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Control charts —

Part 6:
EWMA control charts

Cartes de contrôle —

Partie 6: Cartes de contrôle de EWMA

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Foreword

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For an explanation on the meaning of ISO specific terms and expressions related to conformity assessment, as well as information about ISO's adherence to the WTO principles in the Technical Barriers to Trade (TBT), see the following URL: Foreword — Supplementary information.

The committee responsible for this document is ISO/TC 69, *Applications of statistical methods*, Subcommittee SC 4, *Applications of statistical methods in process management*.

ISO 7870 consists of the following parts, under the general title *Control charts*:

— *Part 1: General guidelines*

— *Part 2: Shewhart control charts*

— *Part 3: Acceptance control charts*

— *Part 4: Cumulative sum charts*

— *Part 5: Specialized control charts*

— *Part 6: EWMA control charts*

A future part on charting techniques for short runs and small mixed batches is planned.

Introduction

Shewhart control charts are the most widespread statistical control methods used for controlling a process, but they are slow in signalling shifts of small magnitude in the process parameters. The exponentially weighted moving average[10] (EWMA) control chart makes possible faster detection of small to moderate shifts.

The Shewhart control chart is simple to implement and it rapidly detects shifts of major magnitude. However, it is fairly ineffective for detecting shifts of small or moderate magnitude. It happens quite often that the shift of the process is slow and progressive (in case of continuous processes in particular); this shift has to be detected very early in order to react before the process deviates seriously from its target value. There are two possibilities for improving the effectiveness of the Shewhart control charts with respect to small and moderate shifts.

— The simplest, but not the most economical possibility is to increase the subgroup size. This may not always be possible due to low production rate; time consuming or too costly testing. As a result, it may not be possible to draw samples of size more than 1 or 2.

— The second possibility is to take into account the results preceding the control under way in order to try to detect the existence of a shift in the production process. The Shewhart control chart takes into account only the information contained in the last sample observation and it ignores any information given by the entire sequence of points. This feature makes the Shewhart control chart relatively insensitive to small process shifts. Its effectiveness may be improved by taking into account the former results.

Where it is desired to detect slow, progressive shifts, it is preferable to use specific charts which take into account the past data and which are effective with a moderate control cost. Two very effective alternatives to the Shewhart control chart in such situations are

a) Cumulative Sum (CUSUM) control chart. This chart is described in ISO 7870-4. The CUSUM control chart reacts more sensitively than the X-bar chart to a shift of the mean value in the range of half or two sigma. If one plots the cumulative sum of deviations of successive sample means from a specified target, even minor, permanent shifts in the process mean will eventually lead to a sizable cumulative sum of deviations. Thus, this chart is particularly well-suited for detecting such small permanent shifts that may go undetected when using the X-bar chart.

b) Exponentially Weighted Moving Average (EWMA) control chart which is covered by this document. This chart is presented like the Shewhart control chart; however, instead of placing on the chart the successive averages of the samples, one monitors a weighted average of the current average and of the previous averages.

EWMA control charts are generally used for detecting small shifts in the process mean. They will detect shifts of half sigma to two sigma much faster. They are, however, slower in detecting large shifts in the process mean. EWMA control charts may also be preferred when the subgroups are of size $n = 1$.

The joint use of an EWMA control chart with a small value of lambda and a Shewhart control chart has been recommended as a means of guaranteeing fast detection of both small and large shifts. The EWMA control chart monitors only the process mean; monitoring the process variability requires the use of some other technique.

Control charts —

Part 6:
EWMA control charts

1 Scope

This International Standard covers EWMA control charts as a statistical process control technique to detect small shifts in the process mean. It makes possible the faster detection of small to moderate shifts in the process average. In this chart, the process average is evaluated in terms of exponentially weighted moving average of all prior sample means. EWMA weights samples in geometrically decreasing order so that the most recent samples are weighted most highly while the most distant samples contribute very little depending upon the smoothing parameter (λ).

NOTE 1 The basic objective is the same as that of the Shewhart control chart described in ISO 7870-2.

The Shewhart control chart's application is worthwhile in the rare situations when

— production rate is slow,

— sampling and inspection procedure is complex and time consuming,

— testing is expensive, and

— it involves safety risks.

NOTE 2 Variables control charts can be constructed for individual observations taken from the production line, rather than samples of observations. This is sometimes necessary when testing samples of multiple observations would be too expensive, inconvenient, or impossible. For example, the number of customer complaints or product returns may only be available on a monthly basis; yet, one would like to chart those numbers to detect quality problems. Another common application of these charts occurs in cases when automated testing devices inspect every single unit that is produced. In that case, one is often primarily interested in detecting small shifts in the product quality (for example, gradual deterioration of quality due to machine wear).

2 Normative references

The following documents, in whole or in part, are normatively referenced in this document and are indispensable for its application. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments) applies.

ISO 7870-1, *Control charts — Part 1: General guidelines*

ISO 7870-2, *Control charts — Part 2: Shewhart control charts*

ISO 7870-4, *Control charts — Part 4: Cumulative sum charts*

3 Symbols and abbreviated terms

μ_0	Target value for the average of the process
U_μ, L_μ	Upper rejectable value of the average, lower rejectable value of the average
$\bar{x}_i$	Mean of the sample *i*
N	Number of units in a sample (sample size)

z_i	EWMA value placed on the control chart
z_0	Initial value of z_i
λ	Value of the smoothing parameter
L_z	Parameter used to establish the control limit for z_i (expressed in number of standard deviations of z)
s	Estimator of the standard deviation σ
σ	True standard deviation of the distribution of x
σ_0	True standard deviation of binomial distribution for $P = p_0$
$\sigma_{\bar{x}}$	Standard deviation of the averages of n individual observations; $\sigma_{\bar{x}} = \sigma / \sqrt{n}$
σ_z	Standard deviation of z_i when i tends towards infinity
δ	Drift related to the average expressed in number of standard deviations
δ_1	Maximum acceptable drift of the average, expressed in number of standard deviations
p	Proportion of nonconforming units of the process
p_0	Target value for the proportion of nonconforming units of the process
p_1	Upper refusable value of the proportion of nonconforming units
p_i	Proportion of nonconforming units in the ith sample
c	Average number of nonconformities
c_0	Target value for the average number of nonconformities
c_1	Refusable average of nonconformities
c_i	Number of nonconforming units in the ith sample
U_{CL}	Upper control limit value for the EWMA control chart
L_{CL}	Lower control limit value for the EWMA control chart. If L_{CL} is negative, then it is taken as zero
ARL	Average Run Length
ARL_0	Average Run Length of the process in control
ARL_1	Average Run Length of the process with setting drift
CL	Centre line of the control limit
MAXRL	Maximum Run Length (5 % overrun probability), expressed as an integer

4 EWMA for inspection by variables

4.1 General

An EWMA control chart plots geometric moving averages of past and current data in which the values being averaged are assigned weights that decrease exponentially from the present into the past. Consequently, the average values are influenced more by recent process performance. The exponentially weighted moving average is defined as Formula (1):

$$z_i = \lambda x_i + (1 - \lambda)\, z_{i-1} \tag{1}$$

NOTE 1 When the EWMA control chart is used with rational subgroups of size $n > 1$ then x_i is simply replaced with $\bar{x}_i$.

Where $0 < \lambda < 1$ is a constant and the starting value (required with the first sample at $i = 1$) is the process target, so that $z_0 = \mu_0$.

NOTE 2 μ_0 can be estimated by the average of preliminary data.

The EWMA control chart becomes an $\bar{X}$ chart for $\lambda = 1$.

4.2 Weighted average explained

To demonstrate that the EWMA is a weighted average of all previous sample means, the right-hand side of Formula (1) in 4.1 can be substituted with $z_{i\text{-}1}$ to obtain Formula (2):

$$\begin{aligned} z_i &= \lambda x_i + (1-\lambda)\left[\lambda x_{i-1} + (1-\lambda) z_{i-2}\right] \\ &= \lambda x_i + \lambda(1-\lambda) x_{i-1} + (1-\lambda)^2 z_{i-2} \end{aligned} \tag{2}$$

Continuing to substitute recursively for $z_{i\text{-}j}$, where j = 2, 3, ..., we obtain Formula (3):

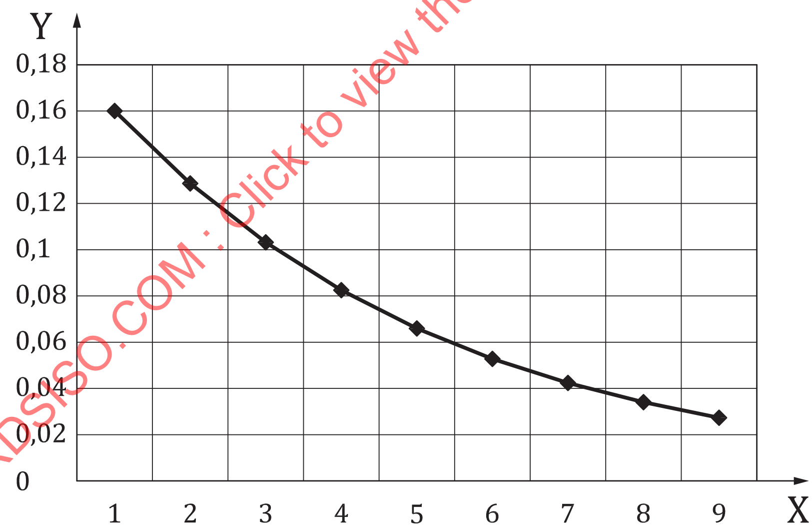
$$z_i = \lambda \sum_{j=0}^{i-1} (1-\lambda)^j x_{i-j} + (1-\lambda)^i z_0 \tag{3}$$

For i = 1, $z_1 = \lambda x_1 + (1 - \lambda)\mu_0$.

The weights, $\lambda(1 - \lambda)^j$, decrease geometrically with the age of the sample mean. Furthermore, the weights sum to unity, since

$$\lambda \sum_{j=0}^{i-1} (1-\lambda)^j = \lambda \left[\frac{1-(1-\lambda)^i}{1-(1-\lambda)}\right] = 1-(1-\lambda)^i \tag{4}$$

If λ = 0,2, then the weight assigned to the current sample mean is 0,2 and the weights given to the preceding means are 0,16; 0,128; 0,102 4 and so forth. These weights are shown in Figure 1. Because these weights decline geometrically, the EWMA is sometimes called a geometric moving average (GMA).



Key

X age of sample mean (EWMA λ = 0,2)

Y weights $\lambda(1\text{-}\lambda)^j$

Figure 1 — Weights of past sample means

Since the EWMA value can be viewed as a weighted average of all past and current observations, it is very insensitive to the normality assumption. It is, therefore an ideal control chart to use with individual observations.

4.3 Control limits for EWMA control chart

If the observations x_i are independent random variables with variance σ^2, then the variance of z_i is represented by Formula (5):

$$\sigma_{z_i}^2 = \sigma^2 \left(\frac{\lambda}{2-\lambda} \right) \left[1 - (1-\lambda)^{2i} \right] \tag{5}$$

Therefore, the EWMA control chart would be constructed by plotting z_i versus the sample number i (or time). The centre line and control limits for the EWMA control chart are as follows:

Centre line = μ_0

$$U_{CL} = \mu_0 + L_z \frac{\sigma}{\sqrt{n}} \sqrt{\frac{\lambda}{(2-\lambda)} \left[1 - (1-\lambda)^{2i} \right]} \tag{6}$$

$$L_{CL} = \mu_0 - L_z \frac{\sigma}{\sqrt{n}} \sqrt{\frac{\lambda}{(2-\lambda)} \left[1 - (1-\lambda)^{2i} \right]} \tag{7}$$

The factor L_z is the width of the control limits and its value depends upon the confidence level. In the case of $\overline{X}$ - R charts, 3σ limits are plotted for 99,73 % ($\pm 3\sigma$) confidence. Similarly, on EWMA control chart, this confidence level can vary depending on the requirements (e.g. L_z = 2,7 gives the confidence of 99,307 %).

No action is taken as long as z_i falls between these limits, and the process is considered to be out of control as soon as z_i overshoots the control limits. In this case, reset the process and resume the EWMA control chart after reinitializing it, i.e. by not taking into account the results obtained prior to this resetting, but by taking z_0 as the initial value.

The term $[1 - (1 - \lambda)^{2i}]$ approaches unity as i gets larger. This means that after the EWMA control chart has been running for several time periods, the control limits will approach steady state values obtained using Formulae (8) and (9):

Centre line = μ_0

$$U_{CL} = \mu_0 - L_z \frac{\sigma}{\sqrt{n}} \sqrt{\frac{\lambda}{(2-\lambda)}} \tag{8}$$

$$L_{CL} = \mu_0 - L_z \frac{\sigma}{\sqrt{n}} \sqrt{\frac{\lambda}{(2-\lambda)}} \tag{9}$$

However, it is strongly recommended to use the exact control limits. This will greatly improve the performance of the control chart in detecting an off-target process immediately after the EWMA control chart is initiated.

NOTE For practical purposes, use the estimate of σ, denoted by s, estimated from the data.

4.4 Construction of EWMA control chart

To illustrate the construction of an EWMA control chart, consider a process with the following parameters calculated from historical data:

$\mu_0 = 50$

$s = 2{,}053\ 9$

with λ chosen to be 0,3; so that

$$\sqrt{\frac{\lambda}{(2-\lambda)}} = \sqrt{\frac{0,3}{1,7}} = 0,420\ 1 \qquad (10)$$

The control limits at steady-state are given, obtained using Formulae (11) and (12):

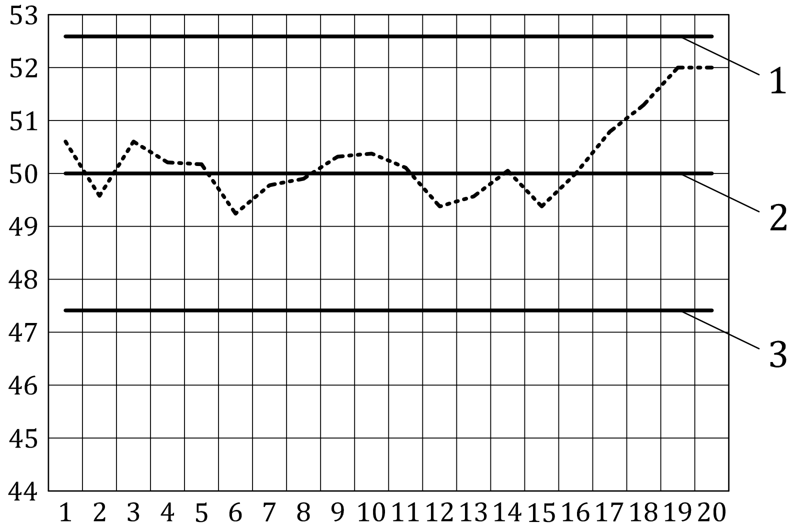
$$U_{CL} = 50 + 3\ (0,420\ 1)(2,053\ 9) = 52,588\ 5 \qquad (11)$$

$$L_{CL} = 50 - 3\ (0,420\ 1)(2,053\ 9) = 47,411\ 5 \qquad (12)$$

Consider the data consisting of 20 points as given in Table 1.

Table 1 — Calculation of EWMA values

Sample	X_i	EWMA values
1	52,0	50,600 0
2	47,0	49,520 0
3	53,0	50,564 0
4	49,3	50,184 8
5	50,1	50,159 4
6	47,0	49,211 6
7	51,0	49,748 1
8	50,1	49,853 7
9	51,2	50,257 6
10	50,5	50,330 3
11	49,6	50,111 2
12	47,6	49,357 8
13	49,9	49,520 5
14	51,3	50,054 3
15	47,8	49,378 0
16	51,2	49,924 6
17	52,6	50,727 2
18	52,4	51,229 1
19	53,6	51,940 3
20	52,1	51,988 2



Key

1 U_{CL} = 52,588 5

2 CL = 50

3 L_{CL} = 47,411 5

Figure 2 — EWMA plot

The EWMA control chart in Figure 2 shows that the process is in control because all EWMA points lie between the control limits.

4.5 Example

Consider the data in Table 2 (observations x_1). The first 20 observations were drawn at random from a normal distribution with mean μ = 10 and standard deviation σ = 1. The last 10 observations were drawn from a normal distribution with mean μ = 11 and standard deviation σ = 1, i.e. after the process has experienced a shift in the mean of one sigma.

Set up an EWMA control chart with λ = 0,10 and L_z = 2,7 to the data in Table 2.

The target value of the mean is μ = 10 and the standard deviation is σ = 1.

The calculations for EWMA control chart are summarized in Table 2 and the control chart is shown in Figure 3.

To illustrate the calculations, consider the first observations, x_i = 9,45.

The first value of the EWMA statistic is shown in Formula (13):

$$\begin{aligned} z_1 = \lambda x_1 + (1-\lambda) z_0 &= 0,1 \times 9,45 + 0,9 \times 10 \\ &= 9,945\ 00 \end{aligned} \quad (13)$$

Therefore, z_1 = 9,945 00 is the first value plotted on the control chart in Figure 3.

The second value of the EWMA is shown in Formula (14):

$$\begin{aligned} z_2 = \lambda x_2 + (1-\lambda) z_1 &= 0,1 \times 7,99 + 0,9 \times 9,945 \\ &= 9,749\ 50 \end{aligned} \quad (14)$$

The other values of the EWMA statistic are computed similarly.

The control limits are calculated following Formulae (15) and (16):

For period $i = 1$:

$$\begin{aligned} U_{\mathrm{CL}} &= \mu_0 + L_z \frac{\sigma}{\sqrt{n}} \sqrt{\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2i}\right]} \\ &= 10 + 2,7 \times 1 \times \sqrt{\frac{0,1}{(2-0,1)}\left[1-(1-0,1)^{2\times 1}\right]} \\ &= 10,270\ 00 \end{aligned} \tag{15}$$

and

$$\begin{aligned} L_{\mathrm{CL}} &= \mu_0 - L_z \frac{\sigma}{\sqrt{n}} \sqrt{\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2i}\right]} \\ &= 10 - 2,7 \times 1 \times \sqrt{\frac{0,1}{(2-0,1)}\left[1-(1-0,1)^{2\times 1}\right]} \\ &= 9,730\ 00 \end{aligned} \tag{16}$$

For period $i = 2$, the limits are shown in Formulae (17) and (18):

$$\begin{aligned} U_{\mathrm{CL}} &= \mu_0 + L_z \frac{\sigma}{\sqrt{n}} \sqrt{\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2i}\right]} \\ &= 10 + 2,7 \times 1 \times \sqrt{\frac{0,1}{(2-0,1)}\left[1-(1-0,1)^{2\times 2}\right]} \\ &= 10,363\ 25 \end{aligned} \tag{17}$$

and

$$\begin{aligned} L_{\mathrm{CL}} &= \mu_0 - L_z \frac{\sigma}{\sqrt{n}} \sqrt{\frac{\lambda}{(2-\lambda)}\left[1-(1-\lambda)^{2i}\right]} \\ &= 10 - 2,7 \times 1 \times \sqrt{\frac{0,1}{(2-0,1)}\left[1-(1-0,1)^{2\times 2}\right]} \\ &= 9,636\ 75 \end{aligned} \tag{18}$$

The calculation of control limits are also summarized in Table 2 and plotted in Figure 3.

Table 2 — EWMA calculations

Sample	x_i	EWMA z_i	U_{CL}	L_{CL}
1	9,45	9,945 00	10,270 00	9,730 00
2	7,99	9,749 50	10,363 25	9,636 75
3	9,29	9,703 55	10,424 00	9,576 00
4	11,66	9,899 20	10,467 46	9,532 54
5	12,16	10,125 28	10,499 90	9,500 10
6	10,18	10,130 75	10,524 71	9,475 29
7	8,04	9,921 67	10,543 98	9,456 02
8	11,46	10,075 51	10,559 09	9,440 90
9	9,20	9,987 96	10,571 05	9,428 95
10	10,34	10,023 16	10,580 55	9,419 45
11	9,03	9,923 84	10,588 13	9,411 87
12	11,47	10,078 46	10,594 20	9,405 80

Table 2 *(continued)*

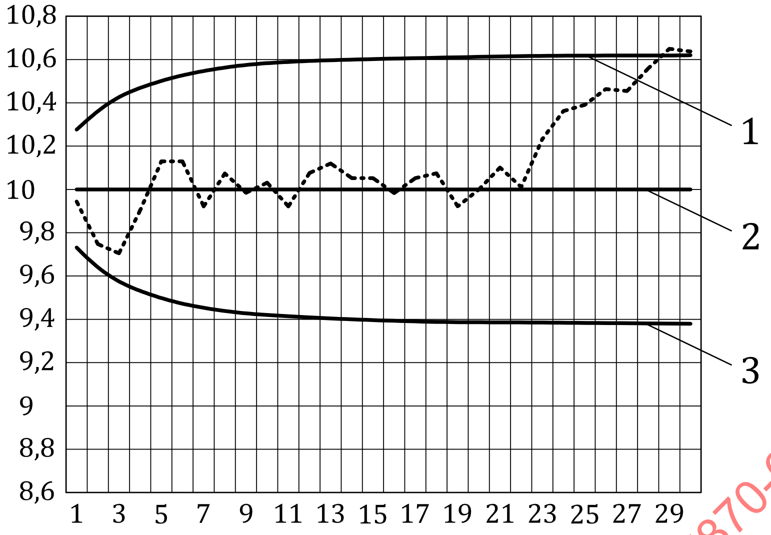
Sample	x_i	EWMA z_i	U_{CL}	L_{CL}
13	10,51	10,121 61	10,599 08	9,400 92
14	9,40	10,049 45	10,603 00	9,397 00
15	10,08	10,052 51	10,606 15	9,393 85
16	9,37	9,984 26	10,608 70	9,391 30
17	10,62	10,047 83	10,670 75	9,389 25
18	10,31	10,074 05	10,612 41	9,876 00
19	8,52	9,918 64	10,613 74	9,386 26
20	10,84	10,010 78	10,614 83	9,385 17
21	10,90	10,099 70	10,615 70	9,384 30
22	9,33	10,027 73	10,616 41	9,383 59
23	12,29	10,249 46	10,616 98	9,383 02
24	11,50	10,374 51	10,617 45	9,382 55
25	10,60	10,397 06	10,617 82	9,382 18
26	11,08	10,465 35	10,618 13	9,381 87
27	10,38	10,456 82	10,618 37	9,381 63
28	11,62	10,573 14	10,618 57	9,381 43
29	11,31	10,646 82	10,618 73	9,381 26
30	10,52	10,634 14	10,618 87	9,381 13

It may be noted from Figure 3 that the control limits increase in width as i increases from i = 1, 2, ..., until they stabilize at the steady-state values given in Formulae (19) and (20):

$$\begin{aligned} U_{CL} &= \mu_0 + L_z \frac{\sigma}{\sqrt{n}} \sqrt{\frac{\lambda}{(2-\lambda)}} \\ &= 10 + 2{,}7 \times 1 \times \sqrt{\frac{0{,}1}{(2-0{,}1)}} \\ &= 10{,}619\ 42 \end{aligned} \tag{19}$$

and

$$\begin{aligned} L_{CL} &= \mu_0 - L_z \frac{\sigma}{\sqrt{n}} \sqrt{\frac{\lambda}{(2-\lambda)}} \\ &= 10 - 2{,}7 \times 1 \times \sqrt{\frac{0{,}1}{(2-0{,}1)}} \\ &= 9{,}380\ 58 \end{aligned} \tag{20}$$



Key

1 U_{CL} = 10,62

2 CL = 10,00

3 L_{CL} = 9,38

Figure 3 — EWMA control chart

The EWMA control chart signals that observation 28 has gone beyond U_{CL}. Hence it is conclude that the process is out of control.

5 Choice of the control chart

5.1 Shewhart control chart versus EWMA control chart

Unlike the Shewhart control chart, it is not possible to find the probability of detecting a shift in the process on the basis of a sample because the probability is not constant. It depends on the number of the samples. One can calculate this probability for each sample, but these probabilities are too numerous to be used in practice.

The effectiveness of the EWMA technique is therefore judged according to the ARL, i.e. the average number of successive samples required for detecting a shift.

If the process is under control, it is expected that there be few false alarms, i.e. that the average number of samples prior to a false alarm be high (in general ARL_0 is taken between 100 and 1 000).

On the other hand, in the event of a shift, it is expected that it be detected as quickly as possible, i.e. that the number of samples between the moment the shift occurred and that of the first point outside the control limits be the lowest possible (low ARL_1).

Compared to the Shewhart control chart, the EWMA technique is extremely effective for minor or moderate shifts: the lower λ is, better is the effectiveness. On the other hand, the Shewhart control chart is more effective for sudden and high drifts.

The effectiveness of the chart depends on the size of the sample: the higher n is, better is the effectiveness (see Annex D).

5.2 Average run length

Table 3 gives the ARL and the MAXRL of the chart as a function of the drift, $\delta\sqrt{n}$. Therefore, the effectiveness for any value of n can be obtained.

For example, the EWMA control chart with λ = 0,5, L_z = 2,979 and n = 1 detects a shift of δ = 1 standard deviation in 14,5 samples on average because $\delta\sqrt{n} = 1$. Whereas, the same chart with n = 4 detects it in 3,2 samples, because $\delta\sqrt{n} = 2$.

In Table 3, the values of L_z for the EWMA techniques have been chosen, so that the ARL (Average Run Length) = 370 (i.e. the same as that of the Shewhart control chart), with control limits established at $\pm 3\sigma\sqrt{n}$ when the shift, δ, is equal to 0. Hence, you can compare the figures in the six columns directly since it is a question of control procedures, which have the same number of false alarms. Table 3 shows that the effectiveness for detecting minor shifts is better for small values of λ (e.g. the ARL goes from 14,9 to 7,6 for $\delta\sqrt{n} = 1$); and, is the contrary for major drifts (e.g. the ARL goes from 1,6 to 1,5 for $\delta\sqrt{n} = 3$).

The choice of λ and L_z is made so as to obtain an Average Run Length which one sets in an a priori manner as the quality objective. One can therefore thus obtain charts which correspond to the practical requirements of industry or services.

Table 3 — Comparison of mean operational periods of EWMA and Shewhart control chart

Shift	Shewhart control chart		EWMA control charts									
	λ = 1,0 L_z = 3,0		λ = 0,5 L_z = 2,979		λ = 0,4 L_z = 2,961		λ = 0,3 L_z = 2,928		λ = 0,2 L_z = 2,864		λ = 0,1 L_z = 2,715	
$\delta\sqrt{n}$	ARL	MAXRL	ARL	MAXRL	ARL	MAXRL	ARL	MAXRL	ARL	MAXRL	ARL	MAXRL
0,00	370,4		370,4		370,8		370,9		370		370,9	
0,25	281,2	842	195,7	584	173,8	518	148,5	441	119,6	353	86,3	248
0,50	155,2	464	71,3	211	58,0	170	45,8	132	35,0	97	25,7	66
0,75	81,2	242	29,9	86	24,0	67	19,2	52	15,4	39	12,5	29
1,00	43,9	130	14,9	41	12,3	33	10,3	26	8,8	21	7,6	17
1,25	25,0	74	8,7	23	7,5	18	6,6	15	5,9	13	5,3	11
1,50	15,0	44	5,7	14	5,1	12	4,7	10	4,3	9	3,9	8
1,75	9,5	27	4,1	9	3,8	8	3,6	7	3,4	7	3,1	6
2,00	6,3	18	3,2	7	3,0	6	2,9	6	2,7	5	2,5	5
2,25	4,4	12	2,6	5	2,5	5	2,4	5	2,3	4	2,1	4
2,50	3,2	9	2,2	4	2,1	4	2,0	4	2,0	4	1,8	3
2,75	2,5	6	1,9	4	1,8	3	1,8	3	1,7	3	1,6	3
3,00	2,0	5	1,6	3	1,6	3	1,6	3	1,5	3	1,5	3

5.3 Choice of parameters for EWMA control chart

5.3.1 Choice of λ

The smaller λ is, the more the past is taken into account and the better minor drifts are detected; on the other hand, major, sudden drifts are less well detected;

The higher λ is, the less the past is taken into account and the better the reactivity to major, sudden drifts will be; on the other hand, minor drifts are less well detected.

The choice of λ shall be made on the basis of the experience that one has of the process. In general, $0{,}05 \leq \lambda \leq 0{,}50$ works well in practice,

— if slow drifts are expected, one should choose a value of λ between 0,05 to 0,25, and

— if one fears sudden, moderate magnitude shifts, one should choose rather a value of λ close to 0,5.

The most commonly used values of λ are between 0,25 and 0,5 inclusive. It is to be noted that one obtains the Shewhart control chart if one takes $\lambda = 1$.

5.3.2 Choice of L_z

The parameter L_z is the multiple of the subgroup standard deviation that establishes the control limits. L_z is typically set at 3 to match other control charts, but it may be necessary to reduce it slightly for small values of λ. L_z between 2,6 and 2,8 is useful when $\lambda \leq 0{,}1$.

NOTE λ and L_z may be determined by plotting the data on exponential probability paper and using standard normal tables.

5.3.3 Calculation for *n*

Table 4 gives the parameters of the EWMA control chart for a given effectiveness: one sets the ARL_0 when the process is controlled and ARL_1 when the process has shifted by a given value δ_1. Table 4 gives the values of L_z and λ which enable to obtain the desired effectiveness.

The maximum acceptable shift δ_1 of the average is shown in Formula (21):

$$\delta_1 = \min\left\{\frac{U_{1/4} - \mu_0}{\sigma_0}, \frac{\mu_0 - L_{1/4}}{\sigma_0}\right\} \tag{21}$$

Proceed as follows:

— *Stage 1*: Select the average number of samples (ARL_0) desired between two false alarms (generally between 100 and 1 000); this determines the choice of the column of Table 4;

— *Stage 2:* Select the average number of samples (ARL_1) required in order to detect the maximum acceptable shift δ_1; then look for, within the table in the previously determined column, the value of ARL_1 which is closest to that sought after; read L_z and λ values associated with ARL_1; the corresponding line gives $\delta_1\sqrt{n}$; hence n;

— *Stage 3*: if n is too high for practical reasons (cost, feasibility, etc.) return to Stage 1 after reducing the requirements on the input parameters of Table 4 (ARL_0, ARL_1, δ_1).

Table 4 — Determination of L_z and λ as a function of the controlled and uncontrolled run lengths (ARL_0 and ARL_1) and of the drift δ_1

$\delta_1\sqrt{n}$	**ARL when the process is controlled**, ARL_0			
	100	**370**	**500**	**1 000**
0,5	$\lambda = 0{,}07$ $L_z = 2{,}01$ $\mathrm{ARL}_1 = 17{,}3$	$\lambda = 0{,}06$ $L_z = 2{,}55$ $\mathrm{ARL}_1 = 26{,}5$	$\lambda = 0{,}05$ $L_z = 2{,}62$ $\mathrm{ARL}_1 = 28{,}7$	$\lambda = 0{,}04$ $L_z = 2{,}82$ $\mathrm{ARL}_1 = 34{,}3$
0,75	$\lambda = 0{,}12$ $L_z = 2{,}21$ $\mathrm{ARL}_1 = 10{,}3$	$\lambda = 0{,}10$ $L_z = 2{,}70$ $\mathrm{ARL}_1 = 14{,}7$	$\lambda = 0{,}09$ $L_z = 2{,}79$ $\mathrm{ARL}_1 = 15{,}8$	$\lambda = 0{,}07$ $L_z = 2{,}97$ $\mathrm{ARL}_1 = 18{,}4$
1,0	$\lambda = 0{,}19$ $L_z = 2{,}35$ $\mathrm{ARL}_1 = 7{,}0$	$\lambda = 0{,}15$ $L_z = 2{,}80$ $\mathrm{ARL}_1 = 9{,}6$	$\lambda = 0{,}15$ $L_z = 2{,}91$ $\mathrm{ARL}_1 = 10{,}2$	$\lambda = 0{,}13$ $L_z = 3{,}11$ $\mathrm{ARL}_1 = 11{,}7$
1,5	$\lambda = 0{,}33$ $L_z = 2{,}47$ $\mathrm{ARL}_1 = 3{,}9$	$\lambda = 0{,}26$ $L_z = 2{,}90$ $\mathrm{ARL}_1 = 5{,}2$	$\lambda = 0{,}24$ $L_z = 2{,}99$ $\mathrm{ARL}_1 = 5{,}5$	$\lambda = 0{,}22$ $L_z = 3{,}20$ $\mathrm{ARL}_1 = 6{,}1$
2,0	$\lambda = 0{,}52$ $L_z = 2{,}54$ $\mathrm{ARL}_1 = 2{,}6$	$\lambda = 0{,}40$ $L_z = 2{,}96$ $\mathrm{ARL}_1 = 3{,}3$	$\lambda = 0{,}37$ $L_z = 3{,}05$ $\mathrm{ARL}_1 = 3{,}5$	$\lambda = 0{,}35$ $L_z = 3{,}25$ $\mathrm{ARL}_1 = 3{,}9$
2,5	$\lambda = 0{,}66$ $L_z = 2{,}56$ $\mathrm{ARL}_1 = 1{,}89$	$\lambda = 0{,}54$ $L_z = 2{,}98$ $\mathrm{ARL}_1 = 2{,}38$	$\lambda = 0{,}52$ $L_z\ L_z = 3{,}07$ $\mathrm{ARL}_1 = 2{,}50$	$\lambda = 0{,}46$ $L_z = 3{,}27$ $\mathrm{ARL}_1 = 2{,}76$
3,0	$\lambda = 0{,}81$ $L_z = 2{,}57$ $\mathrm{ARL}_1 = 1{,}45$	$\lambda = 0{,}70$ $L_z = 2{,}99$ $\mathrm{ARL}_1 = 1{,}78$	$\lambda = 0{,}70$ $L_z = 3{,}09$ $\mathrm{ARL}_1 = 1{,}86$	$\lambda = 0{,}66$ $L_z = 3{,}29$ $\mathrm{ARL}_1 = 2{,}06$
NOTE If ARL_1 is chosen below 1,40, use a Shewhart control chart.				

5.3.4 Example

For a process, the target average value μ_0 = 100 and a standard deviation of σ_0 = 0,8 is desired to get a maximum of one false alarm every 500 samples. It is desired to detect within three or four samples, on average, a shift of ±1 unit (rejectable averages U_μ = 101 and L_μ = 99).

— Select from Table 4 the column which corresponds to ARL_0 = 500.

— Look for, in this column, the ARL_1 which is closest to 3. We find 3,5 which corresponds to L_z = 3,05 and λ = 0,37 for $\delta\sqrt{n} = 2$.

— Calculate δ_1 = min (1/0,8;1/0,8) = 1,25. As $\delta\sqrt{n} = 2$ one deduces from this $n = (2/1{,}25)^2 = 2{,}56 \cong n = 3$, rounding off to the higher integer (which improves the detection effectiveness).

6 Procedure for implementing the EWMA control chart

The implementation of the EWMA control chart is the same as for any other type of control procedure. The procedure is built on the assumption that the "good" historical data are representative of the in-control process, with future data from the same process tested for agreement with the historical data. To start the procedure, a target value (average) and process standard deviation are estimated from historical data. Then the procedure enters the monitoring stage with the EWMA statistics computed and tested against the control limits.

7 Sensitivity of the EWMA to non-normality

For subgroup size one, both individual Shewhart control chart and EWMA control chart may be used. However, an individual control chart is sensitive to non-normality, whereas a properly designed EWMA is less sensitive to the normality assumption.

8 Advantages and limitations

8.1 Advantages

a) Shewhart control chart only use the current observation or sample to monitor the process. EWMA control charts utilize all previous observations, but the weight attached to the data are exponentially declining as the observations get older and older. It gives more weight to the recent past value depending upon the value of λ. By varying the parameter of the EWMA statistic the 'memory' of the EWMA control chart can be influenced.

b) EWMA control chart is relatively robust in the face of non-normally distributed quality characteristics.

8.2 Limitations

a) The EWMA control chart is sensitive to small shifts in the process mean, but does not reflect the out of control situations for larger shifts as quickly as the Shewhart control chart does. It is also recommended to superimpose the EWMA control chart on top of a suitable Shewhart control chart with widened control limits in order to detect both small and large shifts in the process mean.

b) When EWMA control chart is used with a small value of the weight λ then at the beginning of the production, the EWMA control chart is more efficient in detecting the shift. As production proceeds, if a trend develops, the trend is unfortunately shown to be well within control of the EWMA chart's control limits.

Annex A
(informative)

Application of the EWMA control chart

A continuous production process involving the filling of μ_0 = 100 ml dosage bottles with a pharmaceutical product is considered. The target is for customers to have a very low risk, about 0,135 %, of finding a bottle under the lower tolerance T_L = 99,5 ml. Over proportioning should be avoided for economic reasons and since customers use the bottle as a dose. The upper tolerance, T_U, for the individual values is fixed at 100,6 ml.

When the process is in control, the standard deviation of the individual measurements is s_0 = 0,1 ml (value calculated on 150 measurements) and it was ascertained that the distribution could be considered normal. The average may wander within limits set at 3 standard deviations above and below the tolerance limits. This ensures a probability of less than 0,135 % of out of tolerance values as shown in Formulae (A.1) and (A.2):

$$U_\mu = T_U - 3\sigma_0 = 100,6 - 3 \times 0,1 = 100,3 \quad \text{(A.1)}$$

$$L_\mu = T_L - 3\sigma_0 = 99,5 + 3 \times 0,1 = 99,8 \quad \text{(A.2)}$$

Hence, $\delta_1 = \min[(U_\mu - \mu_0)/\sigma_0;(\mu_0 - L_\mu)/\sigma_0] = (100,0 - 99,8)/0,1 = 2,0$

An ARL_0 of 500 may be achieved when the process is properly centred and detected within two or three successive samples when the process has drifted by δ_1 = 2. Table 4 gives for ARL_0 = 500 and ARL_1 between 2 and 3 the following values:

ARL_1 = 2,50;

$\delta_1\sqrt{n} = 2,5$;

L_z = 3,07;

λ = 0,52.

Hence, $n = (2,5/2)^2 = 1,562\,5$, where n = 2 (rounding off to the higher integer, which improves the detection effectiveness).

The control limits are shown in Formulae (A.3) and (A.4):

$$U_{\text{CL}} = 100 + \left(\frac{3,07 \times 0,1}{\sqrt{2}}\sqrt{0,52/(2-0,52)}\right) = 100,129 \quad \text{(A.3)}$$

$$L_{\text{CL}} = 100 - \left(\frac{3,07 \times 0,1}{\sqrt{2}}\sqrt{0,52/(2-0,52)}\right) = 99,871 \quad \text{(A.4)}$$

The initial and target values are μ_0 = 100.

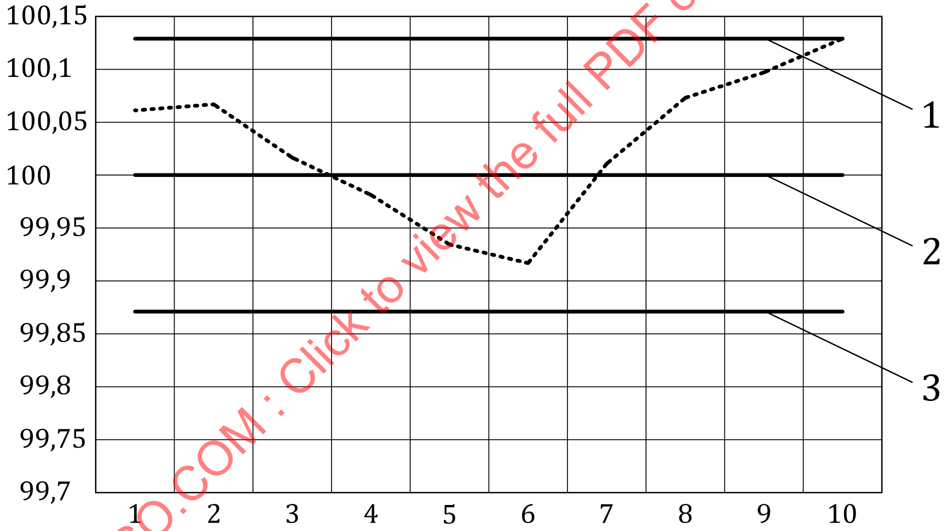
The initial value of σ_0 is obtained by a preliminary study and gives value as 0,1 ml.

The following individual values are obtained on conducting a control (Table A.1); one calculates their means, $\bar{x}_i$, their ranges, R_i, and the statistics, z_i:

Table A.1 — Calculation of Shewhart control chart and EWMA values for n = 2

Sample	Individual values		$\overline{x}_i$	R_i	$z_i = 0,52\overline{x}_i + 0,48z_{i-1}$
1	99,99	100,25	100,12	0,26	100,062
2	100,01	100,13	100,07	0,12	100,066
3	99,98	99,96	99,97	0,02	100,016
4	99,84	100,06	99,95	0,22	99,982
5	99,93	99,85	99,89	0,08	99,934
6	99,86	99,94	99,90	0,08	99,916
7	100,05	100,15	100,10	0,10	100,012
8	100,28	99,98	100,13	0,30	100,073
9	100,17	100,07	100,12	0,10	100,097
10	100,13	100,19	100,16	0,06	100,130

Figure A.1 shows that at the 10th sample, z_i, overshoots the upper control limit, indicating that the process has drifted and should be reset. After resetting, restart a new chart, replacing previous values with $z_0 = \mu_0 = 100$.



Key

1 U_{CL} = 100,129

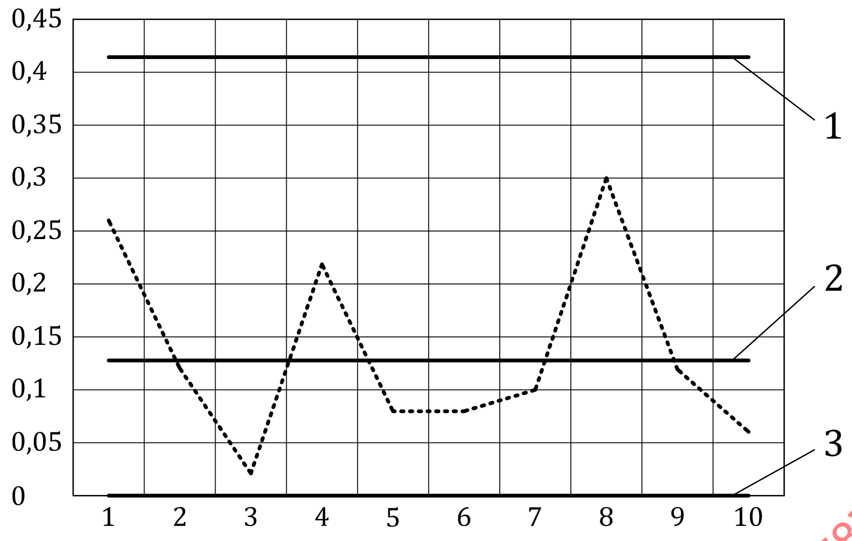
2 CL = 100,000

3 L_{CL} = 99,871

Figure A.1 — EWMA control chart for the control of average

The associated chart of the range R_i of the samples (Figure A.2) does not show any change in the dispersion. The remarked drift corresponds to a drift in the average and not to an increase in the dispersion of the process.

NOTE The calculations of the centreline and the control limit values for the dispersion (range) chart are defined in ISO 7870-2.



Key

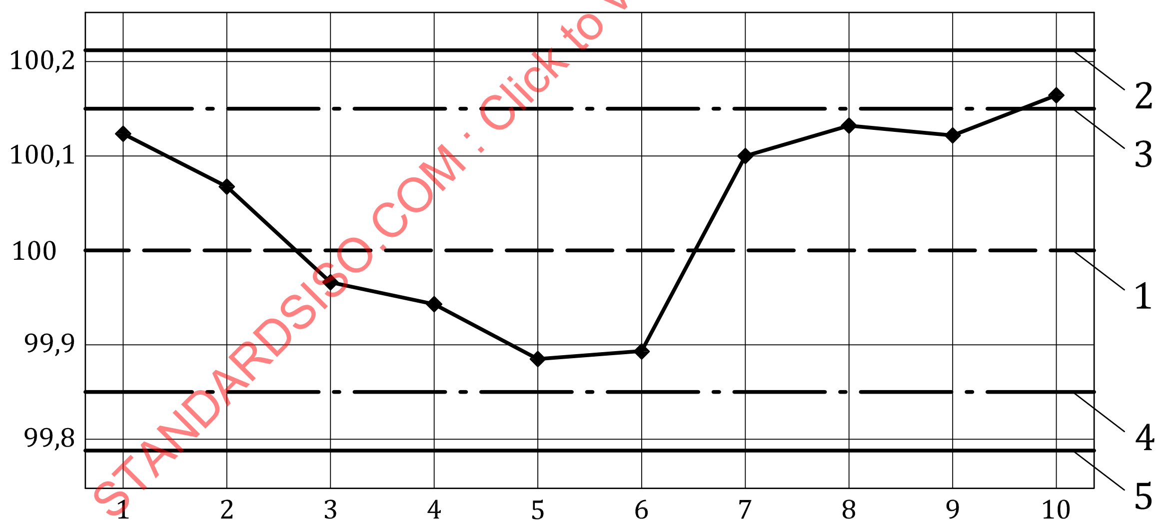
1 upper control limit of the range

2 target range

3 lower control limit of the range

Figure A.2 — Range chart for control of dispersion

The control limits of the corresponding Shewhart mean chart with n = 2 and U_μ = 3,09 are located at 100,22 and 99,78: This chart does not detect any process drift. It would be necessary to double the size of samples (n = 4), i.e. the cost of the control, in order to detect a drift concerning these data (Figure A.3).



Key

1 target

2 upper control limit for n = 2

3 upper control limit for n = 4

4 lower control limit for n = 4

5 lower control limit for n = 2

Figure A.3 — Shewhart mean chart for n = 2 and n = 4

NOTE This example illustrates the fact that the EWMA control chart is more sensitive than the Shewhart control chart for a low drift of the average. If the lag had been sudden and high, the Shewhart control chart would have been more rapid in pointing it out.

Annex B
(normative)

EWMA control chart for controlling a proportion of nonconforming units

B.1 Description of the method

It is possible to construct and use EWMA control charts for the monitoring of a proportion. This chart has the same purpose as the *p*-chart or *np*-chart, as described in ISO 7870-2. It is more effective for detecting minor or moderate magnitude drifts.

From the results of the samples p_1, p_2, ..., p_i, the value of z_i, the weighted average of the previous z_{i-1} and present p_i, is calculated [Formula (B.1)]:

$$z_i = \lambda p_i + \left(1 - \lambda\right) z_{i-1} \tag{B.1}$$

The initial value z_0 is the target value p_0. The standard deviation σ_0 is estimated by s_0:

$$s_0 = \sqrt{p_0\left(1 - p_0\right)} \tag{B.2}$$

NOTE For Bernoulli trials, where p_0 is the probability of failure, the variance is given by $p_0(1-p_0)$.

A control chart on which the values of z_i are plotted should be constructed. This chart should include upper and lower control limits, U_{CL} and L_{CL}, respectively; and are obtained using Formulae (B.3) and (B.4):

$$U_{CL} = p_0 + L_z \frac{s_0}{\sqrt{n}} \sqrt{\frac{\lambda}{2 - \lambda}} \tag{B.3}$$

$$L_{CL} = p_0 - L_z \frac{s_0}{\sqrt{n}} \sqrt{\frac{\lambda}{2 - \lambda}} \tag{B.4}$$

A process is considered under control as long as z_i falls between the abovementioned limits. On the other hand, the process is considered to have drifted when a value goes beyond the limits.

After resetting, the EWMA control chart is resumed and reinitialized, i.e. with $z_0 = p_0$. Previous results which were obtained with another process setting should be discarded.

An EWMA control chart equivalent to the previous one may be constructed by directly using the number of nonconforming units in each sample. In the event all samples have the same size, *n*, all the values for p_0, p_i, z_i, σ_z, σ, U_{CL} and L_{CL} should be multiplied by *n*.

B.2 Choice of control chart

As in the EWMA variables chart, the effectiveness of the EWMA attributes technique is assessed according to the ARL, as described in ISO 7870-1, i.e. the number of successive samples required in order to detect a drift. If the process is properly set, few false alarms may be encountered, i.e. the average number of samples prior to a false alarm may be high (in general ARL_0 between 100 and 1 000).

On the other hand, a drift should be detected as quickly as possible, i.e. that the number of successive samples between the moment the drift occurred and that of the first point outside the control limits be the lowest possible (low ARL_1).

The effectiveness of the EWMA technique is very good compared to that of the p chart, as described in ISO 7870-2, and is comparable to that of the CUSUM technique, as described in ISO 7870-4. The gain in effectiveness over the p chart is to be noted, in particular, for minor or moderate drifts. However, the p chart is more effective for sudden major drifts.

To obtain the ARL, use Table 4. Likewise, the choice of L_z and λ is made by the technique defined in 5.3. But the use of these tables is only valid if $np_0 > 5$.

The maximum acceptable drift, δ_1, is: $\delta_1 = (p_1 - p_0)/s_0$, where p_1 is the maximum permissible proportion of nonconforming units in the production.

B.3 Example

A welding operation is monitored by control chart of the proportion of nonconforming units. The preliminary study enabled to estimate the average proportion p_0 of the properly set and stable process at 0,019 45 (1,945 %). The sample size n is constant and equal to 1 600.

When the condition $np_0 > 5$ is amply fulfilled; the technique defined above and Table 4 can be used. With the same size of samples, an EWMA attributes chart, which has a run length of 370 when $P = p_0$ and which rapidly detects a proportion of nonconforming units equal to 0,028, can be obtained. See Formula (B.5):

$$s_0 = \sqrt{p_0\left(1 - p_0\right)} = 0,138\ 1 \quad \text{(B.5)}$$

Hence,

$$\delta_1\sqrt{n} = \left[\left(0,028 - 0,019\ 45\right)/0,138\ 1\right] \times \sqrt{1\ 600} = 2,48 \quad \text{(B.6)}$$

For $ARL_0 = 370$, the following values are found in Table 4:

$\delta_1\sqrt{n} = 2,5$;

$\lambda = 0,54$;

$L_z = 2,98$;

$ARL_1 = 2,38$.

The control limits are deduced following Formulae (B.7) and (B.8):

$$U_{CL} = 0,019\ 45 + \frac{2,98 \times 0,138\ 1}{\sqrt{1\ 600}}\sqrt{\frac{0,54}{2 - 0,54}} = 0,025\ 0 \quad \text{(B.7)}$$

$$L_{CL} = 0,019\ 45 - \frac{2,98 \times 0,138\ 1}{\sqrt{1\ 600}}\sqrt{\frac{0,54}{2 - 0,54}} = 0,013\ 2 \quad \text{(B.8)}$$

The same EWMA control chart expressed in number of nonconforming units in the samples, provided that the size of samples does not vary or varies little, has the following parameters:

$z_0 = np_0 = 31,12 \cong 31$ units;

$ns_0 = 220,96 \cong 221$ units;

$U_{CL} = 41,12 \cong 41$ units;

$L_{CL} = 21,12 \cong 21$ units.

The calculations for z_i will be made with the number of nonconforming units in each sample.

Annex C
(normative)

EWMA control charts for a number of nonconformities

C.1 Description of the method

It is possible to construct and use EWMA control charts for monitoring a number of nonconformities. This chart has the same purpose as the *c* or *u* charts. It is more effective for detecting drifts of minor or moderate magnitude. It can be applied for the monitoring of quality both in services (accounting, invoicing, dispatch, secretariat, etc.) and in production or laboratories. It is also used for monitoring accident frequency rates (Safety) or complaints (Quality).

From the results of the samples, c_1, c_2, ..., c_i, the values of the weighted average z_i of the previous z_{i-1} and of the present c_i is calculated using Formula (C.1):

$$z_i = \lambda c_i + \left(1 - \lambda\right) z_{i-1} \tag{C.1}$$

The initial value, z_0, is the target value, c_0. The standard deviation is estimated using Formula (C.2):

$$s_0 = \sqrt{c_0} \ . \tag{C.2}$$

A control chart where the values of z_i are plotted should be constructed. As in the *c* chart, this chart should include the upper and lower control limits, U_{CL} and L_{CL}:, respectively, obtained using Formulae (C.3) and (C.4):

$$U_{CL} = c_0 + L_z \sqrt{c_0} \sqrt{\frac{\lambda}{2 - \lambda}} \tag{C.3}$$

$$L_{CL} = c_0 - L_z \sqrt{c_0} \sqrt{\frac{\lambda}{2 - \lambda}} \tag{C.4}$$

A process is considered under control when z_i falls between the abovementioned limits; and is considered to have drifted as soon as a value goes beyond said limits.

It is considered a unilateral EWMA when only the upper control limit is plotted on the chart. The lower control limit can also be plotted on the chart in order to detect an improvement in quality, to identify the reasons for said improvement, and to try to reproduce this improvement.

After resetting, the EWMA control chart should be reinitialized with the value z_0 (usually z_0 = c_0). Previous results obtained with another process setting can be discarded.

C.2 Choice of the control chart

As in a *c* chart of the number of nonconformities, the effectiveness of the EWMA technique is assessed according to the ARL, as described in ISO 7870-1, i.e. the number of successive samples required in order to detect a drift.

If the process is properly set, few false alarms may be encountered, i.e. the average number of samples prior to a false alarm may be high (in general ARL_0 between 100 and 1 000).

A drift should be detected as quickly as possible, i.e. the number of successive samples (ARL_1) between the moment the drift occurred and that of the first point outside the control limits be the lowest possible.

The effectiveness of the EWMA technique is very good compared to that of the *c* chart and is comparable to that of the CUSUM technique. The gain in effectiveness over the *c* chart is to be noted in particular for minor or moderate drifts. On the other hand, the *c* chart of the number of nonconformities is more effective for sudden, high drifts.

To obtain the Average Run Length, use Table 3. The choice of L_z and λ is made by the technique defined in this International Standard; however, the use of these tables is only valid if c_0 is greater than 5.

The maximum acceptable drift δ_1 is:

$$\delta_1 = \frac{c_1 - c_0}{s_0} \tag{C.5}$$

Likewise, an EWMA control chart of the nonconformities per controlled unit can be obtained by replacing c, c_0, c_1 by u, u_0, u_1.

C.3 Example

The following example is considered: $c_0 = 10$; hence $s_0 = \sqrt{10} = 3,16$.

When the condition $c_0 \geq 5$ is fulfilled; the above technique and Table 4 can be used. An EWMA control chart of the number of nonconformities which has a run length, ARL_0, of 370 when $c = c_0$ and which rapidly detects an average number of nonconformities per control unit $c_1 = 15$, can be obtained using Formula (C.6).

$$\delta_1 = (15 - 10)\big/\sqrt{10} = 1,58 \tag{C.6}$$

where $n = 1$: $\delta_1 \sqrt{n} = 1,58$.

In Table 4, for $ARL_0 = 370$ and $\delta_1 \sqrt{n} = 1,5$, the following values can be obtained:

$ARL_1 = 5,2$;

$L_z = 2,90$;

$\lambda = 0,26$.

$$U_{CL} = 10 + 2,9 \times 3,16 \sqrt{\frac{0,26}{2 - 0,26}} = 13,54\,;$$

$$L_{CL} = 10 - 2,9 \times 3,16 \sqrt{\frac{0,26}{2 - 0,26}} = 6,46\,.$$

