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Optics and optical instruments — Field procedures for testing geodetic and surveying instruments —

Part 5: **Electronic tacheometers**

Optique et instruments d'optique — Méthodes d'essai sur site des instruments géodésiques et d'observation —

Partie 5: Tachéomètres électroniques

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Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

International Standards are drafted in accordance with the rules given in the ISO/IEC Directives, Part 2.

The main task of technical committees is to prepare International Standards. Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

Attention is drawn to the possibility that some of the elements of this document may be the subject of patent rights. ISO shall not be held responsible for identifying any or all such patent rights.

ISO 17123-5 was prepared by Technical Committee ISO/TC 172, *Optics and photonics*, Subcommittee SC 6, *Geodetic and surveying instruments*.

ISO 17123 consists of the following parts, under the general title *Optics and optical instruments — Field procedures for testing geodetic and surveying instruments*:

— *Part 1: Theory*

— *Part 2: Levels*

— *Part 3: Theodolites*

— *Part 4: Electro-optical distance meters (EDM instruments)*

— *Part 5: Electronic tacheometers*

— *Part 6: Rotating lasers*

— *Part 7: Optical plumbing instruments*

Optics and optical instruments — Field procedures for testing geodetic and surveying instruments —

Part 5:
Electronic tacheometers

1 Scope

This part of ISO 17123 specifies field procedures to be adopted when determining and evaluating the precision (repeatability) of electronic tacheometers (total stations) and their ancillary equipment when used in building and surveying measurements. Primarily, these tests are intended to be field verifications of the suitability of a particular instrument for the immediate task at hand and to satisfy the requirements of other standards. They are not proposed as tests for acceptance or performance evaluations that are more comprehensive in nature.

This part of ISO 17123 can be thought of as one of the first steps in the process of evaluating the uncertainty of measurements (more specifically of measurands). The uncertainty of a result of a measurement is dependent on a number of factors. These include among others: repeatability, reproducibility (between-day repeatability) and a thorough assessment of all possible error sources, as prescribed by the ISO *Guide to the expression of uncertainty in measurement (GUM)*.

These field procedures have been developed specifically for *in situ* applications without the need for special ancillary equipment and are purposely designed to minimize atmospheric influences.

2 Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 3534-1, *Statistics — Vocabulary and symbols — Part 1: Probability and general statistical terms*

ISO 4463-1, *Measurement methods for building — Setting-out and measurement — Part 1: Planning and organization, measuring procedures, acceptance criteria*

ISO 7077, *Measuring methods for building — General principles and procedures for the verification of dimensional compliance*

ISO 7078, *Building construction — Procedures for setting out, measurement and surveying — Vocabulary and guidance notes*

ISO 9849, *Optics and optical instruments — Geodetic and surveying instruments — Vocabulary*

ISO 17123-1, *Optics and optical instruments — Field procedures for testing geodetic and surveying instruments — Part 1: Theory*

Guide to the expression of uncertainty in measurement (GUM), BIPM, IEC, IFCC, ISO, IUPAC, IUPAP, OIML, 1993, corrected and reprinted in 1995

International vocabulary of basic and general terms in metrology (VIM). BIPM, IEC, IFCC, ISO, IUPAC, IUPAP, OIML, 2nd ed., 1993

3 Terms and definitions

For the purpose of this document, the terms and definitions given in ISO 3534-1, ISO 4463-1, ISO 7077, ISO 7078, ISO 9849, ISO 17123-1, the *GUM* and the *VIM* apply.

4 Requirements

Before commencing surveying, it is important that the operator ensure that the precision in use of the measuring equipment is appropriate for the intended measuring task.

The electronic tacheometer and its ancillary equipment shall be in known and acceptable states of permanent adjustment according to the methods specified in the manufacturer's handbook, and used with tripods and reflectors as recommended by the manufacturer.

The coordinates are considered as observables because on modern electronic tacheometers they are selectable as output quantities.

The results of these tests are influenced by meteorological conditions, especially by the gradient of temperature. An overcast sky and low wind speed guarantee the most favourable weather conditions. Actual meteorological data shall be measured in order to derive atmospheric corrections, which shall be added to the raw distances. The particular conditions to be taken into account may vary depending on where the tasks are to be undertaken. These conditions shall include variations in air temperature, wind speed, cloud cover and visibility. Note should also be taken of the actual weather conditions at the time of measurement and the type of surface above which the measurements are made. The conditions chosen for the tests should match those expected when the intended measuring task is actually carried out (see ISO 7077 and ISO 7078).

Tests performed in laboratories would provide results which are almost unaffected by atmospheric influences, but the costs for such tests are very high, and therefore they are not practicable for most users. In addition, laboratory tests yield precisions much higher than those that can be obtained under field conditions.

This part of ISO 17123 describes two different field procedures as given in the Clauses 6 and 7. The operator shall choose the procedure which is most relevant to the project's particular requirements.

5 Test principle

5.1 Procedure 1: Simplified test procedure

The simplified test procedure provides an estimate as to whether the precision of a given electronic tacheometer equipment is within the specified permitted deviation in accordance with ISO 4463-1.

The simplified test procedure is based on a limited number of measurements. This test procedure relies on measurements of x-, y- and z-coordinates in a test field without nominal values. Due to the influence of atmospheric refraction, the precision of the x- and y-coordinates is not equal to the precision of the z-coordinates. Therefore the precision is calculated separately. The maximum difference is calculated as an indicator for the precision.

A significant standard deviation cannot be obtained. If a more precise assessment of the electronic tacheometer under field conditions is required, it is recommended to adopt the more rigorous full test procedure as given in Clause 7.

5.2 Procedure 2: Full test procedure

The full test procedure shall be adopted to determine the best achievable measure of precision of an electronic tacheometer and its ancillary equipment under field conditions.

This procedure is based on measurements of coordinates in a test field without nominal values. The experimental standard deviation of the coordinate measurement of a single point is determined from least squares adjustments.

When setting up the tacheometer for different series of measurements, special care shall be taken when centring above the ground point. Achievable accuracies of centring expressed in terms of standard deviations are the following:

— plumb bob: 1 mm to 2 mm (worse in windy weather);

— optical or laser plummet: $\leqslant$ 1 mm (the adjustment shall be checked according to the manufacturer's handbook);

— centring rod: 1 mm.

Therefore it is recommended to use forced centring interchange for the test procedures

NOTE With targets at 100 m distance, a miscentring of 2 mm could affect the observed direction by up to 4″ (1,3 mgon). The shorter the distance, the greater the effect.

The full test procedure given in Clause 7 of this part of ISO 17123 is intended for determining the measure of precision in use of a particular electronic tacheometer. This measure of precision in use is expressed in terms of the experimental standard deviations of a coordinate measured once in both face positions of the telescope:

$s_{\text{ISO-TACH-XY}}$, $s_{\text{ISO-TACH-Z}}$

Furthermore, this procedure may be used to determine

— the measure of precision in use of electronic tacheometers by a single survey team with a single instrument and its ancillary equipment at a given time;

— the measure of precision in use of a single instrument over time;

— the measure of precision in use of each of several electronic tacheometers in order to enable a comparison of their respective achievable precisions to be obtained under similar field conditions.

Statistical tests should be applied to determine whether the experimental standard deviations obtained belong to the population of the instrumentation's theoretical standard deviations, σ, and whether two tested samples belong to the same population.

6 Simplified test procedure

6.1 Configuration of the test field

Three instrument stations, S_j (j = 1, 2, 3), shall be set out at the corner points of a triangle (see Figure 1). The side lengths should be chosen according to the intended measuring task (e.g. 100 m to 200 m). The heights, z_j, should be as different as the surface of the ground allows.

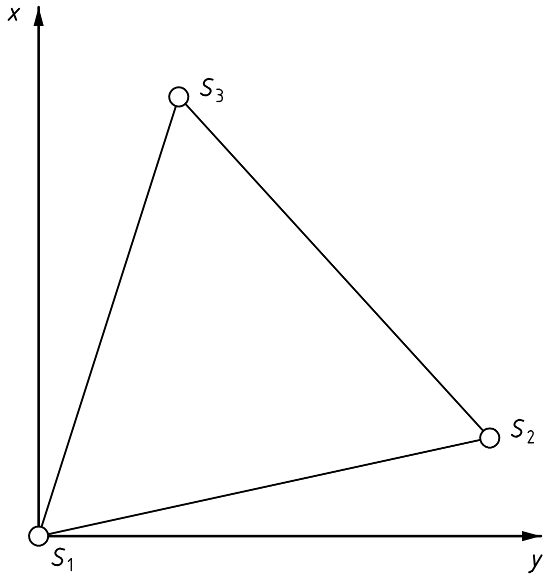

Figure 1 — Configuration of the test field

6.1.1 Measurement

Before commencing the measurements, the instrument shall be adjusted as specified by the manufacturer. All coordinates shall be measured on the same day. The air temperature and the air pressure shall be measured at each instrument station to derive the atmospheric corrections for distance measurements (input of the right value to a factor of 10^{-6}). The distances shall be corrected by a factor of 10^{-6} for any deviation of 1 °C in temperature and/or for any deviation of 3 hPa (3 mbar) in air pressure. The correct zero-point correction according to the reflector prism shall be used.

An arbitrary local coordinate system, (x, y, z), shall be established by assigning to the instrument station, S_1, the coordinates (e.g. 1 000, 2 000, 300). The zero-reading of the horizontal circle defines the x-axis.

From each instrument station, S_j (j = 1, 2, 3), the coordinates of the other two points (target points) in the local coordinates system shall be measured. The results of the measurements from instrument station S_1 shall be used as instrument station coordinates for S_2 and S_3 respectively for the subsequent measurements. Only one backsight (to S_1) shall be used for orientation.

On-board or stand-alone software shall be used for orientation. It is preferable to use the same software which will be used for the practical work. All observations shall be made in one face position of the telescope.

Table 1 provides an observation scheme for the field measurements.

Table 1 — Observation scheme for the simplified test procedure

Target point	x-coordinate (station-, running number) m	y-coordinate (station-, running number) m	z-coordinate (station-, running number) m
Instrument station S_1	Coordinates: (1 000, 2 000, 300) (take into account instrument and reflector height) Orientation: arbitrary		
S_2	$x_{2,1}$	$y_{2,1}$	$z_{2,1}$
S_3	$x_{3,1}$	$y_{3,1}$	$z_{3,1}$
Instrument station S_2	Coordinates: $(x_{2,1}, y_{2,1}\ z_{2,1})$ (take into account instrument and reflector height) Orientation: backsight to S_1 (1 000, 2 000, 300)		
S_3	$x_{3,2}$	$y_{3,2}$	$z_{3,2}$
S_1	$x_{1,1}$	$y_{1,1}$	$z_{1,1}$
Instrument station S_3	Coordinates: $(x_{3,1}, y_{3,1}\ z_{3,1})$ (take into account instrument and reflector height) Orientation: backsight to (1 000, 2 000, 300)		
S_1	$x_{1,2}$	$y_{1,2}$	$z_{1,2}$
S_2	$x_{2,2}$	$y_{2,2}$	$z_{2,2}$

S_j is the instrument station or the target point j (j = 1, 2, 3)

$x_{j,k}$ is the kth measurement (k = 1, 2) of the x-coordinate of point j (j = 1, 2, 3)

$y_{j,k}$ is the kth measurement (k = 1, 2) of the y-coordinate of point j (j = 1, 2, 3)

$z_{j,k}$ is the kth measurement (k = 1, 2) of the z-coordinate of point j (j = 1, 2, 3)

6.1.2 Calculation

The coordinate differences are calculated as follows:

$$
\begin{aligned}
d_1 &= x_{1,1} - x_{1,2} \\
d_2 &= x_{2,1} - x_{2,2} \\
d_3 &= x_{3,1} - x_{3,2} \\
d_4 &= y_{1,1} - y_{1,2} \\
d_5 &= y_{2,1} - y_{2,2} \\
d_6 &= y_{3,1} - y_{3,2} \\
d_7 &= z_{1,1} - z_{1,2} \\
d_8 &= z_{2,1} - z_{2,2} \\
d_9 &= z_{3,1} - z_{3,2}
\end{aligned}
\tag{1}
$$

and the half difference of the maximum differences

$$d_{x,y} = \frac{1}{2} \max_{i=1, \ldots, 6} |d_i| \tag{2}$$

and

$$d_z = \frac{1}{2} \max_{i=7, 8, 9} |d_i| \tag{3}$$

The half differences, $d_{x,\,y}$ and d_z, shall be within the specified permitted deviation, $\pm\, p_{x,\,y}$ and $\pm\, p_z$ respectively, (in accordance with ISO 4463-1) for the intended measuring task. If $\pm\, p_{x,\,y}$ and $\pm\, p_z$ are not given, the half differences shall be $d_{x,\,y} \leqslant 2{,}5 \times s_{\text{ISO-TACH-XY}}$ and $d_z \leqslant 2{,}5 \times s_{\text{ISO-TACH-Z}}$ respectively, where $s_{\text{ISO-TACH-XY}}$ and $s_{\text{ISO-TACH-Z}}$ are the experimental standard deviations of the x, y- and z-measurements respectively, determined according to the full test procedure with the same instrument.

If the half differences, $d_{x,\,y}$ and d_z respectively, are too large for the intended measuring task, it is necessary to make further investigations in order to identify the main sources of the deviations.

7 Full test procedure

7.1 Configuration of the test field

Three tripods, each having a forced centring device, S_j (j = 1, 2, 3), shall be set out at the corner points of a triangle (see Figure 1). The side lengths should be chosen according to the intended measuring task (e.g. 100 m to 200 m). The heights, z_j, should be as different as the surface of the ground allows.

7.2 Measurements

Before commencing the measurements, the instrument shall be adjusted as specified by the manufacturer. All coordinates shall be measured on the same day. Forced centring interchange shall be used to eliminate centring uncertainties.

Three series of measurements (m = 3, for i = 1,…, m) shall be carried out, each of which requires the instrument to be put on one of the n = 3 tripods over point S_j (set j) of the test triangle in a fixed order, e.g. $S_1 \rightarrow S_2 \rightarrow S_3 \rightarrow S_1 \rightarrow S_2$.... The instrument should always be levelled carefully. No procedure of orientation for the coordinate system of the instrument such as "free positioning with scale adaptation" shall be used. The air temperature and pressure should be measured and the values used frequently to correct the electro-optic distance measurements to insure that reliable atmospheric corrections are applied. The coordinates (x_j, y_j, z_j) for each instrument setup shall always be set to zero (0, 0, 0).

The coordinates of the reflectors on the two other points, S_k (k = 1, 2, 3), of the triangle shall be measured in both face positions of the telescope

$$x_{i,j,k,\text{I}},\ y_{i,j,k,\text{I}},\ z_{i,j,k,\text{I}},\ x_{i,j,k,\text{II}},\ y_{i,j,k,\text{II}},\ z_{i,j,k,\text{II}};\ i = 1, 2, 3;\ j = 1, 2, 3;\ k = 1, 2, 3$$

For measuring the coordinate differences z between the reference points of the forced centring device, the difference δ between the instrument height and the target height shall be taken into account. As the exact value of the difference will be an unknown parameter of the adjustment (see 7.3.2), δ shall have the same value in all measurements. Therefore it is necessary to take the same prism or two prisms of the same type.

For easy error-free calculations, it is necessary to follow the measuring sequence given in Table 2.

Table 2 — Sequence of the measurements

Points	i	j	k	Points	i	j	k	Points	i	j	k
$S_1 \rightarrow S_2$	1	1	2	$S_1 \rightarrow S_2$	2	1	2	$S_1 \rightarrow S_2$	3	1	2
$S_1 \rightarrow S_3$	1	1	3	$S_1 \rightarrow S_3$	2	1	3	$S_1 \rightarrow S_3$	3	1	3
$S_2 \rightarrow S_1$	1	2	1	$S_2 \rightarrow S_1$	2	2	1	$S_2 \rightarrow S_1$	3	2	1
$S_2 \rightarrow S_3$	1	2	3	$S_2 \rightarrow S_3$	2	2	3	$S_2 \rightarrow S_3$	3	2	3
$S_3 \rightarrow S_1$	1	3	1	$S_3 \rightarrow S_1$	2	3	1	$S_3 \rightarrow S_1$	3	3	1
$S_3 \rightarrow S_2$	1	3	2	$S_3 \rightarrow S_2$	2	3	2	$S_3 \rightarrow S_2$	3	3	2

The mean values of the readings in both face positions I and II of the telescope are noted as the quasi-observations:

$$
\begin{aligned}
x_{i,j,k} &= \frac{1}{2}\left(x_{i,j,k,\mathrm{I}} + x_{i,j,k,\mathrm{II}}\right) \\
y_{i,j,k} &= \frac{1}{2}\left(y_{i,j,k,\mathrm{I}} + y_{i,j,k,\mathrm{II}}\right) \\
z_{i,j,k} &= \frac{1}{2}\left(z_{i,j,k,\mathrm{I}} + z_{i,j,k,\mathrm{II}}\right) \\
i &= 1, 2, 3;\ \ j = 1, 2, 3;\ \ k = 1, 2, 3
\end{aligned}
\tag{4}
$$

7.3 Calculation

7.3.1 Precision of the x-, y-coordinates

In order to obtain comparable results of the three series of measurements, it is necessary to transform each series to the same position, e.g. the first set of the first series.

Since point S_1 shall get the situation coordinates (0, 0), a translation of each set has to be performed:

$$
\begin{aligned}
x'_{i,j,k} &= x_{i,j,k} - x_{i,j,1} \\
y'_{i,j,k} &= y_{i,j,k} - y_{i,j,1} \\
i &= 1, 2, 3;\ \ j = 1, 2, 3;\ \ k = 1, 2, 3
\end{aligned}
\tag{5}
$$

For the first set of measurements, ($i = 1, j = 1$), no rotation is necessary.

Thus, the transformed coordinates for the rotation of the two corner points, S_2 and S_3, of the test triangle are obtained directly as the translated coordinates of set $j = 1$ of series $i = 1$:

$$
\begin{aligned}
x''_{1,1,k} &= x'_{1,1,k} \\
y''_{1,1,k} &= y'_{1,1,k} \\
k &= 2, 3
\end{aligned}
$$

For each of the following sets, $j = 1, 2, 3$ of the series $i = 1, 2, 3$, a rotation $\varphi_{i,j}$ with the centre, S_1, is necessary.

The most feasible way to rotate is in polar coordinates. For each target, $k = 2, 3$, the rectangular coordinates are transformed to polar coordinates by

$$
t'_{i,j,k} = \arctan \frac{y'_{i,j,k}}{x'_{i,j,k}} \tag{6}
$$

$$
s_{i,j,k} = \sqrt{x'^2_{i,j,k} + y'^2_{i,j,k}} \tag{7}
$$

The orientation of each set j of series i can be expressed by the mean value:

$$
t'_{i,j} = \frac{1}{2}\left(t'_{i,j,2} + t'_{i,j,3}\right) \tag{8}
$$

Therefore, the rotation angle is

$$
\varphi_{i,j} = t'_{1,1} - t'_{i,j};\quad i = 1, 2, 3;\quad j = 1, 2, 3 \tag{9}
$$

and thus, the new orientation is

$$t_{i,j,k} = t'_{i,j,k} + \varphi_{i,j}; \quad i = 1, 2, 3; \quad j = 1, 2, 3; \quad k = 2, 3 \tag{10}$$

The transformed coordinates are then calculated as

$$x''_{i,j,k} = s_{i,j,k} \times \cos t_{i,j,k}; \quad i = 1, 2, 3; \quad j = 1, 2, 3; \quad k = 2, 3 \tag{11}$$

$$y''_{i,j,k} = s_{i,j,k} \times \sin t_{i,j,k}; \quad i = 1, 2, 3; \quad j = 1, 2, 3; \quad k = 2, 3 \tag{12}$$

The adjusted coordinates of S_2 and S_3 are obtained as

$$\overline{x''}_k = \frac{1}{9} \sum_{i=1}^{3} \sum_{j=1}^{3} x''_{i,j,k}; \quad k = 2, 3 \tag{13}$$

$$\overline{y''}_k = \frac{1}{9} \sum_{i=1}^{3} \sum_{j=1}^{3} y''_{i,j,k}; \quad k = 2, 3 \tag{14}$$

With the 36 residuals of the adjustment:

$$r_{x,i,j,k} = \overline{x''}_k - x''_{i,j,k}; \quad i = 1, 2, 3; \quad j = 1, 2, 3; \quad k = 2, 3 \tag{15}$$

$$r_{y,i,j,k} = \overline{y''}_k - y''_{i,j,k}; \quad i = 1, 2, 3; \quad j = 1, 2, 3; \quad k = 2, 3 \tag{16}$$

the sum of the squares of the residuals is

$$\sum r_{XY}^2 = \sum_{i=1}^{3} \sum_{j=1}^{3} \sum_{k=2}^{3} \left(r_{x,i,j,k}^2 + r_{y,i,j,k}^2 \right) \tag{17}$$

Since there are 8 rotation parameters and 4 average coordinates of the corners, point S_2 and point S_3, of the triangle, the number of unknown parameters in the adjustment is $u = 8 + 4 = 12$. Thus the number of degrees of freedom is

$$\nu_{XY} = 36 - 12 = 24 \tag{18}$$

The standard deviation of one x- or y-coordinate observed once in two faces of the telescope is

$$s_{XY} = \sqrt{\frac{\sum r_{XY}^2}{24}} \tag{19}$$

and finally

$$s_{ISO\text{-}TACH\text{-}XY} = s_{XY} \tag{20}$$

7.3.2 Precision of the z-coordinates

Since the z-coordinate of S_1 is set to zero, the unknowns of the adjustment procedure are the coordinates, z_2 and z_3, of the points, S_2 and S_3, and the height difference, δ, of the instrument height and the target height. A least squares adjustment gives a system of normal equations with the explicit solution according to Equations (21) to (23).

The three unknown parameters of the adjustment ($u = 3$) are the coordinates of S_2 and S_3

$$z_2 = \frac{1}{18}\begin{pmatrix} 2z_{1,1,2} + z_{1,1,3} - 2z_{1,2,1} - z_{1,2,3} - z_{1,3,1} + z_{1,3,2} \\ +2z_{2,1,2} + z_{2,1,3} - 2z_{2,2,1} - z_{2,2,3} - z_{2,3,1} + z_{2,3,2} \\ +2z_{3,1,2} + z_{3,1,3} - 2z_{3,2,1} - z_{3,2,3} - z_{3,3,1} + z_{3,3,2} \end{pmatrix} \quad (21)$$

$$z_3 = \frac{1}{18}\begin{pmatrix} z_{1,1,2} + 2z_{1,1,3} - z_{1,2,1} + z_{1,2,3} - 2z_{1,3,1} - z_{1,3,2} \\ +z_{2,1,2} + 2z_{2,1,3} - z_{2,2,1} + z_{2,2,3} - 2z_{2,3,1} - z_{2,3,2} \\ +z_{3,1,2} + 2z_{3,1,3} - z_{3,2,1} + z_{3,2,3} - 2z_{3,3,1} - z_{3,3,2} \end{pmatrix} \quad (22)$$

and the difference, δ:

$$\delta = \frac{1}{18}\begin{pmatrix} -z_{1,1,2} - z_{1,1,3} - z_{1,2,1} - z_{1,2,3} - z_{1,3,1} - z_{1,3,2} \\ -z_{2,1,2} - z_{2,1,3} - z_{2,2,1} - z_{2,2,3} - z_{2,3,1} - z_{2,3,2} \\ -z_{3,1,2} - z_{3,1,3} - z_{3,2,1} - z_{3,2,3} - z_{3,3,1} - z_{3,3,2} \end{pmatrix} \quad (23)$$

With these three parameters, the 18 residuals, $r_{i,j,k}$, of the adjustment are calculated:

$$\begin{array}{lll} r_{1,1,2} = z_2 - \delta - z_{1,1,2} & r_{2,1,2} = z_2 - \delta - z_{2,1,2} & r_{3,1,2} = z_2 - \delta - z_{3,1,2} \\ r_{1,1,3} = z_3 - \delta - z_{1,1,3} & r_{2,1,3} = z_3 - \delta - z_{2,1,3} & r_{3,1,3} = z_3 - \delta - z_{3,1,3} \\ r_{1,2,1} = -z_2 - \delta - z_{1,2,1} & r_{2,2,1} = -z_2 - \delta - z_{2,2,1} & r_{3,2,1} = -z_2 - \delta - z_{3,2,1} \\ r_{1,2,3} = -z_2 + z_3 - \delta - z_{1,2,3} & r_{2,2,3} = -z_2 + z_3 - \delta - z_{2,2,3} & r_{3,2,3} = -z_2 + z_3 - \delta - z_{3,2,3} \\ r_{1,3,1} = -z_3 - \delta - z_{1,3,1} & r_{2,3,1} = -z_3 - \delta - z_{2,3,1} & r_{3,3,1} = -z_3 - \delta - z_{3,3,1} \\ r_{1,3,2} = z_2 - z_3 - \delta - z_{1,3,2} & r_{2,3,2} = z_2 - z_3 - \delta - z_{2,3,2} & r_{3,3,2} = z_2 - z_3 - \delta - z_{3,3,2} \end{array} \quad (24)$$

The sum of the squares of the residuals is obtained by

$$\sum r_Z^2 = \sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{\substack{k=1 \\ k \neq j}}^{3} r_{i,j,k}^2 \quad (25)$$

With the number of degrees of freedom:

$$\nu_Z = 18 - 3 = 15 \quad (26)$$

Finally, the standard deviation of one z-coordinate measured once in both face positions of the telescope is

$$s_{\text{ISO-TACH-Z}} = \sqrt{\frac{\sum r_Z^2}{15}} \quad (27)$$

7.4 Statistical tests

7.4.1 General

Statistical tests are practicable for the full test procedure only.

For the interpretation of the results, statistical tests shall be carried out using the experimental standard deviation of a coordinate measured on the test triangle in order to answer the following questions (see Table 3):

a) Is the calculated experimental standard deviation, s, smaller than or equal to a corresponding value, σ, stated by the manufacturer or smaller than another predetermined value, σ?

b) Do two experimental standard deviations, s and $\tilde{s}$, as determined from two different samples of measurements belong to the same population, assuming that both samples have the same number of degrees of freedom, ν?

The experimental standard deviations, s and $\tilde{s}$, may be obtained from

1) two samples of measurements by the same instrument but different observers;

2) two samples of measurements by the same instrument at different times; or

3) two samples of measurements by different instruments.

For the following tests, a confidence level of $1 - \alpha = 0{,}95$ and, according to the design of measurements, a number of degrees of freedom of $\nu_{XY} = 24$ for the x- and y-coordinates and $\nu_Z = 15$ for the z-coordinate are assumed.

Table 3 — Statistical tests

Question	Null hypothesis	Alternative hypothesis
a)	$s \leqslant \sigma$	$s > \sigma$
b)	$\sigma = \tilde{\sigma}$	$\sigma \neq \tilde{\sigma}$

7.4.2 Response to Question a)

The null hypothesis stating that the experimental standard deviation, s, is smaller than or equal to a theoretical or a predetermined value, σ, is not rejected if the following condition is fulfilled:

for x and y — for z

$$s \leqslant \sigma \times \sqrt{\frac{\chi^2_{1-\alpha}(\nu_{XY})}{\nu_{XY}}} \qquad s \leqslant \sigma \times \sqrt{\frac{\chi^2_{1-\alpha}(\nu_Z)}{\nu_Z}} \tag{28}$$

$$s \leqslant \sigma \times \sqrt{\frac{\chi^2_{0,95}(24)}{24}} \qquad s \leqslant \sigma \times \sqrt{\frac{\chi^2_{0,95}(15)}{15}} \tag{29}$$

$$\chi^2_{0,95}(24) = 36{,}42 \qquad \chi^2_{0,95}(15) = 25{,}00 \tag{30}$$

$$s \leqslant \sigma \times \sqrt{\frac{36{,}42}{24}} \qquad s \leqslant \sigma \times \sqrt{\frac{25{,}00}{15}} \tag{31}$$

$$s \leqslant \sigma \times 1{,}23 \qquad s \leqslant \sigma \times 1{,}29 \tag{32}$$

Otherwise, the null hypothesis is rejected.

7.4.3 Response to Question b)

In the case of two different samples, a test indicates whether the experimental standard deviations, s and $\tilde{s}$, belong to the same population. The corresponding null hypothesis, $\sigma = \tilde{\sigma}$, is not rejected if the following condition is fulfilled:

for x and y	for z	
$\dfrac{1}{F_{1-\alpha/2}(\nu_{XY}, \nu_{XY})} \leqslant \dfrac{s^2}{\tilde{s}^2} \leqslant F_{1-\alpha/2}(\nu_{XY}, \nu_{XY})$	$\dfrac{1}{F_{1-\alpha/2}(\nu_Z, \nu_Z)} \leqslant \dfrac{s^2}{\tilde{s}^2} \leqslant F_{1-\alpha/2}(\nu_Z, \nu_Z)$	(33)
$\dfrac{1}{F_{0,975}(24, 24)} \leqslant \dfrac{s^2}{\tilde{s}^2} \leqslant F_{0,975}(24, 24)$	$\dfrac{1}{F_{0,975}(15, 15)} \leqslant \dfrac{s^2}{\tilde{s}^2} \leqslant F_{0,975}(15, 15)$	(34)
$F_{0,975}(24, 24) = 2,27$	$F_{0,975}(15, 15) = 2,86$	(35)
$0,44 \leqslant \dfrac{s^2}{\tilde{s}^2} \leqslant 2,27$	$0,35 \leqslant \dfrac{s^2}{\tilde{s}^2} \leqslant 2,86$	(36)

Otherwise, the null hypothesis is rejected.

The number of degrees of freedom and, thus, the corresponding test values $\chi^2_{1-\alpha}(\nu)$, $F_{1-\alpha/2}(\nu, \nu)$ and $t_{1-\alpha/2}(\nu)$ (taken from reference books on statistics) change if a different number of measurements is analysed.

Annex A
(informative)

Example of the simplified test procedure

A.1 Measurements

In Table A.1 all measurements are compiled according to the observation scheme given in Table 1.

Table A.1 — Measurements

1 **Instrument station**	2 **Target point**	3 **x-coordinate** m	4 **y-coordinate** m	5 **z-coordinate** m
S_1		1 000,000	2 000,000	300,000
	S_2	984,076	2 082,959	302,227
	S_3	883,478	2 015,557	286,794
S_2		984,076	2 082,959	302,227
	S_3	883,480	2 015,549	286,795
	S_1	1 000,000	1 999,999	300,002
S_3		883,478	2 015,557	286,794
	S_1	1 000,000	2 000,000	300,002
	S_2	984,082	2 082,955	302,228
Measurement conditions:				
Observer:	S. Miller			
Weather:	partly cloudy: (5/8) temperature: +18 °C air pressure: 995 hPa			
Instrument type and number:	NN xxx 630401			
Date:	2001-03-15			

A.2 Calculation

According to Equation (1), the coordinate differences are calculated as follows:

$d_1 = 0{,}000$
$d_2 = -0{,}006$
$d_3 = -0{,}002$
$d_4 = -0{,}001$
$d_5 = 0{,}004$
$d_6 = 0{,}008$
$d_7 = 0{,}000$
$d_8 = -0{,}001$
$d_9 = -0{,}001$

and according to Equation (2), the half difference of the maximum differences

$d_{x,y} = 0{,}004$

and according to Equation (3)

$d_z = 0{,}000\,5$

Annex B
(informative)

Example of the full test procedure

B.1 Measurements of x- and y-coordinates

Table B.1 contains in Columns 2 and 3 the measured x- and y-coordinates.

Table B.1 — Measurements and residuals (HZ)

1			2	3	4	5	6	7	8	9	10	11	12
i	j	k	x	y	x'	y'	t'	t	s	x''	y''	$t'_{1,1}$ =	r_y
			m	m	m	m	rad	rad	m	m	m	m	m
1	1	1	0,000	0,000	0,000	0,000							
		2	−0,007	63,994	−0,007	63,994	1,570 906			−0,007 0	63,994 0	0,001 4	0,005 6
		3	55,003	31,999	55,003	31,999	0,526 906			55,003 0	31,999 0	−0,002 3	0,000 2
						$t'_{1,1}$ =	1,048 906						
1	2	1	30,689	−56,157	0,000	0,000							
		2	0,000	0,000	−30,689	56,157	2,070 937	1,570 911	63,995 5	−0,007 3	63,995 5	0,001 7	0,004 2
		3	63,615	−1,707	32,926	54,450	1,026 927	0,526 901	63,631 2	55,000 8	31,997 4	−0,000 1	0,001 9
						$t'_{1,2}$ =	1,548 932	−0,500 026	= $\varphi_{1,2}$				
1	3	1	−2,791	−63,570	0,000	0,000							
		2	−56,651	−29,000	−53,860	34,570	2,570 969	1,570 930	63,999 9	−0,008 6	63,999 9	0,002 9	−0,000 2
		3	0,000	0,000	2,791	63,570	1,526 920	0,526 882	63,631 2	55,001 5	31,996 3	−0,000 8	0,002 9
						$t'_{1,3}$ =	2,048 944	−1,000 039	= $\varphi_{1,3}$				
2	1	1	0,000	0,000	0,000	0,000							
		2	−18,919	61,133	−18,919	61,133	1,870 921	1,570 909	63,993 5	−0,007 2	63,993 5	0,001 6	0,006 1
		3	43,088	46,823	43,088	46,823	0,826 915	0,526 903	63,631 5	55,001 1	31,997 7	−0,000 4	0,001 6
						$t'_{2,1}$ =	1,348 918	−0,300 012	= $\varphi_{2,1}$				
2	2	1	63,846	−4,519	0,000	0,000							
		2	0,000	0,000	−63,846	4,519	3,070 931	1,570 906	64,005 7	−0,007 0	64,005 7	0,001 4	−0,006 1
		3	35,818	52,606	−28,028	57,125	2,026 931	0,526 906	63,630 5	55,000 1	31,997 3	0,000 6	0,002 0
						$t'_{2,2}$ =	2,548 931	−1,500 025	= $\varphi_{2,2}$				
2	3	1	−56,645	28,992	0,000	0,000							
		2	−2,797	63,567	53,848	34,575	0,570 791	1,570 830	63,992 5	−0,002 2	63,992 5	−0,003 4	0,007 2
		3	0,000	0,000	56,645	−28,992	−0,473 058	0,526 981	63,633 3	55,000 1	32,002 8	0,000 6	−0,003 6
						$t'_{2,3}$ =	0,048 866	1,000 039	= $\varphi_{2,3}$				

Table B.1 (*continued*)

1			2	3	4	5	6	7	8	9	10	11	12
i	j	k	x	y	x'	y'	t'	t	s	x''	y''	$t'_{1,1}$ =	r_y
			m	m	m	m	rad	rad	m	m	m	m	m
3	1	1	0,000	0,000	0,000	0,000							
		2	−9,038	−63,365	−9,038	−63,365	4,570 711	1,570 801	64,006 3	−0,000 3	64,006 3	−0,005 3	−0,006 7
		3	−58,964	−23,916	−58,964	−23,916	3,526 920	0,527 011	63,629 6	54,996 0	32,002 6	0,004 7	−0,003 4
						$t'_{3,1}$ =	4,048 815	−2,999 910	= $\varphi_{3,1}$				
3	2	1	58,201	26,638	0,000	0,000							
		2	0,000	0,000	−58,201	−26,638	3,570 823	1,570 863	64,007 3	−0,004 3	64,007 3	−0,001 3	−0,007 7
		3	6,216	63,335	−51,985	36,697	2,526 908	0,526 948	63,632 6	55,000 6	32,000 7	0,000 1	−0,001 5
						$t'_{3,2}$ =	3,048 865	−1,999 960	= $\varphi_{3,2}$				
3	3	1	−2,791	−63,573	0,000	0,000							
		2	−56,651	−28,999	−53,860	34,574	2,570 916	1,570 903	64,002 0	−0,006 8	64,002 0	0,001 2	−0,002 4
		3	0,000	0,000	2,791	63,573	1,526 922	0,526 909	63,634 2	55,003 2	31,999 4	−0,002 5	−0,000 1
						$t'_{3,3}$ =	2,048 919	−1,000 013	= $\varphi_{3,3}$				
										$\overline{x''}$	$\overline{y''}$		
										−0,005 6	63,999 6		
										55,000 7	31,999 2		
											$\sum r_{XY}^2$ =	4,259 × 10^{-4} m^2	
											$s_{\text{ISO-TACH-XY}}$ =	0,004 2 m	
											ν_{XY} =	24	

Measurement conditions:	
Observer:	S. Miller
Weather:	sunny temperature: +12 °C air pressure: 976 hPa
Instrument type and number:	NN xxx 630401
Date:	2001-03-12

B.2 Calculation

According to Equation (6), the orientation angles, $t'_{i,j,k}$, for each direction are calculated and inserted in Column 6 (in the example given in radians). The distances, $s_{i,j,k}$, are calculated using Equation (7) and inserted in Column 8. Equation (8) gives the orientation angle, $t'_{i,j}$, of each series. With the rotation angle, $\varphi_{i,j,k}$, according to Equation (9) the new orientation, $t_{i,j,k}$, is inserted in Column 7. With $t_{i,j,k}$ and $s_{i,j,k}$, the transformed coordinates, $x''_{i,j,k}$ and $y''_{i,j,k}$, according to Equations (11) and (12) are calculated and inserted in Columns 9 and 10. Equations (13) and (14) give the adjusted coordinates, $\overline{x''}$ and $\overline{y''}$, of S_2 and S_3 (listed at the bottom of Columns 9 and 10). Then the residuals are calculated according to Equations (15) and (16) (Columns 11 and 12). Finally, Equation (17) results in

$$\sum r_{XY}^2 = 4{,}259 \times 10^{-4} \text{ m}^2$$