad \lambda_c = 2.2049 \times 10^7 \frac{lb}{in^2}$$

$$\delta_s := \begin{cases} \frac{D_s^2}{4E_{s1} \cdot t_{s1}} \left(1 - \frac{\nu_s}{2}\right) & \text{if SS = "NO"} \wedge (\text{Config = "a"} \vee \text{Config = "b"} \vee \text{Config = "c"}) \\ 0 \cdot (in^3 \, lb^{-1}) & \text{otherwise} \end{cases} \qquad \delta_s = 2.524 \times 10^{-5} \, in^3 \, lb^{-1}$$

CHAN = "CYL"

SS = "NO"

$$\delta_c := \begin{cases} \frac{D_c^2}{4E_{cD} \cdot t_c} \left(1 - \frac{\nu_c}{2}\right) & \text{if SS = "NO"} \wedge \text{CHAN = "CYL"} \wedge \text{Config = "a"} \\ \frac{D_c^2}{4E_{cD} \cdot t_c} \left(\frac{1 - \nu_c}{2}\right) & \text{if SS = "NO"} \wedge \text{CHAN = "HEMI"} \wedge \text{Config = "a"} \\ 0 \cdot (in^3 \, lb^{-1}) & \text{otherwise} \end{cases} \qquad \delta_c = 3.553 \times 10^{-5} \, in^3 \, lb^{-1}$$

UHX-13.5.3 Step 3 Determine E*/E and ν* relative to h/p from UHX-11.5.2. Calculate X_a

$\frac{h}{p} = 1.100$ $\mu^* = 0.271$ $\frac{E^*}{E_D} = 0.275$ $E^* = 7259614.115\frac{lb}{in^2}$ $\nu^* = 0.3404$ (From right pages above)

$$X_a := \left[24\cdot\left(1-\nu^{*2}\right)\cdot N_t\cdot\frac{E_{tD}\cdot t_t\cdot\left(d_t - t_t\right)\cdot a_o^{\,2}}{E^*\cdot L\cdot h^3}\right]^{.25}$$ $X_a = 7.016$

UHX-13.5.4 Step 4 Calculate diameter ratio K and coefficient F :

$K := \frac{A}{D_o}$ $K = 1.045$

$F := \frac{1-\nu^*}{E^*}\cdot\left(\lambda_s + \lambda_c + E_D\cdot\ln(K)\right)$ $F = 6.732$

$\Phi := (1+\nu^*)\cdot F$ $\Phi = 9.024$

Calculate Z_a , Z_d , Z_v , Z_m

$N := \text{arrondi}\left(4 + \frac{X_a}{2}\right) + 1$ $N = 9.000$

$$ber_x(x) := \sum_{n=0}^{N}\left[(-1)^n\cdot\frac{\left(\frac{x}{2}\right)^{4\cdot n}}{((2\cdot n)!)^2}\right]$$ $ber := ber_x(X_a)$ $ber = -3.432$

$$bei_x(x) := \sum_{n=1}^{N}\left[(-1)^{n-1}\cdot\frac{\left(\frac{x}{2}\right)^{4\cdot n-2}}{((2\cdot n-1)!)^2}\right]$$ $bei := bei_x(X_a)$ $bei = -21.489$

$$ber'_x(x) := \sum_{n=1}^{N}\frac{(-1)^n\cdot(2\cdot n)\cdot\left(\frac{x}{2}\right)^{4\cdot n-1}}{((2\cdot n)!)^2}$$ $ber' := ber'_x(X_a)$ $ber' = 13.068$

$$bei'_x(x) := \sum_{n=1}^{N}\frac{(-1)^{n-1}\cdot(2\cdot n-1)\cdot\left(\frac{x}{2}\right)^{4\cdot n-3}}{((2\cdot n-1)!)^2}$$ $bei' := bei'_x(X_a)$ $bei' = -16.061$

$\Psi_{1x}(x) := bei_x(x) + \left(\frac{1-\nu^*}{x}\right)\cdot ber'_x(x)$ $\Psi_1 := \Psi_{1x}(X_a)$ $\Psi_1 = -20.260$

$\Psi_{2x}(x) := ber_x(x) - \frac{1-\nu^*}{x}\cdot bei'_x(x)$ $\Psi_2 := \Psi_{2x}(X_a)$ $\Psi_2 = -1.922$

$Z_a := bei'\cdot\Psi_2 - ber'\cdot\Psi_1$ $Z_a = 295.626$

$Z_m := \frac{ber'^2 + bei'^2}{X_a\cdot Z_a}$ $Z_m = 0.207$

$Z_v := \frac{ber'\cdot\Psi_2 + bei'\cdot\Psi_1}{X_a^{\,2}\cdot Z_a}$ $Z_v = 0.021$

$Z_d := \frac{ber\cdot\Psi_2 + bei\cdot\Psi_1}{X_a^{\,3}\cdot Z_a}$ $Z_d = 0.004$

$Z_w := \frac{ber'\cdot ber + bei'\cdot bei}{X_a^{\,2}\cdot Z_a}$ $Z_w = 0.021$

Calculate Q_1, Q_{z1}, Q_{z2}, and U:

$$Q_1 := \frac{\rho_s - 1 - \Phi \cdot Z_v}{1 + \Phi \cdot Z_m} \qquad Q_1 = -0.059$$

$$Q_{z1} := \frac{(Z_d + Q_1 \cdot Z_w) \cdot X_a^4}{2} \qquad Q_{z1} = 3.778$$

$$Q_{z2} := \frac{(Z_v + Q_1 \cdot Z_m) \cdot X_a^4}{2} \qquad Q_{z2} = 10.312$$

$$U := \frac{[Z_w + (\rho_s - 1) \cdot Z_m] \cdot X_a^4}{1 + \Phi \cdot Z_m} \qquad U = 20.625$$

UHX-13.5.5 **Step 5** **UHX-13.5.5.(a) Calculate γ :**

$$\gamma^*_p := (T_{tm.} - T_a) \cdot \alpha_{tm} \cdot L - (T_{sm.} - T_a) \cdot [\alpha_{sm} \cdot (L - l_1 - l'_1) + \alpha_{sm1} \cdot (l_1 + l'_1)]$$

$$\gamma^* := \begin{pmatrix} 0 \cdot in \\ 0 \cdot in \\ 0 \cdot in \\ 0 \cdot in \\ \gamma^*_p \\ \gamma^*_p \\ \gamma^*_p \\ \gamma^*_p \end{pmatrix} \qquad \gamma^* = \begin{pmatrix} 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ -0.0809 \\ -0.0809 \\ -0.0809 \\ -0.0809 \end{pmatrix} in$$

UHX-13.5.5(b) Calculate parameters ω_s , ω^*_s , ω_c , ω^*_c, and :

$$\omega_s := \rho_s \cdot k_s \cdot \beta_s \cdot \delta_s \cdot (1 + h'_s) \qquad \omega_c := \rho_c \cdot k_c \cdot \beta_c \cdot \delta_c \cdot (1 + h'_c)$$

$$\omega_s = 4.612\, in^2 \qquad \omega_c = 3.344\, in^2$$

$$\omega^*_s := a_o^2 \cdot \frac{(\rho_s^2 - 1) \cdot (\rho_s - 1)}{4} - \omega_s \qquad \omega^*_c := a_o^2 \cdot \left[\frac{(\rho_c^2 + 1) \cdot (\rho_c - 1)}{4} - \frac{\rho_s - 1}{2}\right] - \omega_c$$

$$\omega^*_s = -4.541\, in^2 \qquad \omega^*_c = -2.603\, in^2$$

UHX-13.5.5(c) Calculate γ_b:

$$\gamma_b := \begin{cases} 0 & \text{if Config = "a"} \\ \dfrac{G_c - C}{D_o} & \text{if Config = "b"} \\ \dfrac{G_c - G_1}{D_o} & \text{if Config = "c"} \\ \dfrac{G_c - G_s}{D_o} & \text{if Config = "d"} \end{cases} \qquad \gamma_b = 0.00000$$

Calculate P^*_s and P^*_c from UHX-13.8 , if required by the user (for **configurations a,b,or c** only)

$$T_r := \begin{cases} \dfrac{T' + T'_s + T'_c}{3} & \text{if } Config = \text{"a"} \\ \dfrac{T' + T'_s}{2} & \text{if } Config = \text{"b"} \vee Config = \text{"c"} \\ T_a & \text{if } Config = \text{"d"} \end{cases} \qquad T_r = 70.0\,degF$$

$$T^*_s := \frac{T'_s + T_r}{2} \quad T^*_s = 70.0\,degF \qquad T^*_c := \frac{T'_c + T_r}{2} \quad T^*_c = 70.0\,degF$$

$$P^*_{sp1} := \frac{E_{s1} \cdot t_{s1}}{a_s} \cdot \left[\alpha'_s \cdot (T^*_s - T_a) - \alpha' \cdot (T_r - T_a) \right]$$

$$P^*_{sp1} = 0.000 \frac{lb}{in^2}$$

$$P^*_{cp1} := \frac{E_{cD} \cdot t_c}{a_c} \cdot \left[\alpha'_c \cdot (T^*_c - T_a) - \alpha' \cdot (T_r - T_a) \right]$$

$$P^*_{cp1} = 0.000 \frac{lb}{in^2}$$

$$P^*_{sp} := \begin{cases} 0 \cdot \dfrac{lb}{in^2} & \text{if } Config = \text{"d"} \\ P^*_{sp1} & \text{otherwise} \end{cases}$$

$$P^*_{cp} := \begin{cases} 0 \cdot \dfrac{lb}{in^2} & \text{if } Config = \text{"d"} \vee Config = \text{"b"} \vee Config = \text{"c"} \\ P^*_{cp1} & \text{otherwise} \end{cases}$$

$$P^*_s := \begin{pmatrix} 0 \cdot \frac{lb}{in^2} \\ 0 \cdot \frac{lb}{in^2} \\ 0 \cdot \frac{lb}{in^2} \\ 0 \cdot \frac{lb}{in^2} \\ P^*_{sp} \\ P^*_{sp} \\ P^*_{sp} \\ P^*_{sp} \end{pmatrix} \quad P^*_s = \begin{pmatrix} 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \end{pmatrix} \frac{lb}{in^2}$$

$$P^*_c := \begin{pmatrix} 0 \cdot \frac{lb}{in^2} \\ 0 \cdot \frac{lb}{in^2} \\ 0 \cdot \frac{lb}{in^2} \\ 0 \cdot \frac{lb}{in^2} \\ P^*_{cp} \\ P^*_{cp} \\ P^*_{cp} \\ P^*_{cp} \end{pmatrix} \quad P^*_c = \begin{pmatrix} 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \end{pmatrix} \frac{lb}{in^2}$$

UHX-13.5.6 Step 6 For each loading case, calculate P'_s and P'_t, P_γ, P_γ^*, P_w, P_{rim}, and effective pressure P_e

$$P'_s := \left[x_s + 2\cdot(1-x_s)\cdot\nu_t + \frac{2\cdot\nu_s}{K_{st}}\cdot\left(\frac{D_s}{D_o}\right)^2 - \frac{\rho_s^{\,2}-1}{J\cdot K_{st}} - \frac{1-J}{2J\cdot K_{st}}\cdot\frac{D_J^{\,2}-D_s^{\,2}}{D_o^{\,2}}\right]\cdot P_s \qquad P'_t := \left[x_t + 2\cdot(1-x_t)\cdot\nu_t + \frac{1}{J\cdot K_{st}}\right]\cdot P_t$$

$$P'_s = \begin{pmatrix} 0.000 \\ 613.671 \\ 613.671 \\ 0.000 \\ 0.000 \\ 613.671 \\ 613.671 \\ 0.000 \end{pmatrix}\frac{lb}{in^2} \qquad P'_t = \begin{pmatrix} 543.676 \\ 0.000 \\ 543.676 \\ 0.000 \\ 543.676 \\ 0.000 \\ 543.676 \\ 0.000 \end{pmatrix}\frac{lb}{in^2}$$

$$P_\gamma := \frac{N_t\cdot K_t}{\pi\cdot a_o^{\,2}}\cdot\gamma^* \qquad P_w := -\frac{U}{a_o^{\,2}}\cdot\frac{\gamma_b}{2\pi}\cdot W^* \qquad P_\omega := \frac{U}{a_o^{\,2}}\cdot\left(\omega_s\cdot P^*_s - \omega_c\cdot P^*_c\right) \qquad P_{rim} := -\frac{U}{a_o^{\,2}}\cdot\left(\omega^*_s\cdot P_s - \omega^*_c\cdot P_t\right)$$

$$P_\gamma = \begin{pmatrix} 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ -962.977 \\ -962.977 \\ -962.977 \\ -962.977 \end{pmatrix}\frac{lb}{in^2} \qquad P_w = \begin{pmatrix} 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \end{pmatrix}\frac{lb}{in^2} \qquad P_\omega = \begin{pmatrix} 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \end{pmatrix}\frac{lb}{in^2} \qquad P_{rim} = \begin{pmatrix} -25.238 \\ 71.560 \\ 46.321 \\ 0.000 \\ -25.238 \\ 71.560 \\ 46.321 \\ 0.000 \end{pmatrix}\frac{lb}{in^2}$$

Calculate P_e for Pressure and Operating Loading Cases 1 through 8:

$$P_e := \frac{J\cdot K_{st}}{1 + J\cdot K_{st}\cdot\left[Q_{z1} + (\rho_s - 1)\cdot Q_{z2}\right]}\cdot\left(P'_s - P'_t + P_\gamma + P_\omega + P_w + P_{rim}\right)$$

PRESSURE DESIGN Loading cases : terms 1,2 3, 4

PRESSURE OPERATING Loading cases : terms 5,6,7, 8

$$P_e = \begin{pmatrix} -96.973 \\ 116.800 \\ 19.826 \\ 0.000 \\ -261.115 \\ -47.343 \\ -144.316 \\ -164.142 \end{pmatrix}\frac{lb}{in^2}$$

UHX-13.5.7 **Step 7** **Determine Q2 and Q3 for Loading Cases 1 through 7:**

$$Q_2 := \overrightarrow{\frac{\left[\left(\omega^*_s \cdot P_s - \omega^*_c \cdot P_t\right) - \left(\omega_s \cdot P^*_s - \omega_c \cdot P^*_c\right)\right] + \frac{\gamma_b}{2 \cdot \pi} \cdot W^*}{1 + \Phi \cdot Z_m}}$$

$$Q_2 = \begin{pmatrix} 181.674 \\ -515.107 \\ -333.433 \\ 0.000 \\ 181.674 \\ -515.107 \\ -333.433 \\ 0.000 \end{pmatrix} \text{lb}$$

$$Q3_1 := \begin{cases} 0 & \text{if } P_{e_1} = 0 \\ \left(Q_1 + \frac{2 \cdot Q2_1}{P_{e_1} \cdot a_o^2}\right) & \text{otherwise} \end{cases} \qquad Q3_2 := \begin{cases} 0 & \text{if } P_{e_2} = 0 \\ \left(Q_1 + \frac{2 \cdot Q2_2}{P_{e_2} \cdot a_o^2}\right) & \text{otherwise} \end{cases} \qquad Q3_3 := \begin{cases} 0 & \text{if } P_{e_3} = 0 \\ \left(Q_1 + \frac{2 \cdot Q2_3}{P_{e_3} \cdot a_o^2}\right) & \text{otherwise} \end{cases}$$

$$Q3_4 := \begin{cases} 0 & \text{if } P_{e_4} = 0 \\ \left(Q_1 + \frac{2 \cdot Q2_4}{P_{e_4} \cdot a_o^2}\right) & \text{otherwise} \end{cases} \qquad Q3_5 := \begin{cases} 0 & \text{if } P_{e_5} = 0 \\ \left(Q_1 + \frac{2 \cdot Q2_5}{P_{e_5} \cdot a_o^2}\right) & \text{otherwise} \end{cases} \qquad Q3_6 := \begin{cases} 0 & \text{if } P_{e_6} = 0 \\ \left(Q_1 + \frac{2 \cdot Q2_6}{P_{e_6} \cdot a_o^2}\right) & \text{otherwise} \end{cases}$$

$$Q3_7 := \begin{cases} 0 & \text{if } P_{e_7} = 0 \\ \left(Q_1 + \frac{2 \cdot Q2_7}{P_{e_7} \cdot a_o^2}\right) & \text{otherwise} \end{cases} \qquad Q3_8 := \begin{cases} 0 & \text{if } P_{e_8} = 0 \\ \left(Q_1 + \frac{2 \cdot Q2_8}{P_{e_8} \cdot a_o^2}\right) & \text{otherwise} \end{cases}$$

$$Q_1 = -0.059 \qquad Q_2 = \begin{pmatrix} 181.674 \\ -515.107 \\ -333.433 \\ 0.000 \\ 181.674 \\ -515.107 \\ -333.433 \\ 0.000 \end{pmatrix} \text{lb} \qquad Q_3 = \begin{pmatrix} -0.06746 \\ -0.07938 \\ -0.13772 \\ 0.00000 \\ -0.06192 \\ -0.00749 \\ -0.04778 \\ -0.05865 \end{pmatrix}$$

Determine Coefficient F_m for Load Cases 1 through 7:

numpoints := 20 $j := 1 .. \text{numpoints}$ $X_j := \frac{j \cdot 1}{\text{numpoints}}$ $x := X \cdot X_a$

$$Q_v(x) := \overrightarrow{\frac{\Psi_{2x}(x) \cdot \Psi_1 - \Psi_{1x}(x) \cdot \Psi_2}{X_a \cdot Z_a}} \qquad Q_m(x) := \overrightarrow{\left(\frac{\Psi_{2x}(x) \cdot bei' - \Psi_{1x}(x) \cdot ber'}{Z_a}\right)}$$

$$F_{mx1}(x) := \begin{cases} 0 & \text{if } P_{e_1} = 0 \\ \frac{Q_v(x) + Q3_1 \cdot Q_m(x)}{2} & \text{otherwise} \end{cases} \qquad F_{mx4}(x) := \begin{cases} 0 & \text{if } P_{e_4} = 0 \\ \frac{Q_v(x) + Q3_4 \cdot Q_m(x)}{2} & \text{otherwise} \end{cases}$$

$$F_{mx2}(x) := \begin{cases} 0 & \text{if } P_{e_2} = 0 \\ \frac{Q_v(x) + Q3_2 \cdot Q_m(x)}{2} & \text{otherwise} \end{cases} \qquad F_{mx5}(x) := \begin{cases} 0 & \text{if } P_{e_5} = 0 \\ \frac{Q_v(x) + Q3_5 \cdot Q_m(x)}{2} & \text{otherwise} \end{cases}$$

$$F_{mx3}(x) := \begin{cases} 0 & \text{if } P_{e_3} = 0 \\ \frac{Q_v(x) + Q3_3 \cdot Q_m(x)}{2} & \text{otherwise} \end{cases} \qquad F_{mx6}(x) := \begin{cases} 0 & \text{if } P_{e_6} = 0 \\ \frac{Q_v(x) + Q3_6 \cdot Q_m(x)}{2} & \text{otherwise} \end{cases}$$

$$F_{mx7}(x) := \begin{cases} 0 & \text{if } P_{e_7} = 0 \\ \frac{Q_v(x) + Q3_7 \cdot Q_m(x)}{2} & \text{otherwise} \end{cases} \qquad F_{mx8}(x) := \begin{cases} 0 & \text{if } P_{e_8} = 0 \\ \frac{Q_v(x) + Q3_8 \cdot Q_m(x)}{2} & \text{otherwise} \end{cases}$$

$$F_{mx}(x) := \begin{pmatrix} F_{mx1}(x) \\ F_{mx2}(x) \\ F_{mx3}(x) \\ F_{mx4}(x) \\ F_{mx5}(x) \\ F_{mx6}(x) \\ F_{mx7}(x) \\ F_{mx8}(x) \end{pmatrix} \quad absF_{mx}(x) := \begin{pmatrix} \overrightarrow{|F_{mx1}(x)|} \\ \overrightarrow{|F_{mx2}(x)|} \\ \overrightarrow{|F_{mx3}(x)|} \\ \overrightarrow{|F_{mx4}(x)|} \\ \overrightarrow{|F_{mx5}(x)|} \\ \overrightarrow{|F_{mx6}(x)|} \\ \overrightarrow{|F_{mx7}(x)|} \\ \overrightarrow{|F_{mx8}(x)|} \end{pmatrix} \quad F_m := \begin{pmatrix} \max(absF_{mx}(x)_1) \\ \max(absF_{mx}(x)_2) \\ \max(absF_{mx}(x)_3) \\ \max(absF_{mx}(x)_4) \\ \max(absF_{mx}(x)_5) \\ \max(absF_{mx}(x)_6) \\ \max(absF_{mx}(x)_7) \\ \max(absF_{mx}(x)_8) \end{pmatrix} \quad F_m = \begin{pmatrix} 0.0337 \\ 0.0397 \\ 0.0689 \\ 0.0000 \\ 0.0310 \\ 0.0318 \\ 0.0239 \\ 0.0293 \end{pmatrix} \quad F_{mx}(X_a) = \begin{pmatrix} -0.034 \\ -0.040 \\ -0.069 \\ 0.000 \\ -0.031 \\ -0.004 \\ -0.024 \\ -0.029 \end{pmatrix}$$

Calculate the Maximum Tubesheet Bending Stress

Effective Groove depth $h'_g := \max(h_g - c_t, 0)$ $\quad h'_g = 0.000\,in$

$$h_{min} := \begin{pmatrix} h - h'_g \\ h - h'_g \\ h - h'_g \\ h - h'_g \\ h \\ h \\ h \\ h \end{pmatrix} \qquad h_{min} = \begin{pmatrix} 1.3750 \\ 1.3750 \\ 1.3750 \\ 1.3750 \\ 1.3750 \\ 1.3750 \\ 1.3750 \\ 1.3750 \end{pmatrix} in$$

$$\sigma_1 := \begin{cases} \overrightarrow{\left[\dfrac{6 \cdot Q_{2_1}}{\mu^* \cdot (h_{min_1})^2}\right]} & \text{if } P_{e_1} = 0 \\ \overrightarrow{\left[\dfrac{1.5 \cdot F_{m_1}}{\mu^*} \cdot \left(\dfrac{2 \cdot a_o}{h_{min_1}}\right)^2 \cdot P_{e_1}\right]} & \text{otherwise} \end{cases}$$

$$\sigma_2 := \begin{cases} \overrightarrow{\left[\dfrac{6 \cdot Q_{2_2}}{\mu^* \cdot (h_{min_2})^2}\right]} & \text{if } P_{e_2} = 0 \\ \overrightarrow{\left[\dfrac{1.5 \cdot F_{m_2}}{\mu^*} \cdot \left(\dfrac{2 \cdot a_o}{h_{min_2}}\right)^2 \cdot P_{e_2}\right]} & \text{otherwise} \end{cases}$$

$$\sigma_1 = -16286.147 \frac{lb}{in^2} \qquad \sigma_2 = 23084.127 \frac{lb}{in^2}$$

$$\sigma_3 := \begin{cases} \overrightarrow{\left[\dfrac{6 \cdot Q_{2_3}}{\mu^* \cdot (h_{min_3})^2}\right]} & \text{if } P_{e_3} = 0 \\ \overrightarrow{\left[\dfrac{1.5 \cdot F_{m_3}}{\mu^*} \cdot \left(\dfrac{2 \cdot a_o}{h_{min_3}}\right)^2 \cdot P_{e_3}\right]} & \text{otherwise} \end{cases}$$

$$\sigma_4 := \begin{cases} \overrightarrow{\left[\dfrac{6 \cdot Q_{2_4}}{\mu^* \cdot (h_{min_4})^2}\right]} & \text{if } P_{e_4} = 0 \\ \overrightarrow{\left[\dfrac{1.5 \cdot F_{m_4}}{\mu^*} \cdot \left(\dfrac{2 \cdot a_o}{h_{min_4}}\right)^2 \cdot P_{e_4}\right]} & \text{otherwise} \end{cases}$$

$$\sigma_3 = 6797.980 \frac{lb}{in^2} \qquad \sigma_4 = 0.000 \frac{lb}{in^2}$$

$$\sigma_5 := \begin{cases} \overrightarrow{\left[\dfrac{6 \cdot Q_{2_5}}{\mu^* \cdot (h_{min_5})^2}\right]} & \text{if } P_{e_5} = 0 \\ \overrightarrow{\left[\dfrac{1.5 \cdot F_{m_5}}{\mu^*} \cdot \left(\dfrac{2 \cdot a_o}{h_{min_5}}\right)^2 \cdot P_{e_5}\right]} & \text{otherwise} \end{cases}$$

$$\sigma_6 := \begin{cases} \overrightarrow{\left[\dfrac{6 \cdot Q_{2_6}}{\mu^* \cdot (h_{min_6})^2}\right]} & \text{if } P_{e_6} = 0 \\ \overrightarrow{\left[\dfrac{1.5 \cdot F_{m_6}}{\mu^*} \cdot \left(\dfrac{2 \cdot a_o}{h_{min_6}}\right)^2 \cdot P_{e_6}\right]} & \text{otherwise} \end{cases}$$

$$\sigma_5 = -40253.396 \frac{lb}{in^2} \qquad \sigma_6 = -7493.724 \frac{lb}{in^2}$$

$$\sigma_7 := \begin{cases} \overrightarrow{\left[\dfrac{6 \cdot Q_{2_7}}{\mu^* \cdot (h_{min_7})^2}\right]} & \text{if } P_{e_7} = 0 \\ \overrightarrow{\left[\dfrac{1.5 \cdot F_{m_7}}{\mu^*} \cdot \left(\dfrac{2 \cdot a_o}{h_{min_7}}\right)^2 \cdot P_{e_7}\right]} & \text{otherwise} \end{cases}$$

$$\sigma_8 := \begin{cases} \overrightarrow{\left[\dfrac{6 \cdot Q_{2_8}}{\mu^* \cdot (h_{min_8})^2}\right]} & \text{if } P_{e_8} = 0 \\ \overrightarrow{\left[\dfrac{1.5 \cdot F_{m_8}}{\mu^*} \cdot \left(\dfrac{2 \cdot a_o}{h_{min_8}}\right)^2 \cdot P_{e_8}\right]} & \text{otherwise} \end{cases}$$

$$\sigma_7 = -17169.269 \frac{lb}{in^2} \qquad \sigma_8 = -23967.249 \frac{lb}{in^2}$$

$$\sigma = \begin{pmatrix} -16286.147 \\ 23084.127 \\ 6797.98 \\ 0 \\ -40253.396 \\ -7493.724 \\ -17169.269 \\ -23967.249 \end{pmatrix} \frac{lb}{in^2}$$

TUBESHEET MAXIMUM STRESS for DESIGN LOADING CASES 1 , 2, 3 , 4

$$\sigma_D := \max\left(|\sigma_1|, |\sigma_2|, |\sigma_3|, |\sigma_4|\right) \qquad \sigma_D = 23084.1 \frac{lb}{in^2}$$

$$\sigma_{allowD} := 1.5 \cdot S_D \qquad \sigma_{allowD} = 23700.0 \frac{lb}{in^2}$$

TUBESHEET MAXIMUM STRESS For OPERATING LOADING CASES 5 , 6 , 7 , 8

$$\sigma_O := \max\left(|\sigma_5|, |\sigma_6|, |\sigma_7|, |\sigma_8|\right) \qquad \sigma_O = 40253.4 \frac{lb}{in^2}$$

$$\sigma_{allowO} := S_{PS} \qquad \sigma_{allowO} = 47400.0 \frac{lb}{in^2}$$

UHX-13.5.8 **Step 8** **Calculate the maximum tubesheet shear stress**

If $|Pe|<1.6S\mu h/a_o$, the TEMA formula is not required to be calculated

$$1.6 \cdot S_D \cdot \mu \cdot \frac{h}{a_o} = 337.067 \frac{lb}{in^2}$$

$$\tau := \left(\frac{1}{\mu}\right) \cdot \frac{1}{h} \left(\frac{A_p}{C_p}\right) \cdot \overrightarrow{|P_e|}$$

$$absP_e := \begin{pmatrix} |P_{e_1}| \\ |P_{e_2}| \\ |P_{e_3}| \\ |P_{e_4}| \\ |P_{e_5}| \\ |P_{e_6}| \\ |P_{e_7}| \\ |P_{e_8}| \end{pmatrix} \qquad absP_e = \begin{pmatrix} 96.973 \\ 116.800 \\ 19.826 \\ 0.000 \\ 261.115 \\ 47.343 \\ 144.316 \\ 164.142 \end{pmatrix} \frac{lb}{in^2}$$

$$\tau = \begin{pmatrix} 3549.7 \\ 4275.5 \\ 725.8 \\ 0.0 \\ 9558.2 \\ 1733.0 \\ 5282.7 \\ 6008.5 \end{pmatrix} \frac{lb}{in^2}$$

$$\tau_{max} := \max\left(\overrightarrow{|\tau|}\right) \qquad \tau_{max} = 9558.2 \frac{lb}{in^2}$$

$$\tau_{allow} := 0.8 \cdot S_D \qquad \tau_{allow} = 12640.0 \frac{lb}{in^2}$$

UHX-13.5.9 **Step 9** **Determine the minimum and maximum stresses in the tubes for Load Cases 1 to 8**

$$Z_{dx}(x) := \frac{\overrightarrow{\Psi_2 \cdot ber_x(x) + \Psi_1 \cdot bei_x(x)}}{X_a^{3} \cdot Z_a} \qquad Z_{wx}(x) := \frac{\overrightarrow{ber' \cdot ber_x(x) + bei' \cdot bei_x(x)}}{X_a^{2} \cdot Z_a}$$

a) :Determine Coefficients $F_{t,min}$ and $F_{t,max}$ for Load Cases 1 to 8:

$$F_{tx1}(x) := \begin{cases} Z_{wx}(x)\cdot\frac{X_a^{4}}{2} & \text{if } P_{e_1} = 0 \\ \left(Z_{dx}(x) + Q3_1\cdot Z_{wx}(x)\right)\cdot\frac{X_a^{4}}{2} & \text{otherwise}\end{cases}$$

$$F_{tx4}(x) := \begin{cases} Z_{wx}(x)\cdot\frac{X_a^{4}}{2} & \text{if } P_{e_4} = 0 \\ \left(Z_{dx}(x) + Q3_4\cdot Z_{wx}(x)\right)\cdot\frac{X_a^{4}}{2} & \text{otherwise}\end{cases}$$

$$F_{tx2}(x) := \begin{cases} Z_{wx}(x)\cdot\frac{X_a^{4}}{2} & \text{if } P_{e_2} = 0 \\ \left(Z_{dx}(x) + Q3_2\cdot Z_{wx}(x)\right)\cdot\frac{X_a^{4}}{2} & \text{otherwise}\end{cases}$$

$$F_{tx5}(x) := \begin{cases} Z_{wx}(x)\cdot\frac{X_a^{4}}{2} & \text{if } P_{e_5} = 0 \\ \left(Z_{dx}(x) + Q3_5\cdot Z_{wx}(x)\right)\cdot\frac{X_a^{4}}{2} & \text{otherwise}\end{cases}$$

$$F_{tx3}(x) := \begin{cases} Z_{wx}(x)\cdot\frac{X_a^{4}}{2} & \text{if } P_{e_3} = 0 \\ \left(Z_{dx}(x) + Q3_3\cdot Z_{wx}(x)\right)\cdot\frac{X_a^{4}}{2} & \text{otherwise}\end{cases}$$

$$F_{tx6}(x) := \begin{cases} Z_{wx}(x)\cdot\frac{X_a^{4}}{2} & \text{if } P_{e_6} = 0 \\ \left(Z_{dx}(x) + Q3_6\cdot Z_{wx}(x)\right)\cdot\frac{X_a^{4}}{2} & \text{otherwise}\end{cases}$$

$$F_{tx7}(x) := \begin{cases} Z_{wx}(x)\cdot\frac{X_a^{4}}{2} & \text{if } P_{e_7} = 0 \\ \left(Z_{dx}(x) + Q3_7\cdot Z_{wx}(x)\right)\cdot\frac{X_a^{4}}{2} & \text{otherwise}\end{cases}$$

$$F_{tx8}(x) := \begin{cases} Z_{wx}(x)\cdot\frac{X_a^{4}}{2} & \text{if } P_{e_8} = 0 \\ \left(Z_{dx}(x) + Q3_8\cdot Z_{wx}(x)\right)\cdot\frac{X_a^{4}}{2} & \text{otherwise}\end{cases}$$

$$F_{tx}(x) := \begin{pmatrix} F_{tx1}(x) \\ F_{tx2}(x) \\ F_{tx3}(x) \\ F_{tx4}(x) \\ F_{tx5}(x) \\ F_{tx6}(x) \\ F_{tx7}(x) \\ F_{tx8}(x) \end{pmatrix} \quad F_{tmin} := \begin{pmatrix} \min(F_{tx1}(x)) \\ \min(F_{tx2}(x)) \\ \min(F_{tx3}(x)) \\ \min(F_{tx4}(x)) \\ \min(F_{tx5}(x)) \\ \min(F_{tx6}(x)) \\ \min(F_{tx7}(x)) \\ \min(F_{tx8}(x)) \end{pmatrix} \quad F_{tmax} := \begin{pmatrix} \max(F_{tx1}(x)) \\ \max(F_{tx2}(x)) \\ \max(F_{tx3}(x)) \\ \max(F_{tx4}(x)) \\ \max(F_{tx5}(x)) \\ \max(F_{tx6}(x)) \\ \max(F_{tx7}(x)) \\ \max(F_{tx8}(x)) \end{pmatrix}$$

$$F_{tx}(X_a) = \begin{pmatrix} 3.558 \\ 3.260 \\ 1.802 \\ 24.996 \\ 3.696 \\ 5.057 \\ 4.050 \\ 3.778 \end{pmatrix} \quad F_{tmin} = \begin{pmatrix} -0.270 \\ -0.242 \\ -0.190 \\ -7.048 \\ -0.282 \\ -0.489 \\ -0.329 \\ -0.293 \end{pmatrix} \quad F_{tmax} = \begin{pmatrix} 3.558 \\ 3.260 \\ 2.097 \\ 24.996 \\ 3.696 \\ 5.057 \\ 4.050 \\ 3.778 \end{pmatrix}$$

b) Determine tube stresses σ_{tmin} and σ_{tmax} for Load Cases 1 to 8 (continued)

$$\sigma_{t1_1} := \left| \begin{array}{ll} \overrightarrow{\left[\frac{1}{x_t - x_s}\cdot\left[\left(P_{s_1}\cdot x_s - P_{t_1}\cdot x_t\right) - \frac{2\cdot Q_{2_1}}{a_o^{2}}\cdot F_{tmin_1}\right]\right]} & \text{if } P_{e_1} = 0 \\ \overrightarrow{\left[\frac{1}{x_t - x_s}\cdot\left[\left(P_{s_1}\cdot x_s - P_{t_1}\cdot x_t\right) - P_{e_1}\cdot F_{tmin_1}\right]\right]} & \text{otherwise} \end{array}\right.$$

$$\sigma_{t1_1} = -1288.776\frac{lb}{in^2}$$

$$\sigma_{t1_2} := \left| \begin{array}{ll} \overrightarrow{\left[\frac{1}{x_t - x_s}\cdot\left[\left(P_{s_2}\cdot x_s - P_{t_2}\cdot x_t\right) - \frac{2\cdot Q_{2_2}}{a_o^{2}}\cdot F_{tmin_2}\right]\right]} & \text{if } P_{e_2} = 0 \\ \overrightarrow{\left[\frac{1}{x_t - x_s}\cdot\left[\left(P_{s_2}\cdot x_s - P_{t_2}\cdot x_t\right) - P_{e_2}\cdot F_{tmin_2}\right]\right]} & \text{otherwise} \end{array}\right.$$

$$\sigma_{t1_2} = 1633.647\frac{lb}{in^2}$$

$$\sigma_{t1_3} := \left| \begin{array}{ll} \overrightarrow{\left[\frac{1}{x_t - x_s}\cdot\left[\left(P_{s_3}\cdot x_s - P_{t_3}\cdot x_t\right) - \frac{2\cdot Q_{2_3}}{a_o^{2}}\cdot F_{tmin_3}\right]\right]} & \text{if } P_{e_3} = 0 \\ \overrightarrow{\left[\frac{1}{x_t - x_s}\cdot\left[\left(P_{s_3}\cdot x_s - P_{t_3}\cdot x_t\right) - P_{e_3}\cdot F_{tmin_3}\right]\right]} & \text{otherwise} \end{array}\right.$$

$$\sigma_{t1_3} = 360.212\frac{lb}{in^2}$$

$$\sigma_{t1_4} := \left| \begin{array}{ll} \overrightarrow{\left[\frac{1}{x_t - x_s}\cdot\left[\left(P_{s_4}\cdot x_s - P_{t_4}\cdot x_t\right) - \frac{2\cdot Q_{2_4}}{a_o^{2}}\cdot F_{tmin_4}\right]\right]} & \text{if } P_{e_4} = 0 \\ \overrightarrow{\left[\frac{1}{x_t - x_s}\cdot\left[\left(P_{s_4}\cdot x_s - P_{t_4}\cdot x_t\right) - P_{e_4}\cdot F_{tmin_4}\right]\right]} & \text{otherwise} \end{array}\right.$$

$$\sigma_{t1_4} = 0.000\frac{lb}{in^2}$$

$$\sigma_{t1_5} := \left| \begin{array}{ll} \overrightarrow{\left[\frac{1}{x_t - x_s}\cdot\left[\left(P_{s_5}\cdot x_s - P_{t_5}\cdot x_t\right) - \frac{2\cdot Q_{2_5}}{a_o^{2}}\cdot F_{tmin_5}\right]\right]} & \text{if } P_{e_5} = 0 \\ \overrightarrow{\left[\frac{1}{x_t - x_s}\cdot\left[\left(P_{s_5}\cdot x_s - P_{t_5}\cdot x_t\right) - P_{e_5}\cdot F_{tmin_5}\right]\right]} & \text{otherwise} \end{array}\right.$$

$$\sigma_{t1_5} = -1743.547\frac{lb}{in^2}$$

$$\sigma_{t1_6} := \left| \begin{array}{ll} \overrightarrow{\left[\frac{1}{x_t - x_s}\cdot\left[\left(P_{s_6}\cdot x_s - P_{t_6}\cdot x_t\right) - \frac{2\cdot Q_{2_6}}{a_o^{2}}\cdot F_{tmin_6}\right]\right]} & \text{if } P_{e_6} = 0 \\ \overrightarrow{\left[\frac{1}{x_t - x_s}\cdot\left[\left(P_{s_6}\cdot x_s - P_{t_6}\cdot x_t\right) - P_{e_6}\cdot F_{tmin_6}\right]\right]} & \text{otherwise} \end{array}\right.$$

$$\sigma_{t1_6} = 1141.626\frac{lb}{in^2}$$

$$\sigma_{t1_7} := \left| \begin{array}{ll} \overrightarrow{\left[\frac{1}{x_t - x_s}\cdot\left[\left(P_{s_7}\cdot x_s - P_{t_7}\cdot x_t\right) - \frac{2\cdot Q_{2_7}}{a_o^{2}}\cdot F_{tmin_7}\right]\right]} & \text{if } P_{e_7} = 0 \\ \overrightarrow{\left[\frac{1}{x_t - x_s}\cdot\left[\left(P_{s_7}\cdot x_s - P_{t_7}\cdot x_t\right) - P_{e_7}\cdot F_{tmin_7}\right]\right]} & \text{otherwise} \end{array}\right.$$

$$\sigma_{t1_7} = -129.351\frac{lb}{in^2}$$

$$\sigma_{t1_8} := \begin{cases} \overrightarrow{\left[\dfrac{1}{x_t - x_s}\cdot\left[\left(P_{s_8}\cdot x_s - P_{t_8}\cdot x_t\right) - \dfrac{2\cdot Q_{2_8}}{a_o^{\,2}}\cdot F_{tmin_8}\right]\right]} & \text{if } P_{e_8} = 0 \\ \overrightarrow{\left[\dfrac{1}{x_t - x_s}\cdot\left[\left(P_{s_8}\cdot x_s - P_{t_8}\cdot x_t\right) - P_{e_8}\cdot F_{tmin_8}\right]\right]} & \text{otherwise} \end{cases}$$

$$\sigma_{t1_8} = -459.775\,\frac{lb}{in^2}$$

$$\sigma_{t2_1} := \begin{cases} \overrightarrow{\left[\dfrac{1}{x_t - x_s}\cdot\left[\left(P_{s_1}\cdot x_s - P_{t_1}\cdot x_t\right) - \dfrac{2\cdot Q_{2_1}}{a_o^{\,2}}\cdot F_{tmax_1}\right]\right]} & \text{if } P_{e_1} = 0 \\ \overrightarrow{\left[\dfrac{1}{x_t - x_s}\cdot\left[\left(P_{s_1}\cdot x_s - P_{t_1}\cdot x_t\right) - P_{e_1}\cdot F_{tmax_1}\right]\right]} & \text{otherwise} \end{cases}$$

$$\sigma_{t2_1} = 2259.338\,\frac{lb}{in^2}$$

$$\sigma_{t2_2} := \begin{cases} \overrightarrow{\left[\dfrac{1}{x_t - x_s}\cdot\left[\left(P_{s_2}\cdot x_s - P_{t_2}\cdot x_t\right) - \dfrac{2\cdot Q_{2_2}}{a_o^{\,2}}\cdot F_{tmax_2}\right]\right]} & \text{if } P_{e_2} = 0 \\ \overrightarrow{\left[\dfrac{1}{x_t - x_s}\cdot\left[\left(P_{s_2}\cdot x_s - P_{t_2}\cdot x_t\right) - P_{e_2}\cdot F_{tmax_2}\right]\right]} & \text{otherwise} \end{cases}$$

$$\sigma_{t2_2} = -2276.563\,\frac{lb}{in^2}$$

$$\sigma_{t2_3} := \begin{cases} \overrightarrow{\left[\dfrac{1}{x_t - x_s}\cdot\left[\left(P_{s_3}\cdot x_s - P_{t_3}\cdot x_t\right) - \dfrac{2\cdot Q_{2_3}}{a_o^{\,2}}\cdot F_{tmax_3}\right]\right]} & \text{if } P_{e_3} = 0 \\ \overrightarrow{\left[\dfrac{1}{x_t - x_s}\cdot\left[\left(P_{s_3}\cdot x_s - P_{t_3}\cdot x_t\right) - P_{e_3}\cdot F_{tmax_3}\right]\right]} & \text{otherwise} \end{cases}$$

$$\sigma_{t2_3} = -73.142\,\frac{lb}{in^2}$$

$$\sigma_{t2_4} := \begin{cases} \overrightarrow{\left[\dfrac{1}{x_t - x_s}\cdot\left[\left(P_{s_4}\cdot x_s - P_{t_4}\cdot x_t\right) - \dfrac{2\cdot Q_{2_4}}{a_o^{\,2}}\cdot F_{tmax_4}\right]\right]} & \text{if } P_{e_4} = 0 \\ \overrightarrow{\left[\dfrac{1}{x_t - x_s}\cdot\left[\left(P_{s_4}\cdot x_s - P_{t_4}\cdot x_t\right) - P_{e_4}\cdot F_{tmax_4}\right]\right]} & \text{otherwise} \end{cases}$$

$$\sigma_{t2_4} = 0.000\,\frac{lb}{in^2}$$

$$\sigma_{t2_5} := \begin{cases} \overrightarrow{\left[\dfrac{1}{x_t - x_s}\cdot\left[\left(P_{s_5}\cdot x_s - P_{t_5}\cdot x_t\right) - \dfrac{2\cdot Q_{2_5}}{a_o^{\,2}}\cdot F_{tmax_5}\right]\right]} & \text{if } P_{e_5} = 0 \\ \overrightarrow{\left[\dfrac{1}{x_t - x_s}\cdot\left[\left(P_{s_5}\cdot x_s - P_{t_5}\cdot x_t\right) - P_{e_5}\cdot F_{tmax_5}\right]\right]} & \text{otherwise} \end{cases}$$

$$\sigma_{t2_5} = 8187.390\,\frac{lb}{in^2}$$

$$\sigma_{t2_6} := \begin{cases} \overrightarrow{\left[\dfrac{1}{x_t - x_s}\cdot\left[\left(P_{s_6}\cdot x_s - P_{t_6}\cdot x_t\right) - \dfrac{2\cdot Q_{2_6}}{a_o^{\,2}}\cdot F_{tmax_6}\right]\right]} & \text{if } P_{e_6} = 0 \\ \overrightarrow{\left[\dfrac{1}{x_t - x_s}\cdot\left[\left(P_{s_6}\cdot x_s - P_{t_6}\cdot x_t\right) - P_{e_6}\cdot F_{tmax_6}\right]\right]} & \text{otherwise} \end{cases}$$

$$\sigma_{t2_6} = 3651.489\,\frac{lb}{in^2}$$

$$\sigma_{t2_7} := \begin{cases} \overrightarrow{\left[\frac{1}{x_t - x_s}\cdot\left[\left(P_{s_7}\cdot x_s - P_{t_7}\cdot x_t\right) - \frac{2\cdot Q_{2_7}}{a_o^{\,2}}\cdot F_{tmax_7}\right]\right]} & \text{if } P_{e_7} = 0 \\ \overrightarrow{\left[\frac{1}{x_t - x_s}\cdot\left[\left(P_{s_7}\cdot x_s - P_{t_7}\cdot x_t\right) - P_{e_7}\cdot F_{tmax_7}\right]\right]} & \text{otherwise} \end{cases}$$

$$\sigma_{t2_7} = 5910.827\frac{lb}{in^2}$$

$$\sigma_{t2_8} := \begin{cases} \overrightarrow{\left[\frac{1}{x_t - x_s}\cdot\left[\left(P_{s_8}\cdot x_s - P_{t_8}\cdot x_t\right) - \frac{2\cdot Q_{2_8}}{a_o^{\,2}}\cdot F_{tmax_8}\right]\right]} & \text{if } P_{e_8} = 0 \\ \overrightarrow{\left[\frac{1}{x_t - x_s}\cdot\left[\left(P_{s_8}\cdot x_s - P_{t_8}\cdot x_t\right) - P_{e_8}\cdot F_{tmax_8}\right]\right]} & \text{otherwise} \end{cases}$$

$$\sigma_{t2_8} = 5928.052\frac{lb}{in^2}$$

$$\sigma_{t1} = \begin{pmatrix} -1288.8 \\ 1633.6 \\ 360.2 \\ 0.0 \\ -1743.5 \\ 1141.6 \\ -129.4 \\ -459.8 \end{pmatrix}\frac{lb}{in^2} \qquad \sigma_{t2} = \begin{pmatrix} 2259.3 \\ -2276.6 \\ -73.1 \\ 0.0 \\ 8187.4 \\ 3651.5 \\ 5910.8 \\ 5928.1 \end{pmatrix}\frac{lb}{in^2}$$

$$\sigma_{tmax} := \begin{pmatrix} \max\left(\left|\sigma_{t1_1}\right|, \left|\sigma_{t2_1}\right|\right) \\ \max\left(\left|\sigma_{t1_2}\right|, \left|\sigma_{t2_2}\right|\right) \\ \max\left(\left|\sigma_{t1_3}\right|, \left|\sigma_{t2_3}\right|\right) \\ \max\left(\left|\sigma_{t1_4}\right|, \left|\sigma_{t2_4}\right|\right) \\ \max\left(\left|\sigma_{t1_5}\right|, \left|\sigma_{t2_5}\right|\right) \\ \max\left(\left|\sigma_{t1_6}\right|, \left|\sigma_{t2_6}\right|\right) \\ \max\left(\left|\sigma_{t1_7}\right|, \left|\sigma_{t2_7}\right|\right) \\ \max\left(\left|\sigma_{t1_8}\right|, \left|\sigma_{t2_8}\right|\right) \end{pmatrix} \qquad \sigma_{tmin} := \begin{pmatrix} \min\left(\sigma_{t1_1}, \sigma_{t1_1}\right) \\ \min\left(\sigma_{t1_2}, \sigma_{t2_2}\right) \\ \min\left(\sigma_{t1_3}, \sigma_{t2_3}\right) \\ \min\left(\sigma_{t1_4}, \sigma_{t2_4}\right) \\ \min\left(\sigma_{t1_5}, \sigma_{t2_5}\right) \\ \min\left(\sigma_{t1_6}, \sigma_{t2_6}\right) \\ \min\left(\sigma_{t1_7}, \sigma_{t2_7}\right) \\ \min\left(\sigma_{t1_8}, \sigma_{t2_8}\right) \end{pmatrix}$$

$$\sigma_{tmax} = \begin{pmatrix} 2259.3 \\ 2276.6 \\ 360.2 \\ 0.0 \\ 8187.4 \\ 3651.5 \\ 5910.8 \\ 5928.1 \end{pmatrix}\frac{lb}{in^2} \qquad \sigma_{tmin} = \begin{pmatrix} -1288.8 \\ -2276.6 \\ -73.1 \\ 0.0 \\ -1743.5 \\ 1141.6 \\ -129.4 \\ -459.8 \end{pmatrix}\frac{lb}{in^2}$$

TUBE MAXIMUM STRESS for DESIGN LOADING CASES 1,2,3,4

$$\sigma_{tD\_max} := \max\left(\left|\sigma_{tmax_1}\right|, \left|\sigma_{tmax_2}\right|, \left|\sigma_{tmax_3}\right|, \left|\sigma_{tmax_4}\right|, \left|\sigma_{tmin_1}\right|, \left|\sigma_{tmin_2}\right|, \left|\sigma_{tmin_3}\right|, \left|\sigma_{tmin_4}\right|\right) \qquad \sigma_{tD\_max} = 2276.6\frac{lb}{in^2}$$

$$\sigma t_{allowD} := 1.0\cdot S_{tD} \qquad \sigma t_{allowD} = 16706.0\frac{lb}{in^2}$$

TUBE MAXIMUM STRESS for OPERATING LOADING CASES 5,6,7,8

$$\sigma_{tO\_max} := \max\left(\left|\sigma_{tmax_5}\right|, \left|\sigma_{tmax_6}\right|, \left|\sigma_{tmax_7}\right|, \left|\sigma_{tmax_8}\right|, \left|\sigma_{tmin_5}\right|, \left|\sigma_{tmin_6}\right|, \left|\sigma_{tmin_7}\right|, \left|\sigma_{tmin_8}\right|\right) \qquad \sigma_{tO\_max} = 8187.4\frac{lb}{in^2}$$

$$\sigma t_{allowO} := 2.0\cdot S_{tD} \qquad \sigma t_{allowO} = 33412.0\frac{lb}{in^2}$$

Step (b) : check the tubes for buckling if σ_{t1} <0 or σ_{t2} <0

$l_t := kl \quad l_t = 48.000\,in \qquad r_t := \frac{\sqrt{d_t^2 + (d_t - 2\cdot t_t)^2}}{4} \quad r_t = 0.337\,in \qquad C_t := \sqrt{\frac{2\cdot\pi^2\cdot E_{tD}}{S_{ytD}}} \quad C_t = 166.608 \qquad F_t := \frac{l_t}{r_t} \quad F_t = 142.571$

$$F_{S1} := \begin{cases} 1.25 & \text{if } P_{e_1} = 0 \\ \max\left[3.25 - .25\cdot\left[(Z_d + Q_{3_1}\cdot Z_v)\cdot X_a^4\right], 1.25\right] & \text{otherwise} \end{cases} \qquad F_{s1} := si(F_{S1} > 2, 2, F_{S1}) \quad F_{s1} = 1.471$$

$$F_{S2} := \begin{cases} 1.25 & \text{if } P_{e_2} = 0 \\ \max\left[3.25 - .25\cdot\left[(Z_d + Q_{3_2}\cdot Z_v)\cdot X_a^4\right], 1.25\right] & \text{otherwise} \end{cases} \qquad F_{s2} := si(F_{S2} > 2, 2, F_{S2}) \quad F_{s2} = 1.620$$

$$F_{S3} := \begin{cases} 1.25 & \text{if } P_{e_3} = 0 \\ \max\left[3.25 - .25\cdot\left[(Z_d + Q_{3_3}\cdot Z_v)\cdot X_a^4\right], 1.25\right] & \text{otherwise} \end{cases} \qquad F_{s3} := si(F_{S3} > 2, 2, F_{S3}) \quad F_{s3} = 2.000$$

$$F_{S4} := \begin{cases} 1.25 & \text{if } P_{e_4} = 0 \\ \max\left[3.25 - .25\cdot\left[(Z_d + Q_{3_4}\cdot Z_v)\cdot X_a^4\right], 1.25\right] & \text{otherwise} \end{cases} \qquad F_{s4} := si(F_{S4} > 2, 2, F_{S4}) \quad F_{s4} = 1.250$$

$$F_{S5} := \begin{cases} 1.25 & \text{if } P_{e_5} = 0 \\ \max\left[3.25 - .25\cdot\left[(Z_d + Q_{3_5}\cdot Z_v)\cdot X_a^4\right], 1.25\right] & \text{otherwise} \end{cases} \qquad F_{s5} := si(F_{S5} > 2, 2, F_{S5}) \quad F_{s5} = 1.402$$

$$F_{S6} := \begin{cases} 1.25 & \text{if } P_{e_6} = 0 \\ \max\left[3.25 - .25\cdot\left[(Z_d + Q_{3_6}\cdot Z_v)\cdot X_a^4\right], 1.25\right] & \text{otherwise} \end{cases} \qquad F_{s6} := si(F_{S6} > 2, 2, F_{S6}) \quad F_{s6} = 1.250$$

$$F_{S7} := \begin{cases} 1.25 & \text{if } P_{e_7} = 0 \\ \max\left[3.25 - .25\cdot\left[(Z_d + Q_{3_7}\cdot Z_v)\cdot X_a^4\right], 1.25\right] & \text{otherwise} \end{cases} \qquad F_{s7} := si(F_{S7} > 2, 2, F_{S7}) \quad F_{s7} = 1.250$$

$$F_{S8} := \begin{cases} 1.25 & \text{if } P_{e_8} \\ \max\left[3.25 - .2 \ldots {}_8\cdot Z_v \ldots\right] & \text{otherwise} \end{cases} \qquad F_{s8} := si(F_{S8} > 2, 2, F_{S8}) \quad F_{s8} = 1.361$$

$$F_s := \begin{pmatrix} F_{s1} \\ F_{s2} \\ F_{s3} \\ F_{s4} \\ F_{s5} \\ F_{s6} \\ F_{s7} \\ F_{s8} \end{pmatrix} \qquad F_s = \begin{pmatrix} 1.471 \\ 1.620 \\ 2.000 \\ 1.250 \\ 1.402 \\ 1.250 \\ 1.250 \\ 1.361 \end{pmatrix}$$

$$S_{tb} := si\left[C_t > F_t, \frac{S_{ytD}}{F_s}\cdot\left(1 - \frac{F_t}{2\cdot C_t}\right), \frac{1}{F_s}\cdot\pi^2\cdot\frac{E_{tD}}{F_t^2}\right]$$

Allowable Buckling Stress, S_{tb}

$$\sigma_{tmin} = \begin{pmatrix} -1288.8 \\ -2276.6 \\ -73.1 \\ 0.0 \\ -1743.5 \\ 1141.6 \\ -129.4 \\ -459.8 \end{pmatrix} \frac{lb}{in^2} \qquad S_{tb} = \begin{pmatrix} 7467.8 \\ 6780.7 \\ 5492.5 \\ 8788.0 \\ 7836.5 \\ 8788.0 \\ 8788.0 \\ 8071.9 \end{pmatrix} \frac{lb}{in^2}$$

UHX 13.5.10 **Step 10** **Determine the shell membrane stress :**

In the main shell

$$\sigma_{sm} := \frac{a_o^2}{(D_s + t_s)\cdot t_s}\cdot\left[P_e + \left(\rho_s^2 - 1\right)\cdot(P_s - P_t)\right] + \frac{a_s^2}{(D_s + t_s)t_s}\cdot P_t \qquad \sigma_{sm} = \begin{pmatrix} 1830.579 \\ 2287.188 \\ 4117.767 \\ 0.000 \\ -1085.899 \\ -629.290 \\ 1201.289 \\ -2916.478 \end{pmatrix}\frac{lb}{in^2}$$

SHELL MAXIMUM STRESS for DESIGN LOADING CASES 1 , 2, 3 , 4

$$\sigma_{smD} := \max\left(\left|\sigma_{sm_1}\right|, \left|\sigma_{sm_2}\right|, \left|\sigma_{sm_3}\right|, \left|\sigma_{sm_4}\right|\right) \qquad \sigma_{smD} = 4117.8\frac{lb}{in^2} \qquad \sigma_{sallowD} := E_{sw}\cdot S_D \qquad \sigma_{sallowD} = 13430.0\frac{lb}{in^2}$$

SHELL MAXIMUM STRESS for OPERATING LOADING CASES 5 , 6 , 7 , 8

$$\sigma_{smO} := \max\left(\left|\sigma_{sm_5}\right|, \left|\sigma_{sm_6}\right|, \left|\sigma_{sm_7}\right|, \left|\sigma_{sm_8}\right|\right) \qquad \sigma_{smO} = 2916.5\frac{lb}{in^2} \qquad \sigma_{sallowO} := S_{PS} \qquad \sigma_{sallowO} = 47400.0\frac{lb}{in^2}$$

In the shell band

$$t_{s1} := \begin{cases} 0in & \text{if Config = "d"} \\ t_{s1} & \text{otherwise} \end{cases} \qquad t_{s1} = 0.563\,in$$

$$\sigma_{sm1} := \frac{a_o^2}{(D_s + t_{s1})\cdot t_{s1}}\cdot\left[P_e + \left(\rho_s^2 - 1\right)\cdot(P_s - P_t)\right] + \frac{a_s^2}{(D_s + t_{s1})t_{s1}}\cdot P_t \qquad \sigma_{sm1} = \begin{pmatrix} 1830.579 \\ 2287.188 \\ 4117.767 \\ 0.000 \\ -1085.899 \\ -629.290 \\ 1201.289 \\ -2916.478 \end{pmatrix}\frac{lb}{in^2}$$

SHELL MAXIMUM STRESS for DESIGN LOADING CASES 1 , 2, 3 , 4

$$\sigma_{smD} := \max\left(\left|\sigma_{sm_1}\right|, \left|\sigma_{sm_2}\right|, \left|\sigma_{sm_3}\right|, \left|\sigma_{sm_4}\right|\right) \qquad \sigma_{smD} = 4117.8\frac{lb}{in^2} \qquad \sigma_{sallowD} := E_{sw}\cdot S_D \qquad \sigma_{sallowD} = 13430.0\frac{lb}{in^2}$$

SHELL MAXIMUM STRESS for OPERATING LOADING CASES 5 , 6 , 7 , 8

$$\sigma_{smO} := \max\left(\left|\sigma_{sm_5}\right|, \left|\sigma_{sm_6}\right|, \left|\sigma_{sm_7}\right|, \left|\sigma_{sm_8}\right|\right) \qquad \sigma_{smO} = 2916.5\frac{lb}{in^2} \qquad \sigma_{sallowO} := S_{PS} \qquad \sigma_{sallowO} = 47400.0\frac{lb}{in^2}$$

UHX 13.5.11 **Step 11** **Determine the shell stresses if integral to the tubesheet :**

$l_1 = 0.000\,in \qquad l_{smin} := 1.8\cdot\sqrt{D_s\cdot t_{s1}}$

bending stress: minimum lengths of shell band for config. a,b or c $\qquad l'_1 = 0.000\,in \qquad l_{smin} = 8.749\,in$

$$\sigma_{sb1} := \overrightarrow{\frac{6}{t_{s1}^2}\cdot k_s\cdot\left[\beta_s\cdot\left(\delta_s\cdot P_s + \frac{a_s^2}{E_{sD}\cdot t_{s1}}\cdot P^*_s\right) + \frac{6\cdot\left(1 - \nu^{*2}\right)}{E^*}\cdot\left(\frac{a_o^3}{h^3}\right)\cdot\left(1 + \frac{h'_s}{2}\right)\cdot\left[P_e\cdot\left(Z_v + Z_m\cdot Q_1\right) + \frac{2}{a_o^2}\cdot Z_m\cdot Q_2\right]\right]}$$

total shell stress:

$$\sigma_s := \overrightarrow{\left(\left|\sigma_{sm1}\right| + \left|\sigma_{sb1}\right|\right)} \qquad \sigma_s = \begin{pmatrix} 14014.879 \\ 30035.554 \\ 19681.834 \\ 0.000 \\ 39504.060 \\ 2143.795 \\ 11871.084 \\ 29150.339 \end{pmatrix}\frac{lb}{in^2} \qquad \sigma_{sb1} = \begin{pmatrix} -12184.300 \\ 27748.366 \\ 15564.066 \\ 0.000 \\ -38418.161 \\ 1514.505 \\ -10669.795 \\ -26233.861 \end{pmatrix}\frac{lb}{in^2}$$

SHELL MAXIMUM STRESS for DESIGN LOADING CASES 1 , 2, 3 , 4

$\sigma_{sD} := \max\left(\left|\sigma_{s_1}\right|, \left|\sigma_{s_2}\right|, \left|\sigma_{s_3}\right|, \left|\sigma_{s_4}\right|\right)$ $\sigma_{sD} = 30035.6 \frac{lb}{in^2}$ $\sigma_{sallowD} := 1.5 \cdot S_D$ $\sigma_{allowD} = 23700.0 \frac{lb}{in^2}$

SHELL MAXIMUM STRESS for OPERATING LOADING CASES 5 , 6 , 7 , 8

$\sigma_{sO} := \max\left(\left|\sigma_{s_5}\right|, \left|\sigma_{s_6}\right|, \left|\sigma_{s_7}\right|, \left|\sigma_{s_8}\right|\right)$ $\sigma_{sO} = 39504.1 \frac{lb}{in^2}$ $\sigma_{sallowO} := S_{PSs}$ $\sigma_{allowO} = 47400.0 \frac{lb}{in^2}$

UHX-13.5.11 **Step 11 (cont'd).** **Determine the channel stresses if integral to the tubesheet**

channel membrane stress:

$$\sigma_{cm} := \begin{cases} \frac{a_c^2}{(D_c + t_c) \cdot t_c} P_t & \text{if Config = "a"} \\ 0 \cdot \frac{lb}{in^2} & \text{otherwise} \end{cases}$$

$$\sigma_{cm} = \begin{pmatrix} 5567.108 \\ 0.000 \\ 5567.108 \\ 0.000 \\ 5567.108 \\ 0.000 \\ 5567.108 \\ 0.000 \end{pmatrix} \frac{lb}{in^2}$$

channel bending stress:

$$\sigma_{cb} := \overrightarrow{\frac{6}{t_c^2} \cdot k_c \cdot \left[\beta_c \cdot \left(\delta_c \cdot P_t + \frac{a_c^2}{E_{cD} \cdot t_c} P^*_c\right) - \frac{6 \cdot (1 - \nu^{*2})}{E^*} \cdot \left(\frac{a_o^3}{h^3}\right) \cdot \left(1 + \frac{h'_c}{2}\right) \cdot \left[P_e \cdot (Z_v + Z_m \cdot Q_1) + \frac{2}{a_o^2} \cdot Z_m \cdot Q_2\right]\right]}$$

$$\sigma_{cb} = \begin{pmatrix} 28345.980 \\ -8492.472 \\ 19853.508 \\ 0.000 \\ 52379.190 \\ 15540.738 \\ 43886.718 \\ 24033.210 \end{pmatrix} \frac{lb}{in^2}$$

total channel stress:

$$\sigma_c := \overrightarrow{\left(\left|\sigma_{cm}\right| + \left|\sigma_{cb}\right|\right)}$$

$$\sigma_c = \begin{pmatrix} 33913.088 \\ 8492.472 \\ 25420.616 \\ 0.000 \\ 57946.298 \\ 15540.738 \\ 49453.826 \\ 24033.210 \end{pmatrix} \frac{lb}{in^2}$$

CHANNEL MAXIMUM STRESS for DESIGN LOADING CASES 1 , 2, 3 , 4

$\sigma_{cD} := \max\left(\left|\sigma_{c_1}\right|, \left|\sigma_{c_2}\right|, \left|\sigma_{c_3}\right|, \left|\sigma_{c_4}\right|\right)$ $\sigma_{cD} = 33913.1 \frac{lb}{in^2}$ $\sigma_{callowD} := 1.5 \cdot S_{cD}$ $\sigma_{callowD} = 30000.0 \frac{lb}{in^2}$

CHANNEL MAXIMUM STRESS for OPERATING LOADING CASES 5 , 6 , 7 , 8

$\sigma_{cO} := \max\left(\left|\sigma_{c_5}\right|, \left|\sigma_{c_6}\right|, \left|\sigma_{c_7}\right|, \left|\sigma_{c_8}\right|\right)$ $\sigma_{cO} = 57946.3 \frac{lb}{in^2}$ $\sigma_{callowO} := S_{PSc}$ $\sigma_{callowO} = 67200.0 \frac{lb}{in^2}$

_UHX13.7 Simplified Elastic Plastic Procedure

Calculation procedure for the effect of plasticity at the tubesheet, channel or shell joint.

This procedure applies only to Configurations a,b,c and Design Loading Cases 1, 2 , 3 , 4 in the following conditions: -for integral shell (config. a,b,c) when $1.5S_s < \sigma_s \le S_{PS,s}$

-for integral channel (config. a) when $1.5S_c < \sigma_c \le S_{PS,c}$

$$S^*_s := \min\left(S_{ys1}, \frac{S_{PSs}}{2}\right) \qquad S^*_c := \min\left(S_{ycD}, \frac{S_{PSc}}{2}\right)$$

$$S^*_s = 17500.000\frac{lb}{in^2} \qquad S^*_c = 33600.000\frac{lb}{in^2}$$

$$fact_{sv} := \overrightarrow{\left(1.4 - \frac{0.4\cdot|\sigma_{sb1}|}{S^*_s}\right)} \qquad fact_{sv} = \begin{pmatrix}1.122\\0.766\\1.044\\1.400\\0.522\\1.365\\1.156\\0.800\end{pmatrix} \qquad fact_{cv} := \overrightarrow{\left(1.4 - \frac{0.4\cdot|\sigma_{cb}|}{S^*_c}\right)} \qquad fact_{cv} = \begin{pmatrix}1.0625\\1.2989\\1.1636\\1.4\\0.7764\\1.215\\0.8775\\1.1139\end{pmatrix}$$

$$fact_s := \begin{pmatrix}\min(fact_{sv_1}, 1)\\\min(fact_{sv_2}, 1)\\\min(fact_{sv_3}, 1)\\\min(fact_{sv_4}, 1)\end{pmatrix} \qquad fact_s = \begin{pmatrix}1.000\\0.766\\1.000\\1.000\end{pmatrix} \qquad fact_c := \begin{pmatrix}\min(fact_{cv_1}, 1)\\\min(fact_{cv_2}, 1)\\\min(fact_{cv_3}, 1)\\\min(fact_{cv_4}, 1)\end{pmatrix} \qquad fact_c = \begin{pmatrix}1.000\\1.000\\1.000\\1.000\end{pmatrix}$$

Calculate reduced values of E_s and E_c for each loading case:

$$E^*_{s1} := E_{s1}\cdot fact_s \qquad E^*_{s1} = \begin{pmatrix}2.6400\times10^7\\2.0216\times10^7\\2.6400\times10^7\\2.6400\times10^7\end{pmatrix}\frac{lb}{in^2} \qquad E^*_{c1} := E_{cD}\cdot fact_c \qquad E^*_{c1} = \begin{pmatrix}2.830\times10^7\\2.830\times10^7\\2.830\times10^7\\2.830\times10^7\end{pmatrix}\frac{lb}{in^2}$$

From Step 2 , recalculate:

$$kk_s := \beta_s\cdot\frac{E^*_{s1}\cdot t_{s1}^3}{6\cdot(1-\nu_s^2)} \qquad kk_s = \begin{pmatrix}3.1971\times10^5\\2.4482\times10^5\\3.1971\times10^5\\3.1971\times10^5\end{pmatrix}lb \qquad \lambda\lambda_s := \frac{6\cdot D_s}{h^3}\cdot kk_s\cdot\left(1 + h'_s + \frac{h'^2_s}{2}\right) \qquad \lambda\lambda_s = \begin{pmatrix}5.0868\times10^7\\3.8952\times10^7\\5.0868\times10^7\\5.0868\times10^7\end{pmatrix}\frac{lb}{in^2}$$

$$kk_c := \beta_c\cdot\frac{E^*_{c1}\cdot t_c^3}{6\cdot(1-\nu_c^2)} \qquad kk_c = \begin{pmatrix}1.2446\times10^5\\1.2446\times10^5\\1.2446\times10^5\\1.2446\times10^5\end{pmatrix}lb \qquad \lambda\lambda_c := \frac{6\cdot D_c}{h^3}\cdot kk_c\cdot\left(1 + h'_c + \frac{h'^2_c}{2}\right) \qquad \lambda\lambda_c = \begin{pmatrix}2.2049\times10^7\\2.2049\times10^7\\2.2049\times10^7\\2.2049\times10^7\end{pmatrix}\frac{lb}{in^2}$$

From Step 4 , recalculate:

$$F := \frac{1-\nu^*}{E^*}\cdot\left(\lambda\lambda_s + \lambda\lambda_c + E_D\cdot\ln(K)\right) \qquad F = \begin{pmatrix}6.732\\5.649\\6.732\\6.732\end{pmatrix}$$

$$\Phi := (1+\nu^*)\cdot F \qquad \Phi = \begin{pmatrix}9.024\\7.572\\9.024\\9.024\end{pmatrix}$$

$$Q_{1e} := \overrightarrow{\left(\frac{\rho_s - 1 - \Phi\cdot Z_v}{1+\Phi\cdot Z_m}\right)} \qquad Q_{1e} = \begin{pmatrix}-0.059\\-0.054\\-0.059\\-0.059\end{pmatrix}$$

$$Q_{z1e} := \frac{\left(Z_d + Q_{1e}\cdot Z_w\right)\cdot X_a^{\,4}}{2} \qquad Q_{z1e} = \begin{pmatrix}3.778\\3.899\\3.778\\3.778\end{pmatrix}$$

$$Q_{z2e} := \frac{\left(Z_v + Q_{1e}\cdot Z_m\right)\cdot X_a^{\,4}}{2} \qquad Q_{z2e} = \begin{pmatrix}10.312\\11.518\\10.312\\10.312\end{pmatrix}$$

$$U_e := \frac{\left[Z_w + \left(\rho_s - 1\right)\cdot Z_m\right]\cdot X_a^{\,4}}{1+\Phi\cdot Z_m} \qquad U_e = \begin{pmatrix}20.625\\23.037\\20.625\\20.625\end{pmatrix}$$

$$W^{*\prime} := \begin{pmatrix}W^*_1\\W^*_2\\W^*_3\\W^*_4\end{pmatrix} \qquad W^{*\prime} = \begin{pmatrix}0.000\\0.000\\0.000\\0.000\end{pmatrix}\text{lb}$$

From Step 6 , recalculate P_w, P_{rim}, and P_e :

$$P'_w := \begin{pmatrix}\frac{U_{e_1}}{a_o^{\,2}}\cdot\frac{\gamma_b}{2\pi}\cdot W^{*\prime}_1\\ \frac{U_{e_2}}{a_o^{\,2}}\cdot\frac{\gamma_b}{2\pi}\cdot W^{*\prime}_2\\ \frac{U_{e_3}}{a_o^{\,2}}\cdot\frac{\gamma_b}{2\pi}\cdot W^{*\prime}_3\\ \frac{U_{e_4}}{a_o^{\,2}}\cdot\frac{\gamma_b}{2\pi}\cdot W^{*\prime}_4\end{pmatrix} \qquad P'_w = \begin{pmatrix}0.000\\0.000\\0.000\\0.000\end{pmatrix}\frac{\text{lb}}{\text{in}^2}$$

$$P_{se} := \begin{pmatrix}P_{s_1}\\P_{s_2}\\P_{s_3}\\P_{s_4}\end{pmatrix} \qquad P_{te} := \begin{pmatrix}P_{t_1}\\P_{t_2}\\P_{t_3}\\P_{t_4}\end{pmatrix} \qquad P'_{se} := \begin{pmatrix}P'_{s_1}\\P'_{s_2}\\P'_{s_3}\\P'_{s_4}\end{pmatrix} \qquad P'_{te} := \begin{pmatrix}P'_{t_1}\\P'_{t_2}\\P'_{t_3}\\P'_{t_4}\end{pmatrix}$$

$$P'_{rim} := \overrightarrow{\left[\frac{U_e}{a_o^{\,2}}\cdot\left(\omega^*_s\cdot P_{se} - \omega^*_c\cdot P_{te}\right)\right]} \qquad P'_{rim} = \begin{pmatrix}-25.238\\79.928\\46.321\\0.000\end{pmatrix}\frac{\text{lb}}{\text{in}^2}$$

$$P'_e := \overrightarrow{\left[\frac{J\cdot K_{st}}{1+J\cdot K_{st}\cdot\left[Q_{z1e} + \left(\rho_s - 1\right)\cdot Q_{z2e}\right]}\cdot\left(P'_{se} - P'_{te} + P'_w + P'_{rim}\right)\right]} \qquad P'_e = \begin{pmatrix}-96.973\\115.426\\19.826\\0.000\end{pmatrix}\frac{\text{lb}}{\text{in}^2}$$

From Step 7 , recalculate Q_2 ,Q_3 , F_m and the tubesheet bending stress for load cases 1, 2, and 3

$$Q'_2 := \overrightarrow{\left[\frac{(\omega^*_s \cdot P_{se} - \omega^*_c \cdot P_{te}) + \frac{\gamma_b}{2 \cdot \pi} \cdot W^{*\prime}}{1 + \Phi \cdot Z_m}\right]} \qquad Q'_2 = \begin{pmatrix} 181.674 \\ -575.344 \\ -333.433 \\ 0.000 \end{pmatrix} lb$$

$$Q'_3 := \overrightarrow{\left(Q_{1e} + \frac{2 \cdot Q'_2}{P'_e \cdot a_o^2}\right)} \qquad Q'_3 = \begin{pmatrix} -0.0675 \\ -0.0773 \\ -0.1377 \\ -0.0586 \end{pmatrix}$$

$$F'_{m1}(x) := \begin{cases} 0 & \text{if } P'_{e_1} = 0 \\ \frac{Q_v(x) + Q'_{3_1} \cdot Q_m(x)}{2} & \text{otherwise} \end{cases} \qquad F'm1_j := \left|F'_{m1}(x_j)\right| \qquad F'_{m1} := \max(F'm1) \qquad F'_{m1} = 0.034$$

$$F'_{m2}(x) := \begin{cases} 0 & \text{if } P'_{e_2} = 0 \\ \frac{Q_v(x) + Q'_{3_2} \cdot Q_m(x)}{2} & \text{otherwise} \end{cases} \qquad F'm2_j := \left|F'_{m2}(x_j)\right| \qquad F'_{m2} := \max(F'm2) \qquad F'_{m2} = 0.039$$

$$F'_{m3}(x) := \begin{cases} 0 & \text{if } P'_{e_3} = 0 \\ \frac{Q_v(x) + Q'_{3_3} \cdot Q_m(x)}{2} & \text{otherwise} \end{cases} \qquad F'm3_j := \left|F'_{m3}(x_j)\right| \qquad F'_{m3} := \max(F'm3) \qquad F'_{m3} = 0.069$$

$$F'_{m4}(x) := \begin{cases} 0 & \text{if } P'_{e_4} = 0 \\ \frac{Q_v(x) + Q'_{3_4} \cdot Q_m(x)}{2} & \text{otherwise} \end{cases} \qquad F'm4_j := \left|F'_{m4}(x_j)\right| \qquad F'_{m4} := \max(F'm4) \qquad F'_{m4} = 0.000$$

$$F'_m := \begin{pmatrix} F'_{m1} \\ F'_{m2} \\ F'_{m3} \\ F'_{m4} \end{pmatrix} \qquad F'_m = \begin{pmatrix} 0.0337 \\ 0.0386 \\ 0.0689 \\ 0 \end{pmatrix}$$

Tubesheet Bending Stress for the Elastic-Plastic Solution

$$\sigma' := \overrightarrow{\left[\frac{1.5 \cdot F'_m}{\mu^*} \cdot \left(\frac{2 \cdot a_o}{h - h'_g}\right)^2 \cdot P'_e\right]} \qquad \sigma' = \begin{pmatrix} -16286.147 \\ 22204.445 \\ 6797.980 \\ 0.000 \end{pmatrix} \frac{lb}{in^2}$$

$$\sigma'_{max} := \max\left(\left|\sigma'_1\right|, \left|\sigma'_2\right|, \left|\sigma'_3\right|\right) \qquad \sigma'_{max} = 22204.445 \frac{lb}{in^2}$$

$$\sigma_{allow} := 1.5 \cdot S_D \qquad \sigma_{allow} = 23700.0 \frac{lb}{in^2}$$

UHX13.9 Simply Supported Tubesheet Procedure

This procedure applies only when the TS is integral with the shell or channel, i.e. :
- shell of configurations a, b or c
- channel of configuration a

To perform this calculation, use the option : SS = "NO"

Calculation must be performed in 2 phases :

Phase 1) Perform Steps 1 to 11 with **SS="NON"** (normal calculation) with the following modifications in Step 11:

- minimum length requirement l_{smin} of shell band for configurations a,b,c do not apply
- minimum length requirement l_{cmin} of channel band for configuration a do not apply

if $\sigma_s \leq S_{PS,s}$ and $\sigma_c \leq S_{PS,c}$, the shell and/or channel designs are acceptable.
Otherwise increase the thickness of the overstressed components (shell and/or channel) and return to Step 1.

Phase 2) Perform Steps 1 to 7 using **SS="OUI"** (Simply Supported calculation) for loading cases 1,2 and 3 only

If $|\sigma| \leq 1.5S$, the calculation procedure is complete.
Otherwise, increase the assumed tubesheet thickness *h* and repeat Steps 1 to 7.

Note: If $|\sigma|<1.5S$, the tubesheet thickness can be optimized to a value h_0 till $|\sigma|=1.5S$ provided that, for that optimized thickness h_0, the stresses in the tubesheet,shell and channel , calculated by the normal calculation (see Phase 1), remain respectively below S_{PS} , $S_{PS,s}$, $S_{PS,c}$ for each of the 8 loading cases.

Distribution of Moments and Loads in the Tubesheet

The 2nd part of this Mathcad software provides general basic equations without using coefficient Q_3 They are taken from Item 04-1401 and enable to calculate q(x) , w(x) , θ(x) , σ(x) , τ(x) , σ_t(x) at any radius of the Tubesheet , depending on Loads V_a and M_a acting at periphery of the perforated tubesheet (r = a_o) . It provides also the axial deplacement and axial load acting in the shell.

Δ_s = elastic stretch of shell

N_s = axial force in shell (per unit of circumference)

N_c = axial force in channel (per unit of circumference)

K_w = elastic foundation modulus of the heat exchanger (based on L)

k_w = 2K_w = elastic foundation modulus of the half H.E.(based on L/2)

D* = Tubesheet flexural rigidity

YELLOW :most important data and results   Limit Conditions   Results obtained fromUHX-13

Perforated Tubesheet

$K_w := \frac{N_t \cdot K_t}{\pi a_o^2}$ $k_w := 2 \cdot K_w$ $D^* := \frac{E^* h^3}{12 \cdot (1 - \nu^{*2})}$ $k := \left(\frac{k_w}{D^*}\right)^{0.25}$ numpoints = 20

$K_w = 11905.554 \frac{lb}{in^3}$ $k_w = 23811.107 \frac{lb}{in^3}$ $D^* = 1.779 \times 10^6$ lb·in $k = 0.340\,in^{-1}$

$Z_{mx}(x) := \frac{ber' \cdot ber'_x(x) + bei' \cdot bei'_x(x)}{X_a \cdot Z_a}$ $Z_{vx}(x) := \frac{\Psi_2 \cdot ber'_x(x) + \Psi_1 \cdot bei'_x(x)}{X_a^2 \cdot Z_a}$

$Z_{mx}(X_a) = 0.207$ $Z_m = 0.207$ $Z_{vx}(X_a) = 0.021$ $Z_v = 0.021$

$Z_{dx}(x) := \frac{\Psi_2 \cdot ber_x(x) + \Psi_1 \cdot bei_x(x)}{X_a^3 \cdot Z_a}$ $Z_{wx}(x) := \frac{ber' \cdot ber_x(x) + bei' \cdot bei_x(x)}{X_a^2 \cdot Z_a}$

$Z_{dx}(X_a) = 0.004$ $Z_d = 0.004$ $Z_{wx}(X_a) = 0.021$ $Z_w := Z_{wx}(X_a)$ $Z_w = 0.021$

Loads V_a and M_a acting at periphery of perforated tubesheet (r = a_o)

$V_a := \overrightarrow{\left(\frac{a_o}{2} P_e\right)}$ $M_a := \overrightarrow{(a_o \cdot V_a \cdot Q_3)}$

$$V_a = \begin{pmatrix} -1000.036 \\ 1204.496 \\ 204.460 \\ 0.000 \\ -2692.753 \\ -488.221 \\ -1488.257 \\ -1692.717 \end{pmatrix} \frac{lb}{in} \qquad M_a = \begin{pmatrix} 1391.316 \\ -1972.064 \\ -580.747 \\ 0.000 \\ 3438.824 \\ 75.445 \\ 1466.761 \\ 2047.508 \end{pmatrix} lb$$

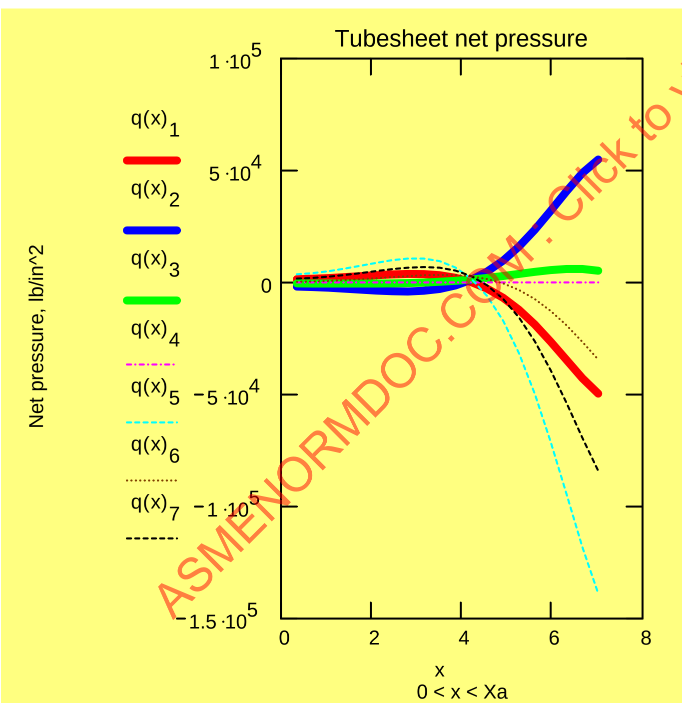
Replace V_a by: $P_e a_o/2$ and M_a by: $a_o V_a Q_3$ in formulae below to obtain UHX-13 formulae used in previous section

Net Tubesheet Pressure

$$q(x) := \begin{bmatrix} a_o^2 \cdot k^4 \cdot \left[M_{a_1} \cdot Z_{wx}(x) + \left(a_o \cdot V_{a_1}\right) \cdot Z_{dx}(x) \right] \\ a_o^2 \cdot k^4 \cdot \left[M_{a_2} \cdot Z_{wx}(x) + \left(a_o \cdot V_{a_2}\right) \cdot Z_{dx}(x) \right] \\ a_o^2 \cdot k^4 \cdot \left[M_{a_3} \cdot Z_{wx}(x) + \left(a_o \cdot V_{a_3}\right) \cdot Z_{dx}(x) \right] \\ a_o^2 \cdot k^4 \cdot \left[M_{a_4} \cdot Z_{wx}(x) + \left(a_o \cdot V_{a_4}\right) \cdot Z_{dx}(x) \right] \\ a_o^2 \cdot k^4 \cdot \left[M_{a_5} \cdot Z_{wx}(x) + \left(a_o \cdot V_{a_5}\right) \cdot Z_{dx}(x) \right] \\ a_o^2 \cdot k^4 \cdot \left[M_{a_6} \cdot Z_{wx}(x) + \left(a_o \cdot V_{a_6}\right) \cdot Z_{dx}(x) \right] \\ a_o^2 \cdot k^4 \cdot \left[M_{a_7} \cdot Z_{wx}(x) + \left(a_o \cdot V_{a_7}\right) \cdot Z_{dx}(x) \right] \\ a_o^2 \cdot k^4 \cdot \left[M_{a_8} \cdot Z_{wx}(x) + \left(a_o \cdot V_{a_8}\right) \cdot Z_{dx}(x) \right] \end{bmatrix}$$

$$q_{max} := \begin{pmatrix} \max(q(x)_1) \\ \max(q(x)_2) \\ \max(q(x)_3) \\ \max(q(x)_4) \\ \max(q(x)_5) \\ \max(q(x)_6) \\ \max(q(x)_7) \\ \max(q(x)_8) \end{pmatrix} \qquad q_{min} := \begin{pmatrix} \min(q(x)_1) \\ \min(q(x)_2) \\ \min(q(x)_3) \\ \min(q(x)_4) \\ \min(q(x)_5) \\ \min(q(x)_6) \\ \min(q(x)_7) \\ \min(q(x)_8) \end{pmatrix}$$

$$q_{max} = \begin{pmatrix} 26.152 \\ 380.756 \\ 41.573 \\ 0.000 \\ 73.727 \\ 23.163 \\ 47.453 \\ 48.099 \end{pmatrix} \frac{lb}{in^2} \qquad q_{min} = \begin{pmatrix} -345.033 \\ -28.309 \\ -3.762 \\ 0.000 \\ -965.193 \\ -239.405 \\ -584.437 \\ -620.160 \end{pmatrix} \frac{lb}{in^2}$$



$$F_q := \frac{\left(Z_d + Q_3 \cdot Z_v\right) \cdot X_a^4}{2}$$

$$q(X_a) = \begin{pmatrix} -345.033 \\ 380.756 \\ 35.723 \\ 0.000 \\ -965.193 \\ -239.405 \\ -584.437 \\ -620.160 \end{pmatrix} \frac{lb}{in^2} \qquad \overrightarrow{\left(F_q \cdot P_e\right)} = \begin{pmatrix} -345.033 \\ 380.756 \\ 35.723 \\ 0.000 \\ -965.193 \\ -239.405 \\ -584.437 \\ -620.160 \end{pmatrix} \frac{lb}{in^2}$$

Deflection w(x) of the perforated tubesheet (0 <= x <= Xa)

$\rho_1 := Z_w + (\rho_s - 1)\cdot Z_m \quad \rho_1 = 0.024 \qquad \rho_2 := Z_d + (\rho_s - 1)\cdot Z_v \quad \rho_2 = 0.005$

Determination of Q

$$Q := k_w \cdot \frac{a_o^2}{D^*} \cdot \left[M_a \cdot \rho_1 + (a_o \cdot V_a) \cdot \rho_2 \right] \qquad Q = \begin{pmatrix} -359.327 \\ 391.631 \\ 32.304 \\ 0.000 \\ -1010.264 \\ -259.306 \\ -618.633 \\ -650.937 \end{pmatrix} \frac{lb}{in^2}$$

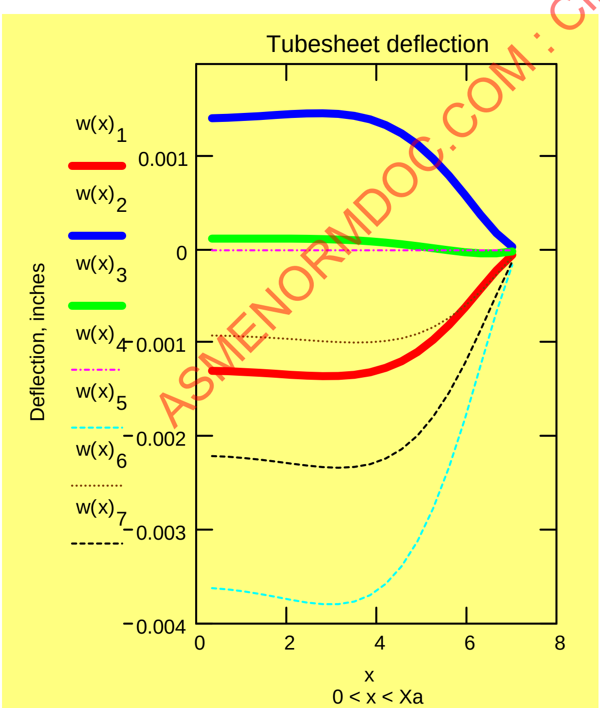
$$w(x) := \begin{bmatrix} \left(\frac{1}{k_w}\right)\cdot Q_1 - \frac{a_o^2}{D^*}\cdot\left[M_{a_1}\cdot Z_{wx}(x) + (a_o\cdot V_{a_1})\cdot Z_{dx}(x)\right] \\ \left(\frac{1}{k_w}\right)\cdot Q_2 - \frac{a_o^2}{D^*}\cdot\left[M_{a_2}\cdot Z_{wx}(x) + (a_o\cdot V_{a_2})\cdot Z_{dx}(x)\right] \\ \left(\frac{1}{k_w}\right)\cdot Q_3 - \frac{a_o^2}{D^*}\cdot\left[M_{a_3}\cdot Z_{wx}(x) + (a_o\cdot V_{a_3})\cdot Z_{dx}(x)\right] \\ \left(\frac{1}{k_w}\right)\cdot Q_4 - \frac{a_o^2}{D^*}\cdot\left[M_{a_4}\cdot Z_{wx}(x) + (a_o\cdot V_{a_4})\cdot Z_{dx}(x)\right] \\ \left(\frac{1}{k_w}\right)\cdot Q_5 - \frac{a_o^2}{D^*}\cdot\left[M_{a_5}\cdot Z_{wx}(x) + (a_o\cdot V_{a_5})\cdot Z_{dx}(x)\right] \\ \left(\frac{1}{k_w}\right)\cdot Q_6 - \frac{a_o^2}{D^*}\cdot\left[M_{a_6}\cdot Z_{wx}(x) + (a_o\cdot V_{a_6})\cdot Z_{dx}(x)\right] \\ \left(\frac{1}{k_w}\right)\cdot Q_7 - \frac{a_o^2}{D^*}\cdot\left[M_{a_7}\cdot Z_{wx}(x) + (a_o\cdot V_{a_7})\cdot Z_{dx}(x)\right] \\ \left(\frac{1}{k_w}\right)\cdot Q_8 - \frac{a_o^2}{D^*}\cdot\left[M_{a_8}\cdot Z_{wx}(x) + (a_o\cdot V_{a_8})\cdot Z_{dx}(x)\right] \end{bmatrix}$$

x =

	1
1	0.351
2	0.702
3	1.052
4	1.403
5	1.754
6	2.105
7	2.455
8	2.806
9	3.157
10	3.508
11	3.859
12	4.209
13	4.560
14	4.911
15	5.262
16	5.612

$w(x)_1 =$ (in)

	1
1	-1.550·10 -2
2	-1.556·10 -2
3	-1.564·10 -2
4	-1.576·10 -2
5	-1.589·10 -2
6	-1.602·10 -2
7	-1.613·10 -2
8	-1.619·10 -2
9	-1.617·10 -2
10	-1.601·10 -2
11	-1.569·10 -2
12	-1.512·10 -2
13	-1.427·10 -2
14	-1.308·10 -2
15	-1.153·10 -2
16	-9.609·10 -3



$$w(0.0) = \begin{pmatrix} -0.015 \\ 0.017 \\ 0.002 \\ 0.000 \\ -0.043 \\ -0.011 \\ -0.026 \\ -0.028 \end{pmatrix} in \qquad w(X_a) = \begin{pmatrix} -6.003\times10^{-4} \\ 4.567\times10^{-4} \\ -1.436\times10^{-4} \\ 0.000 \\ -1.893\times10^{-3} \\ -8.358\times10^{-4} \\ -1.436\times10^{-3} \\ -1.293\times10^{-3} \end{pmatrix} in$$

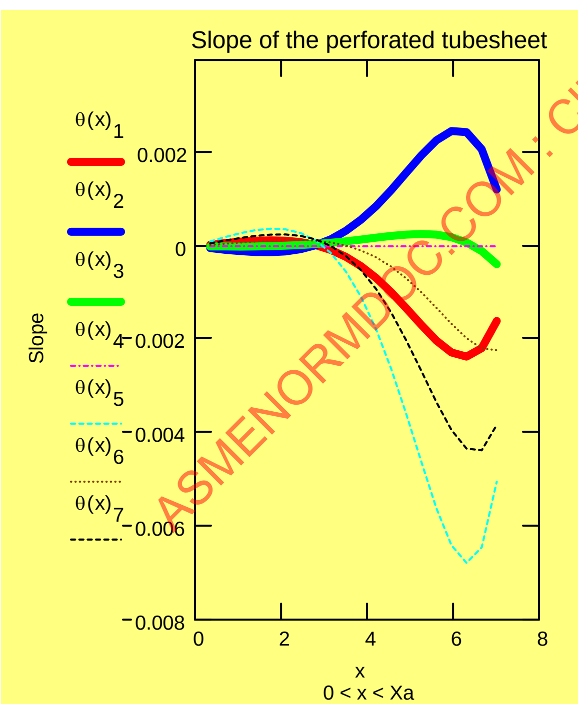
For use on Page 40:

$ww(x) := w(x)$

Slope θ(x) of the perforated tubesheet (0 <= x <= Xa)

$$\theta(x) := \begin{bmatrix} \frac{a_o}{D^*}\cdot\left[M_{a_1}\cdot Z_{mx}(x) + \left(a_o\cdot V_{a_1}\right)\cdot Z_{vx}(x)\right] \\ \frac{a_o}{D^*}\cdot\left[M_{a_2}\cdot Z_{mx}(x) + \left(a_o\cdot V_{a_2}\right)\cdot Z_{vx}(x)\right] \\ \frac{a_o}{D^*}\cdot\left[M_{a_3}\cdot Z_{mx}(x) + \left(a_o\cdot V_{a_3}\right)\cdot Z_{vx}(x)\right] \\ \frac{a_o}{D^*}\cdot\left[M_{a_4}\cdot Z_{mx}(x) + \left(a_o\cdot V_{a_4}\right)\cdot Z_{vx}(x)\right] \\ \frac{a_o}{D^*}\cdot\left[M_{a_5}\cdot Z_{mx}(x) + \left(a_o\cdot V_{a_5}\right)\cdot Z_{vx}(x)\right] \\ \frac{a_o}{D^*}\cdot\left[M_{a_6}\cdot Z_{mx}(x) + \left(a_o\cdot V_{a_6}\right)\cdot Z_{vx}(x)\right] \\ \frac{a_o}{D^*}\cdot\left[M_{a_7}\cdot Z_{mx}(x) + \left(a_o\cdot V_{a_7}\right)\cdot Z_{vx}(x)\right] \\ \frac{a_o}{D^*}\cdot\left[M_{a_8}\cdot Z_{mx}(x) + \left(a_o\cdot V_{a_8}\right)\cdot Z_{vx}(x)\right] \end{bmatrix}$$

$$\theta(X_a) = \begin{pmatrix} -1.601\times10^{-3} \\ 1.218\times10^{-3} \\ -3.829\times10^{-4} \\ 0.000 \\ -5.048\times10^{-3} \\ -2.229\times10^{-3} \\ -3.830\times10^{-3} \\ -3.447\times10^{-3} \end{pmatrix}$$



Limit condition : $a_o\cdot\theta(X_a)\cdot(\rho_s - 1) = w(X_a)$

$$a_o\cdot\theta(X_a)\cdot(\rho_s - 1) = \begin{pmatrix} -6.003\times10^{-4} \\ 4.567\times10^{-4} \\ -1.436\times10^{-4} \\ 0.000 \\ -1.893\times10^{-3} \\ -8.358\times10^{-4} \\ -1.436\times10^{-3} \\ -1.293\times10^{-3} \end{pmatrix} \text{in} \qquad w(X_a) = \begin{pmatrix} -6.003\times10^{-4} \\ 4.567\times10^{-4} \\ -1.436\times10^{-4} \\ 0.000 \\ -1.893\times10^{-3} \\ -8.358\times10^{-4} \\ -1.436\times10^{-3} \\ -1.293\times10^{-3} \end{pmatrix} \text{in}$$

Limit condition : $\theta(0)=0$

Limit Condition : $w(a_s)=0$

$$w(a_s) := w(X_a) - a_o\cdot\theta(X_a)\cdot(\rho_s - 1)$$

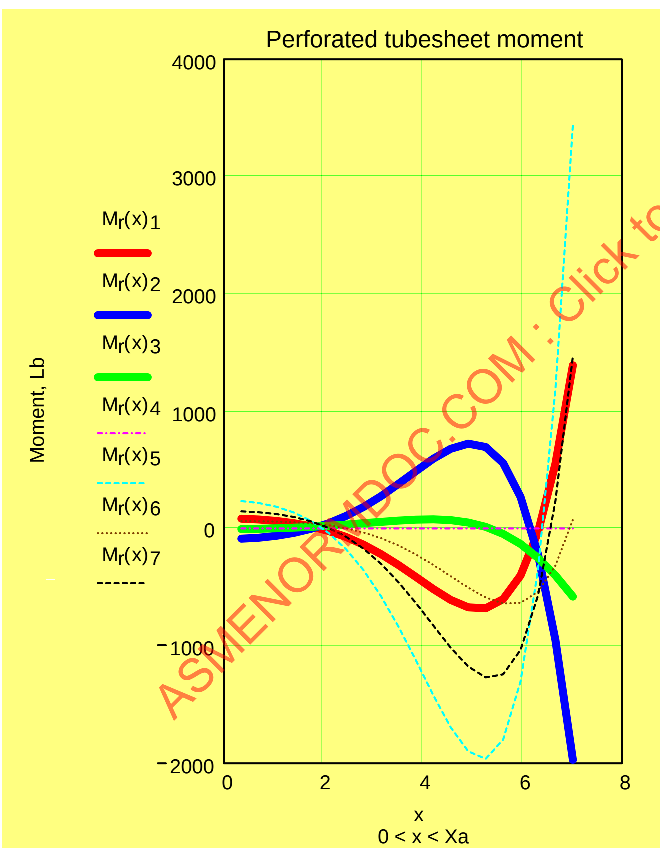
$$\theta(0) = \begin{pmatrix} 0.00000 \\ 0.00000 \\ 0.00000 \\ 0.00000 \\ 0.00000 \\ 0.00000 \\ 0.00000 \\ 0.00000 \end{pmatrix} \qquad w(a_s) = \begin{pmatrix} 0.00000 \\ 0.00000 \\ 0.00000 \\ 0.00000 \\ 0.00000 \\ 0.00000 \\ 0.00000 \\ 0.00000 \end{pmatrix} \text{in}$$

Bending Moment $M_r(x)$ of the perforated tubesheet (0 <= x <= Xa)

$$M_r(x) := \begin{bmatrix} M_{a_1} \cdot Q_m(x) + \left(a_o \cdot V_{a_1}\right) \cdot Q_v(x) \\ M_{a_2} \cdot Q_m(x) + \left(a_o \cdot V_{a_2}\right) \cdot Q_v(x) \\ M_{a_3} \cdot Q_m(x) + \left(a_o \cdot V_{a_3}\right) \cdot Q_v(x) \\ M_{a_4} \cdot Q_m(x) + \left(a_o \cdot V_{a_4}\right) \cdot Q_v(x) \\ M_{a_5} \cdot Q_m(x) + \left(a_o \cdot V_{a_5}\right) \cdot Q_v(x) \\ M_{a_6} \cdot Q_m(x) + \left(a_o \cdot V_{a_6}\right) \cdot Q_v(x) \\ M_{a_7} \cdot Q_m(x) + \left(a_o \cdot V_{a_7}\right) \cdot Q_v(x) \\ M_{a_8} \cdot Q_m(x) + \left(a_o \cdot V_{a_8}\right) \cdot Q_v(x) \end{bmatrix}$$

$$M_{rmax} := \begin{pmatrix} \max\left(M_r(x)_1\right) \\ \max\left(M_r(x)_2\right) \\ \max\left(M_r(x)_3\right) \\ \max\left(M_r(x)_4\right) \\ \max\left(M_r(x)_5\right) \\ \max\left(M_r(x)_6\right) \\ \max\left(M_r(x)_7\right) \\ \max\left(M_r(x)_8\right) \end{pmatrix} \qquad M_{rmin} := \begin{pmatrix} \min\left(M_r(x)_1\right) \\ \min\left(M_r(x)_2\right) \\ \min\left(M_r(x)_3\right) \\ \min\left(M_r(x)_4\right) \\ \min\left(M_r(x)_5\right) \\ \min\left(M_r(x)_6\right) \\ \min\left(M_r(x)_7\right) \\ \min\left(M_r(x)_8\right) \end{pmatrix}$$

$$M_{rmax} = \begin{pmatrix} 1391.316 \\ 722.607 \\ 76.920 \\ 0.000 \\ 3438.824 \\ 75.445 \\ 1466.761 \\ 2047.508 \end{pmatrix} lb \qquad M_{rmin} = \begin{pmatrix} -680.624 \\ -1972.064 \\ -580.747 \\ 0.000 \\ -1963.244 \\ -640.185 \\ -1269.260 \\ -1282.621 \end{pmatrix} lb$$



$$M_r\left(X_a\right) = \begin{pmatrix} 1391.316 \\ -1972.064 \\ -580.747 \\ 0.000 \\ 3438.824 \\ 75.445 \\ 1466.761 \\ 2047.508 \end{pmatrix} lb$$

Limit Condition : $M_r(X_a)=M_a$

$$M_r\left(X_a\right) = \begin{pmatrix} 1391.316 \\ -1972.064 \\ -580.747 \\ 0.000 \\ 3438.824 \\ 75.445 \\ 1466.761 \\ 2047.508 \end{pmatrix} lb \qquad M_a = \begin{pmatrix} 1391.316 \\ -1972.064 \\ -580.747 \\ 0.000 \\ 3438.824 \\ 75.445 \\ 1466.761 \\ 2047.508 \end{pmatrix} lb$$

Bending Stress $\sigma_r(x)$ of the perforated tubesheet (0 <= x <= Xa)

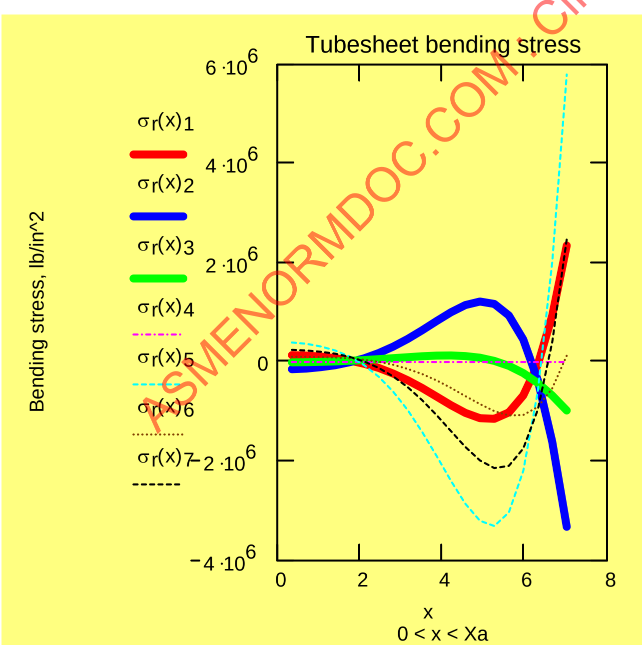
$$\sigma_r(x) := \begin{bmatrix} \frac{6}{\mu^* \cdot \left(h_{min_1}\right)^2} \cdot M_r(x)_1 \\ \frac{6}{\mu^* \cdot \left(h_{min_2}\right)^2} \cdot M_r(x)_2 \\ \frac{6}{\mu^* \cdot \left(h_{min_3}\right)^2} \cdot M_r(x)_3 \\ \frac{6}{\mu^* \cdot \left(h_{min_4}\right)^2} \cdot M_r(x)_4 \\ \frac{6}{\mu^* \cdot \left(h_{min_5}\right)^2} \cdot M_r(x)_5 \\ \frac{6}{\mu^* \cdot \left(h_{min_6}\right)^2} \cdot M_r(x)_6 \\ \frac{6}{\mu^* \cdot \left(h_{min_7}\right)^2} \cdot M_r(x)_7 \\ \frac{6}{\mu^* \cdot \left(h_{min_8}\right)^2} \cdot M_r(x)_8 \end{bmatrix}$$

$\sigma_r(x)_1 =$ (lb/in²)

	1
1	963.111
2	885.777
3	744.88
4	523.371
5	199.45
6	-251.022
7	-851.001
8	$-1.617 \cdot 10^3$
9	$-2.554 \cdot 10^3$
10	$-3.645 \cdot 10^3$
11	$-4.841 \cdot 10^3$
12	$-6.052 \cdot 10^3$
13	$-7.131 \cdot 10^3$
14	$-7.867 \cdot 10^3$
15	$-7.967 \cdot 10^3$
16	$-7.061 \cdot 10^3$

$$\sigma_{rmax} := \begin{pmatrix} \max(\sigma_r(x)_1) \\ \max(\sigma_r(x)_2) \\ \max(\sigma_r(x)_3) \\ \max(\sigma_r(x)_4) \\ \max(\sigma_r(x)_5) \\ \max(\sigma_r(x)_6) \\ \max(\sigma_r(x)_7) \\ \max(\sigma_r(x)_8) \end{pmatrix} \qquad \sigma_{rmin} := \begin{pmatrix} \min(\sigma_r(x)_1) \\ \min(\sigma_r(x)_2) \\ \min(\sigma_r(x)_3) \\ \min(\sigma_r(x)_4) \\ \min(\sigma_r(x)_5) \\ \min(\sigma_r(x)_6) \\ \min(\sigma_r(x)_7) \\ \min(\sigma_r(x)_8) \end{pmatrix}$$

$$\sigma_{rmax} = \begin{pmatrix} 16286.147 \\ 8458.521 \\ 900.392 \\ 0.000 \\ 40253.396 \\ 883.122 \\ 17169.269 \\ 23967.249 \end{pmatrix} \frac{lb}{in^2} \qquad \sigma_{rmin} = \begin{pmatrix} -7967.088 \\ -23084.127 \\ -6797.980 \\ 0.000 \\ -22980.892 \\ -7493.724 \\ -14857.408 \\ -15013.804 \end{pmatrix} \frac{lb}{in^2}$$



From Step 7

$$\sigma_r(X_a) = \begin{pmatrix} 16286.147 \\ -23084.127 \\ -6797.980 \\ 0.000 \\ 40253.396 \\ 883.122 \\ 17169.269 \\ 23967.249 \end{pmatrix} \frac{lb}{in^2} \qquad \sigma = \begin{pmatrix} -16286.147 \\ 23084.127 \\ 6797.980 \\ 0.000 \\ -40253.396 \\ -7493.724 \\ -17169.269 \\ -23967.249 \end{pmatrix} \frac{lb}{in^2}$$

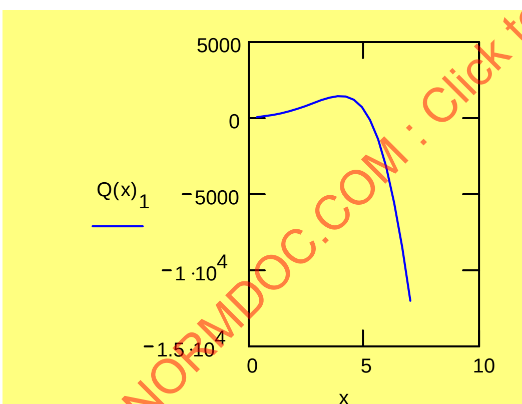
Shear Load Q(x) of the perforated tubesheet (0 <= x <= Xa)

$$Q_\alpha(x) := X_a \cdot \frac{ber' \cdot bei'_x(x) - bei' \cdot ber'_x(x)}{Z_a} \qquad Q_\beta(x) := \frac{\Psi_2 \cdot bei'_x(x) - \Psi_1 \cdot ber'_x(x)}{Z_a}$$

$$Q(x) := \begin{bmatrix} \frac{1}{a_o} \cdot \left[M_{a_1} \cdot Q_\alpha(x) + \left(a_o \cdot V_{a_1}\right) \cdot Q_\beta(x)\right] \\ \frac{1}{a_o} \cdot \left[M_{a_2} \cdot Q_\alpha(x) + \left(a_o \cdot V_{a_2}\right) \cdot Q_\beta(x)\right] \\ \frac{1}{a_o} \cdot \left[M_{a_3} \cdot Q_\alpha(x) + \left(a_o \cdot V_{a_3}\right) \cdot Q_\beta(x)\right] \\ \frac{1}{a_o} \cdot \left[M_{a_4} \cdot Q_\alpha(x) + \left(a_o \cdot V_{a_4}\right) \cdot Q_\beta(x)\right] \\ \frac{1}{a_o} \cdot \left[M_{a_5} \cdot Q_\alpha(x) + \left(a_o \cdot V_{a_5}\right) \cdot Q_\beta(x)\right] \\ \frac{1}{a_o} \cdot \left[M_{a_6} \cdot Q_\alpha(x) + \left(a_o \cdot V_{a_6}\right) \cdot Q_\beta(x)\right] \\ \frac{1}{a_o} \cdot \left[M_{a_7} \cdot Q_\alpha(x) + \left(a_o \cdot V_{a_7}\right) \cdot Q_\beta(x)\right] \\ \frac{1}{a_o} \cdot \left[M_{a_8} \cdot Q_\alpha(x) + \left(a_o \cdot V_{a_8}\right) \cdot Q_\beta(x)\right] \end{bmatrix}$$

$Q(x)_1 =$ (lb/in)

	1
1	4.925
2	10.531
3	17.452
4	26.218
5	37.187
6	50.451
7	65.731
8	82.232
9	98.496
10	112.232
11	120.166
12	117.926
13	100.008
14	59.874
15	-9.748
16	-116.288



$$Q_{rmin} := \begin{pmatrix} \min(Q(x)_1) \\ \min(Q(x)_2) \\ \min(Q(x)_3) \\ \min(Q(x)_4) \\ \min(Q(x)_5) \\ \min(Q(x)_6) \\ \min(Q(x)_7) \\ \min(Q(x)_8) \end{pmatrix} \qquad Q_{rmax} := \begin{pmatrix} \max(Q(x)_1) \\ \max(Q(x)_2) \\ \max(Q(x)_3) \\ \max(Q(x)_4) \\ \max(Q(x)_5) \\ \max(Q(x)_6) \\ \max(Q(x)_7) \\ \max(Q(x)_8) \end{pmatrix}$$

Limit Condition : $Q(X_a)=V_a$

$$Q(X_a) = \begin{pmatrix} -1000.036 \\ 1204.496 \\ 204.460 \\ 0.000 \\ -2692.753 \\ -488.221 \\ -1488.257 \\ -1692.717 \end{pmatrix} \frac{lb}{in} \qquad V_a = \begin{pmatrix} -1000.036 \\ 1204.496 \\ 204.460 \\ 0.000 \\ -2692.753 \\ -488.221 \\ -1488.257 \\ -1692.717 \end{pmatrix} \frac{lb}{in}$$

$$Q_{rmin} = \begin{pmatrix} -1000.036 \\ -128.010 \\ -13.919 \\ 0.000 \\ -2692.753 \\ -488.221 \\ -1488.257 \\ -1692.717 \end{pmatrix} \frac{lb}{in} \qquad Q_{rmax} = \begin{pmatrix} 120.166 \\ 1204.496 \\ 204.460 \\ 0.000 \\ 341.939 \\ 112.018 \\ 223.416 \\ 224.013 \end{pmatrix} \frac{lb}{in}$$

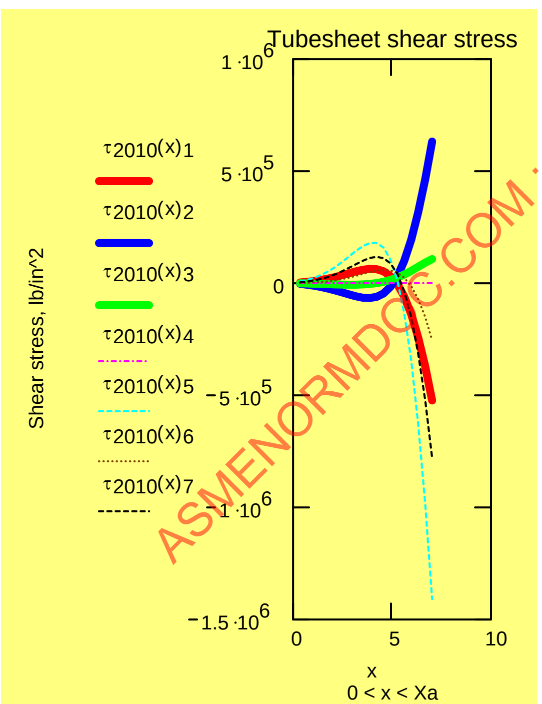
Shear Stress τ(x) of the perforated tubesheet (0 <= x <= Xa)

$$\tau_{2010}(x) := \begin{pmatrix} \frac{Q(x)_1}{\mu \cdot h} \\ \frac{Q(x)_2}{\mu \cdot h} \\ \frac{Q(x)_3}{\mu \cdot h} \\ \frac{Q(x)_4}{\mu \cdot h} \\ \frac{Q(x)_5}{\mu \cdot h} \\ \frac{Q(x)_6}{\mu \cdot h} \\ \frac{Q(x)_7}{\mu \cdot h} \\ \frac{Q(x)_8}{\mu \cdot h} \end{pmatrix}$$

$\tau_{2010}(x)_1 =$ (lb/in²)

	1
1	17.908
2	38.294
3	63.462
4	95.340
5	135.225
6	183.459
7	239.021
8	299.027
9	358.168
10	408.117
11	436.968
12	428.823
13	363.666
14	217.723
15	-35.448
16	-422.864

$$\tau_{max2010} := \begin{pmatrix} \max\left(\frac{Q(x)_1}{\mu \cdot h}\right) \\ \max\left(\frac{Q(x)_2}{\mu \cdot h}\right) \\ \max\left(\frac{Q(x)_3}{\mu \cdot h}\right) \\ \max\left(\frac{Q(x)_4}{\mu \cdot h}\right) \\ \max\left(\frac{Q(x)_5}{\mu \cdot h}\right) \\ \max\left(\frac{Q(x)_6}{\mu \cdot h}\right) \\ \max\left(\frac{Q(x)_7}{\mu \cdot h}\right) \\ \max\left(\frac{Q(x)_8}{\mu \cdot h}\right) \end{pmatrix} \qquad \tau_{min2010} := \begin{pmatrix} \min\left(\frac{Q(x)_1}{\mu \cdot h}\right) \\ \min\left(\frac{Q(x)_2}{\mu \cdot h}\right) \\ \min\left(\frac{Q(x)_3}{\mu \cdot h}\right) \\ \min\left(\frac{Q(x)_4}{\mu \cdot h}\right) \\ \min\left(\frac{Q(x)_5}{\mu \cdot h}\right) \\ \min\left(\frac{Q(x)_6}{\mu \cdot h}\right) \\ \min\left(\frac{Q(x)_7}{\mu \cdot h}\right) \\ \min\left(\frac{Q(x)_8}{\mu \cdot h}\right) \end{pmatrix}$$



$$\tau_{max2010} = \begin{pmatrix} 436.968 \\ 4379.985 \\ 743.490 \\ 0.000 \\ 1243.416 \\ 407.336 \\ 812.423 \\ 814.593 \end{pmatrix} \frac{lb}{in^2} \qquad \tau_{min2010} = \begin{pmatrix} -3636.495 \\ -465.492 \\ -50.615 \\ 0.000 \\ -9791.830 \\ -1775.350 \\ -5411.845 \\ -6155.335 \end{pmatrix} \frac{lb}{in^2}$$

$$\max\left(\overrightarrow{|\tau_{max2010}|}\right) = 4379.985 \frac{lb}{in^2} \qquad \max\left(\overrightarrow{|\tau_{min2010}|}\right) = 9791.830 \frac{lb}{in^2}$$

$$\tau_{max2010} := \max\left(\max\left(\overrightarrow{|\tau_{max2010}|}\right), \max\left(\overrightarrow{|\tau_{min2010}|}\right)\right) \qquad \tau_{max2010} = 9791.8 \frac{lb}{in^2}$$

$$\tau_{max} = 9558.2 \frac{lb}{in^2}$$

Tube Sress $\sigma_t(x)$ of the Tube-Bundle (0 <= x <= Xa)

$$\sigma_{tx}(x) := \begin{bmatrix} \frac{1}{x_t - x_s}\cdot\left[\left(P_{s_1}\cdot x_s - P_{t_1}\cdot x_t\right) - q(x)_1\right] \\ \frac{1}{x_t - x_s}\cdot\left[\left(P_{s_2}\cdot x_s - P_{t_2}\cdot x_t\right) - q(x)_2\right] \\ \frac{1}{x_t - x_s}\cdot\left[\left(P_{s_3}\cdot x_s - P_{t_3}\cdot x_t\right) - q(x)_3\right] \\ \frac{1}{x_t - x_s}\cdot\left[\left(P_{s_4}\cdot x_s - P_{t_4}\cdot x_t\right) - q(x)_4\right] \\ \frac{1}{x_t - x_s}\cdot\left[\left(P_{s_5}\cdot x_s - P_{t_5}\cdot x_t\right) - q(x)_5\right] \\ \frac{1}{x_t - x_s}\cdot\left[\left(P_{s_6}\cdot x_s - P_{t_6}\cdot x_t\right) - q(x)_6\right] \\ \frac{1}{x_t - x_s}\cdot\left[\left(P_{s_7}\cdot x_s - P_{t_7}\cdot x_t\right) - q(x)_7\right] \\ \frac{1}{x_t - x_s}\cdot\left[\left(P_{s_8}\cdot x_s - P_{t_8}\cdot x_t\right) - q(x)_8\right] \end{bmatrix}$$

$$\sigma_{txmin} := \begin{pmatrix} \min\left(\sigma_{tx}(x)_1\right) \\ \min\left(\sigma_{tx}(x)_2\right) \\ \min\left(\sigma_{tx}(x)_3\right) \\ \min\left(\sigma_{tx}(x)_4\right) \\ \min\left(\sigma_{tx}(x)_5\right) \\ \min\left(\sigma_{tx}(x)_6\right) \\ \min\left(\sigma_{tx}(x)_7\right) \\ \min\left(\sigma_{tx}(x)_8\right) \end{pmatrix} \qquad \sigma_{txmax} := \begin{pmatrix} \max\left(\sigma_{tx}(x)_1\right) \\ \max\left(\sigma_{tx}(x)_2\right) \\ \max\left(\sigma_{tx}(x)_3\right) \\ \max\left(\sigma_{tx}(x)_4\right) \\ \max\left(\sigma_{tx}(x)_5\right) \\ \max\left(\sigma_{tx}(x)_6\right) \\ \max\left(\sigma_{tx}(x)_7\right) \\ \max\left(\sigma_{tx}(x)_8\right) \end{pmatrix}$$

$$\sigma_{txmin} = \begin{pmatrix} -1288.776 \\ -2276.563 \\ -73.142 \\ 0.000 \\ -1743.547 \\ 1141.626 \\ -129.351 \\ -459.775 \end{pmatrix} \frac{lb}{in^2} \qquad \sigma_{txmax} = \begin{pmatrix} 2259.338 \\ 1633.647 \\ 360.212 \\ 0.000 \\ 8187.390 \\ 3651.489 \\ 5910.827 \\ 5928.052 \end{pmatrix} \frac{lb}{in^2}$$

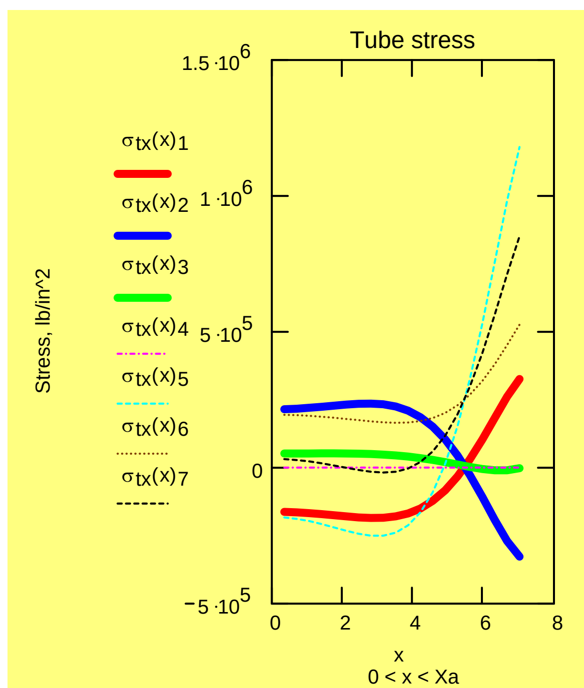
$$\sigma_{tx}(X_a) = \begin{pmatrix} 2259.338 \\ -2276.563 \\ -17.225 \\ 0.000 \\ 8187.390 \\ 3651.489 \\ 5910.827 \\ 5928.052 \end{pmatrix} \frac{lb}{in^2}$$

$$\sigma_{tx}(0) = \begin{pmatrix} -1127.96 \\ 1484.92 \\ 356.96 \\ 0 \\ -1263.84 \\ 1349.03 \\ 221.08 \\ -135.88 \end{pmatrix} \frac{lb}{in^2}$$

$$\sigma_{tx}(0)_1 = -1127.96\frac{lb}{in^2}$$

$\sigma_{tx}(x)_1 =$ (lb/in²)

	1
1	-1132.22
2	-1144.74
3	-1164.71
4	-1190.64
5	-1220.15
6	-1249.81
7	-1274.82
8	-1288.78
9	-1283.53
10	-1249.14
11	-1174.08
12	-1045.82
13	-851.85
14	-581.34
15	-227.56
16	208.92



Rigid Ring

$$R := \frac{A + 2 \cdot a_o}{2} \qquad R = 42.188\ \text{in}$$

Limit Condition : $\theta_R = \theta(X_a)$

$$\theta_R := \frac{12}{h^3 \cdot (\lambda_s + \lambda_c + E_D \cdot \ln(K))} \cdot a_o \cdot \left[-M_a + a_o \cdot V_a \cdot (\rho_s - 1) + (\omega^*{}_s \cdot P_s - \omega^*{}_c \cdot P_t) + \frac{\gamma_b}{2 \cdot \pi} \cdot W^* \right]$$

$$\theta_R = \begin{pmatrix} -0.00160 \\ 0.00122 \\ -0.00038 \\ 0.00000 \\ -0.00505 \\ -0.00223 \\ -0.00383 \\ -0.00345 \end{pmatrix} \qquad \theta(X_a) = \begin{pmatrix} -0.00160 \\ 0.00122 \\ -0.00038 \\ 0.00000 \\ -0.00505 \\ -0.00223 \\ -0.00383 \\ -0.00345 \end{pmatrix}$$

$$M_R := \frac{E_D \cdot h^3 \cdot \ln(K)}{12 \cdot R} \cdot \theta_R \qquad M_R = \begin{pmatrix} -9.647 \\ 7.339 \\ -2.307 \\ 0.000 \\ -30.417 \\ -13.431 \\ -23.078 \\ -20.770 \end{pmatrix} \text{lb}$$

Shell and Channel

Verification from σ_{sb}

$$\theta_s := \theta(X_a)$$

$$M_s := \left(1 + \frac{h'_s}{2}\right)\cdot k_s\cdot\theta_s + \beta_s\cdot k_s\cdot\delta_s\cdot P_s \qquad M_s = \begin{pmatrix} -642.531 \\ 1463.293 \\ 820.761 \\ 0.000 \\ -2025.958 \\ 79.866 \\ -562.665 \\ -1383.426 \end{pmatrix} \text{lb} \qquad \frac{t_{s1}^{\ 2}\cdot\sigma_{sb1}}{6} = \begin{pmatrix} -642.531 \\ 1463.293 \\ 820.761 \\ 0.000 \\ -2025.958 \\ 79.866 \\ -562.665 \\ -1383.426 \end{pmatrix} \text{lb}$$

Verification from σ_{cb}

$$\theta_c := -\theta(X_a)$$

$$M_c := \left(1 + \frac{h'_c}{2}\right)\cdot k_c\cdot\theta_c + \beta_c\cdot k_c\cdot\delta_c\cdot P_t \qquad M_c = \begin{pmatrix} 664.359 \\ -199.042 \\ 465.317 \\ 0.000 \\ 1227.637 \\ 364.236 \\ 1028.595 \\ 563.278 \end{pmatrix} \text{lb} \qquad \frac{t_c^{\ 2}\cdot\sigma_{cb}}{6} = \begin{pmatrix} 664.359 \\ -199.042 \\ 465.317 \\ 0.000 \\ 1227.637 \\ 364.236 \\ 1028.595 \\ 563.278 \end{pmatrix} \text{lb}$$

$$Q_s := -\left(1 + h'_s\right)\cdot\beta_s\cdot k_s\cdot\theta_s - 2\beta_s^{\ 2}\, k_s\cdot\delta_s\cdot P_s \qquad Q_c := -\left(1 + h'_c\right)\cdot\beta_c\cdot k_c\cdot\theta_c - 2\beta_c^{\ 2}\, k_c\cdot\delta_c\cdot P_t$$

$$Q_s = \begin{pmatrix} 287.279 \\ -942.614 \\ -655.335 \\ 0.000 \\ 905.817 \\ -324.077 \\ -36.797 \\ 618.538 \end{pmatrix} \text{lb in}^{-1} \qquad Q_c = \begin{pmatrix} -514.307 \\ 112.243 \\ -402.065 \\ 0.000 \\ -831.948 \\ -205.397 \\ -719.705 \\ -317.640 \end{pmatrix} \text{lb in}^{-1}$$

$$w_s := \frac{Q_s}{\beta_s^2 \cdot k_s} + \frac{M_s}{\beta_s \cdot k_s} + \delta_s \cdot P_s \qquad w_s = \begin{pmatrix} 0.0011 \\ -0.0008 \\ 0.0003 \\ 0.0000 \\ 0.0035 \\ 0.0015 \\ 0.0026 \\ 0.0024 \end{pmatrix} in$$

$$w_c := \frac{Q_c}{\beta_c^2 \cdot k_c} + \frac{M_c}{\beta_c \cdot k_c} + \delta_c \cdot P_t \qquad w_c = \begin{pmatrix} -0.0011 \\ 0.0008 \\ -0.0003 \\ 0.0000 \\ -0.0035 \\ -0.0015 \\ -0.0026 \\ -0.0024 \end{pmatrix} in$$

Limit Condition for SHELL : w_s=-(h/2)θ_s

$$\theta_s := \frac{Q_s}{\beta_s \cdot k_s} + \frac{2M_s}{k_s} \qquad \theta_s := \theta(X_a) \qquad ww_s := \frac{-h}{2} \cdot \theta_s$$

$$\theta_s = \begin{pmatrix} -0.0016 \\ 0.0012 \\ -0.0004 \\ 0.0000 \\ -0.0050 \\ -0.0022 \\ -0.0038 \\ -0.0034 \end{pmatrix} \quad \theta_s = \begin{pmatrix} -0.0016 \\ 0.0012 \\ -0.0004 \\ 0.0000 \\ -0.0050 \\ -0.0022 \\ -0.0038 \\ -0.0034 \end{pmatrix} \quad ww_s = \begin{pmatrix} 0.0011 \\ -0.0008 \\ 0.0003 \\ 0.0000 \\ 0.0035 \\ 0.0015 \\ 0.0026 \\ 0.0024 \end{pmatrix} in$$

Limit Condition for CHANNEL:w_c=-(h/2)θ_c

$$\theta_c := \frac{Q_c}{\beta_c \cdot k_c} + \frac{2M_c}{k_c} \qquad \theta_c := -\theta(X_a) \qquad ww_c := \frac{-h}{2} \cdot \theta_c$$

$$\theta_c = \begin{pmatrix} 0.0016 \\ -0.0012 \\ 0.0004 \\ 0.0000 \\ 0.0050 \\ 0.0022 \\ 0.0038 \\ 0.0034 \end{pmatrix} \quad \theta_c = \begin{pmatrix} 0.0016 \\ -0.0012 \\ 0.0004 \\ 0.0000 \\ 0.0050 \\ 0.0022 \\ 0.0038 \\ 0.0034 \end{pmatrix} \quad ww_c = \begin{pmatrix} -0.0011 \\ 0.0008 \\ -0.0003 \\ 0.0000 \\ -0.0035 \\ -0.0015 \\ -0.0026 \\ -0.0024 \end{pmatrix} in$$

Verification from σ_{sb}

$$\sigma\sigma_{sb} := \frac{6 \cdot M_s}{t_{s1}^2} \qquad \sigma\sigma_{sb} = \begin{pmatrix} -12184.300 \\ 27748.366 \\ 15564.066 \\ 0.000 \\ -38418.161 \\ 1514.505 \\ -10669.795 \\ -26233.861 \end{pmatrix} lb\,in^{-2}$$

$$\sigma_{sb1} = \begin{pmatrix} -12184.300 \\ 27748.366 \\ 15564.066 \\ 0.000 \\ -38418.161 \\ 1514.505 \\ -10669.795 \\ -26233.861 \end{pmatrix} \frac{lb}{in^2} \qquad \sigma_{sm} = \begin{pmatrix} 1830.579 \\ 2287.188 \\ 4117.767 \\ 0.000 \\ -1085.899 \\ -629.290 \\ 1201.289 \\ -2916.478 \end{pmatrix} \frac{lb}{in^2}$$

Verification from σ_{cb}

$$\sigma\sigma_{cb} := 6 \cdot \frac{M_c}{t_c^2} \qquad \sigma\sigma_{cb} = \begin{pmatrix} 28345.980 \\ -8492.472 \\ 19853.508 \\ 0.000 \\ 52379.190 \\ 15540.738 \\ 43886.718 \\ 24033.210 \end{pmatrix} lb\,in^{-2}$$

$$\sigma_{cb} = \begin{pmatrix} 28345.980 \\ -8492.472 \\ 19853.508 \\ 0.000 \\ 52379.190 \\ 15540.738 \\ 43886.718 \\ 24033.210 \end{pmatrix} \frac{lb}{in^2}$$

Axial force in the SHELL

Calculation from direct formula

$$V_s := \frac{a_o \cdot V_a + \frac{a_o^2}{2} \cdot P_t + \frac{a_s^2 - a_o^2}{2} \cdot P_s}{\frac{D_s + t_{s1}}{2}}$$

$$V_s = \begin{pmatrix} 1029.701 \\ 1286.543 \\ 2316.244 \\ 0.000 \\ -610.818 \\ -353.975 \\ 675.725 \\ -1640.519 \end{pmatrix} \frac{lb}{in}$$

$$KK := \frac{J \cdot K_{st}}{1 + J \cdot K_{st} \cdot \left[Q_{z1} + (\rho_s - 1) \cdot Q_{z2} \right]}$$

Verification from σ_{sm}

$$V_{\sigma sm} := \sigma_{sm} \cdot t_{s1}$$

$$V_{\sigma sm} = \begin{pmatrix} 1029.701 \\ 1286.543 \\ 2316.244 \\ 0.000 \\ -610.818 \\ -353.975 \\ 675.725 \\ -1640.519 \end{pmatrix} \frac{lb}{in}$$

V_s due to pressures P_s , P_t and thermal expansion γ

$$V_{s1} := \frac{a_o^2}{D_s + t_{s1}} \cdot \left[KK \cdot \left[\left[x_s + 2 \cdot (1 - x_s) \cdot \nu_t - \frac{\rho_s^2 - 1}{J \cdot K_{st}} \right] \cdot P_s - P'_t + P_\gamma + P_\omega + P_W + P_{rim} \right] + P_t + (\rho_s^2 - 1) P_s \right]$$

V_s due to Poisson's ratio ν_s of the shell

$$V_{s2} := \frac{a_o^2}{D_s + t_{s1}} \cdot KK \cdot \left[\frac{2}{K_{st}} \cdot \left(\frac{D_s}{D_o} \right)^2 \cdot \nu_s \right] \cdot P_s$$

V_s due to pressure P_s acting on Bellows

$$V_{s3} := \frac{a_o^2}{D_s + t_{s1}} \cdot KK \cdot \frac{1 - J}{2 \cdot J \cdot K_{st}} \cdot \frac{D_J^2 - D_s^2}{D_o^2} \cdot P_s$$

V_s TOTAL

$$V_{sT} := V_{s1} + V_{s2} + V_{s3}$$

$$V_{s1} = \begin{pmatrix} 1029.701 \\ 631.846 \\ 1661.547 \\ 0.000 \\ -610.818 \\ -1008.673 \\ 21.028 \\ -1640.519 \end{pmatrix} \frac{lb}{in} \quad V_{s2} = \begin{pmatrix} 0.000 \\ 654.698 \\ 654.698 \\ 0.000 \\ 0.000 \\ 654.698 \\ 654.698 \\ 0.000 \end{pmatrix} \frac{lb}{in} \quad V_{s3} = \begin{pmatrix} 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.000 \end{pmatrix} \frac{lb}{in} \quad V_{sT} = \begin{pmatrix} 1029.701 \\ 1286.543 \\ 2316.244 \\ 0.000 \\ -610.818 \\ -353.975 \\ 675.725 \\ -1640.519 \end{pmatrix} \frac{lb}{in}$$

$$\sigma_{sm} = \begin{pmatrix} 1830.579 \\ 2287.188 \\ 4117.767 \\ 0.000 \\ -1085.899 \\ -629.290 \\ 1201.289 \\ -2916.478 \end{pmatrix} \frac{lb}{in^2}$$

Verification σ_{sm}

$$\sigma_{sm} := \frac{V_{sT}}{t_s} \qquad \sigma_{sm} = \begin{pmatrix} 1830.579 \\ 2287.188 \\ 4117.767 \\ 0.000 \\ -1085.899 \\ -629.290 \\ 1201.289 \\ -2916.478 \end{pmatrix} \frac{lb}{in^2}$$

SHELL Axial displacement

a) Total Axial displacement Δ_{sTotal} of the shell of length L

$$T_{sm} := \begin{pmatrix} T_a \\ T_a \\ T_a \\ T_a \\ T_{sm.} \\ T_{sm.} \\ T_{sm.} \\ T_{sm.} \end{pmatrix} \qquad \theta_s := T_{sm} - T_a \qquad \Delta_{sTotal} := \frac{L}{J \cdot E_{sD} \cdot t_s} \cdot V_s - \nu_s \cdot \frac{L \cdot D_s^2}{2E_{sD} \cdot t_s \cdot (D_s + t_s)} \cdot P_s + \frac{\pi}{8} \cdot \frac{D_J^2 - D_s^2}{K_J} \cdot P_s + L \cdot \alpha_{sm} \cdot \theta_s$$

$$\theta_s = \begin{pmatrix} 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \\ 81.0 \\ 81.0 \\ 81.0 \\ 81.0 \end{pmatrix} \text{degF} \qquad \Delta_{sTotal} = \begin{pmatrix} 0.0165 \\ -0.0117 \\ 0.0047 \\ 0.0000 \\ 0.1594 \\ 0.1312 \\ 0.1477 \\ 0.1429 \end{pmatrix} \text{in}$$

b)Total Axial displacement of the half-shell of length l=L/2 : $\delta_{sTotal}=\delta_s(V_s)+\delta_s(\theta_s)+\delta_s(\nu_s)+\delta_s(J)$ $\frac{L}{2}$ $l = 118.625$ in

δ_s due to axial load V_s	δ_s due to θ_s	δ_s due to Poisson's ratio ν_s	δ_s due to pressure P_s acting on Bellows J
$\delta_{sVs} := \frac{l}{J \cdot E_{sD} \cdot t_s} \cdot V_s$	$\delta_{s\theta s} := l \cdot \alpha_{sm} \cdot \theta_s$	$\delta_{s\nu s} := -\nu_s \cdot \frac{l \cdot D_s^2}{2E_{sD} \cdot t_s \cdot (D_s + t_s)} \cdot P_s$	$\delta_{sJ} := \frac{\pi}{8} \cdot \frac{D_J^2 - D_s^2}{2K_J} \cdot P_s$

$$\delta_{sVs} = \begin{pmatrix} 0.008225 \\ 0.010277 \\ 0.018503 \\ 0.000000 \\ -0.004879 \\ -0.002828 \\ 0.005398 \\ -0.013105 \end{pmatrix} \text{in} \qquad \delta_{s\theta s} = \begin{pmatrix} 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0846 \\ 0.0846 \\ 0.0846 \\ 0.0846 \end{pmatrix} \text{in} \qquad \delta_{s\nu s} = \begin{pmatrix} 0.0000 \\ -0.0161 \\ -0.0161 \\ 0.0000 \\ 0.0000 \\ -0.0161 \\ -0.0161 \\ 0.0000 \end{pmatrix} \text{in} \qquad \delta_{sJ} = \begin{pmatrix} 0.0000 \\ -0.0000 \\ -0.0000 \\ 0.0000 \\ 0.0000 \\ -0.0000 \\ -0.0000 \\ 0.0000 \end{pmatrix} \text{in}$$

TOTAL DISPLACEMENT δ_{sTotal} OF HALF SHELL

$$\delta_{sTotal} := \delta_{sVs} + \delta_{s\theta s} + \delta_{s\nu s} + \delta_{sJ} \qquad \delta_{sTotal} = \begin{pmatrix} 0.0082 \\ -0.0059 \\ 0.0024 \\ 0.0000 \\ 0.0797 \\ 0.0656 \\ 0.0738 \\ 0.0715 \end{pmatrix} \text{in} \qquad \frac{\Delta_{sTotal}}{2} = \begin{pmatrix} 0.008 \\ -0.006 \\ 0.002 \\ 0.000 \\ 0.080 \\ 0.066 \\ 0.074 \\ 0.071 \end{pmatrix} \text{in}$$

Axial displacement of the HALF TUBES of length l=L/2

$$T_{tm} := \begin{pmatrix} T_a \\ T_a \\ T_a \\ T_a \\ T_{tm.} \\ T_{tm.} \\ T_{tm.} \\ T_{tm.} \end{pmatrix} \quad T_{tm} = \begin{pmatrix} 70.0 \\ 70.0 \\ 70.0 \\ 70.0 \\ 113.0 \\ 113.0 \\ 113.0 \\ 113.0 \end{pmatrix} degF \quad \theta_t := T_{tm} - T_a \quad \theta_t = \begin{pmatrix} 0.0 \\ 0.0 \\ 0.0 \\ 0.0 \\ 43.0 \\ 43.0 \\ 43.0 \\ 43.0 \end{pmatrix} degF$$

$$l := \frac{L}{2} \quad l = 118.625\,in$$

$$AA := \frac{\pi a_o^2}{N_t} \quad AA = 1.399\,in^2$$

$$s_t := \pi t_t \cdot (d_t - t_t) \quad s_t = 0.1464\,in^2$$

$$V_t(x) := \begin{bmatrix} AA\cdot\left[\left(P_{s_1}\cdot x_s - P_{t_1}\cdot x_t\right) - q(x)_1\right] \\ AA\cdot\left[\left(P_{s_2}\cdot x_s - P_{t_2}\cdot x_t\right) - q(x)_2\right] \\ AA\cdot\left[\left(P_{s_3}\cdot x_s - P_{t_3}\cdot x_t\right) - q(x)_3\right] \\ AA\cdot\left[\left(P_{s_4}\cdot x_s - P_{t_4}\cdot x_t\right) - q(x)_4\right] \\ AA\cdot\left[\left(P_{s_5}\cdot x_s - P_{t_5}\cdot x_t\right) - q(x)_5\right] \\ AA\cdot\left[\left(P_{s_6}\cdot x_s - P_{t_6}\cdot x_t\right) - q(x)_6\right] \\ AA\cdot\left[\left(P_{s_7}\cdot x_s - P_{t_7}\cdot x_t\right) - q(x)_7\right] \\ AA\cdot\left[\left(P_{s_8}\cdot x_s - P_{t_8}\cdot x_t\right) - q(x)_8\right] \end{bmatrix} \quad V_t(0) = \begin{pmatrix} -165.1272 \\ 217.3845 \\ 52.2574 \\ 0.0000 \\ -185.0198 \\ 197.4919 \\ 32.3647 \\ -19.8927 \end{pmatrix} lb$$

$$\delta_{tVt}(x) := \frac{l}{E_{tD}\cdot s_t}\cdot V_t(x) \quad \delta_{tVt}(0) = \begin{pmatrix} -0.004956 \\ 0.006524 \\ 0.001568 \\ 0.000000 \\ -0.005553 \\ 0.005927 \\ 0.000971 \\ -0.000597 \end{pmatrix} in$$

From Page 33 :

$$\sigma_{tx}(0)_1 = -1127.96\frac{lb}{in^2}$$

$$\sigma_t(x) := \frac{\delta_{tVt}(x)}{l}\cdot E_{tD} \quad \sigma_t(0) = \begin{pmatrix} -1127.96 \\ 1484.92 \\ 356.96 \\ 0.00 \\ -1263.84 \\ 1349.03 \\ 221.08 \\ -135.88 \end{pmatrix} \frac{lb}{in^2}$$

Total Axial displacement of the half-tubes of length l=L/2 at TS center (x=0) : $\delta_{tTotal}=\delta_t(V_t) + \delta_t(\theta_t) + \delta_t(\nu_t)$

δ_t due to axial load V_t

$$\delta_{tVt} := \delta_{tVt}(0)$$

δ_t due to θ_t

$$\delta_{t\theta t} := l\cdot\alpha_{tm}\cdot\theta_t$$

δ_t due to Poisson's ratio ν_t

$$\delta_{tvt} := \frac{-2}{k_w}\left[\left(1 - x_t\right)\cdot P_t - \left(1 - x_s\right)\cdot P_s\right]\nu_t$$

$$\delta_{tTotal} := \delta_{tVt} + \delta_{t\theta t} + \delta_{tvt}$$

$$\delta_{tVt} = \begin{pmatrix} -0.00496 \\ 0.00652 \\ 0.00157 \\ 0.00000 \\ -0.00555 \\ 0.00593 \\ 0.00097 \\ -0.00060 \end{pmatrix} in \quad \delta_{t\theta t} = \begin{pmatrix} 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0441 \\ 0.0441 \\ 0.0441 \\ 0.0441 \end{pmatrix} in \quad \delta_{tvt} = \begin{pmatrix} -0.0023 \\ 0.0046 \\ 0.0023 \\ 0.0000 \\ -0.0023 \\ 0.0046 \\ 0.0023 \\ 0.0000 \end{pmatrix} in \quad \delta_{tTotal} = \begin{pmatrix} -0.0073 \\ 0.0111 \\ 0.0039 \\ 0.0000 \\ 0.0363 \\ 0.0547 \\ 0.0474 \\ 0.0435 \end{pmatrix} in$$