INTERNATIONAL STANDARD

ISO 9039

First edition
1994-08-01

Optics and optical instruments — Quality evaluation of optical systems — Determination of distortion

Optique et instruments d'optique — Évaluation de la qualité des systèmes optiques — Détermination de la distorsion

Reference number
ISO 9039:1994(E)

Foreword

ISO (the International Organization for Standardization) is a worldwide federation of national standards bodies (ISO member bodies). The work of preparing International Standards is normally carried out through ISO technical committees. Each member body interested in a subject for which a technical committee has been established has the right to be represented on that committee. International organizations, governmental and non-governmental, in liaison with ISO, also take part in the work. ISO collaborates closely with the International Electrotechnical Commission (IEC) on all matters of electrotechnical standardization.

Draft International Standards adopted by the technical committees are circulated to the member bodies for voting. Publication as an International Standard requires approval by at least 75 % of the member bodies casting a vote.

International Standard ISO 9039 was prepared by Technical Committee ISO/TC 172, *Optics and optical instruments*, Subcommittee SC 1, *Fundamental standards*.

Annexes A and B of this International Standard are for information only.

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Case Postale 56 • CH-1211 Genève 20 • Switzerland

Printed in Switzerland

Introduction

Generally, the function of rotationally symmetric optical systems is to form an image that is geometrically similar to the object, except some particular systems such as fish-eye lenses and eyepieces, where this condition is deliberately not maintained. This function is accomplished ideally according to the geometry of perspective projection. Departures from the ideal image geometry are called distortion. The distortion is a position-dependent quantity which generally has a vectorial character. In a given image plane (which may also lie at infinity), this vector, representing the difference between theoretical and real image position, has a radial and a tangential component. In optical systems, the tangential component is basically conditioned by imperfect rotational symmetry. The systems manufactured in accordance with the present state of the art have a negligible tangential distortion. A tangential component of the distortion appears, however, as primary aberration in the case of electromagnetically focused electrooptical systems. This International Standard deals only with the radial distortion. For special systems, e.g. certain electrooptical systems, an expansion may become necessary to include vectorial representation.

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Optics and optical instruments — Quality evaluation of optical systems — Determination of distortion

1 Scope

This International Standard specifies methods of determining distortion in optical systems for the purposes of quality evaluation.

It applies to optical imaging systems in the optical spectral range from 100 nm to 15 000 nm which, by their design, aim at a rotationally symmetric image geometry. It is applicable to electrooptical imaging systems provided that adequate rotational symmetry of the image is guaranteed. It does not apply, therefore, to anamorphic and fibre optic systems.

2 Normative reference

The following standard contains provisions which, through reference in this text, constitute provisions of this International Standard. At the time of publication, the edition indicated was valid. All standards are subject to revision, and parties to agreements based on this International Standard are encouraged to investigate the possibility of applying the most recent edition of the standard indicated below. Members of IEC and ISO maintain registers of currently valid International Standards.

ISO 9334:—[1], *Optics and optical instruments — Optical transfer function — Definitions and mathematical relationships.*

3 Definitions

For the purposes of this International Standard, the following definitions apply.

3.1 distortion: Measure of the deviation of the extra-axial image points from the ideal image points in a given plane lying parallel to the reference plane of the system. If the image plane is at infinity, the image positions are given in terms of tangents of field angles.

3.2 reference plane: Plane corresponding to a physical feature of the device under test which is used for alignment e.g. a mounting flange or a fixture specially mounted for that purpose.

3.3 absolute distortion, V_a: Distance in the radial direction between the observed image point and the ideal image point, expressed in millimetres or micrometres.

3.4 relative distortion, V_r: Distance in the radial direction between the observed image point and the ideal image point, expressed as a percentage of the ideal image height h'_0. With the image at infinity, relative distortion is the difference between the tangents of the observed field angle and the ideal field angle, expressed as a percentage of the tangent of the ideal field angle ω'_0.

3.5 object height, h: Distance between an object point and the axis of rotational symmetry of the test specimen, expressed in millimetres.

3.6 image height, h': Distance between an image point and the axis of rotational symmetry of the test specimen, expressed in millimetres.

3.7 object pupil field angle, ω_p: Absolute value of the angle, expressed in radians or degrees, between the axis of rotational symmetry and the direction of travel of radiation from the object to the entrance pupil of the test specimen.

1) To be published.

3.8 image pupil field angle, ω'_p: Absolute value of the angle, expressed in radians or degrees, between the axis of rotational symmetry and the direction of travel of radiation from the exit pupil of the test specimen to the image.

3.9 object distance, a: Distance between the object plane and the first principal point, expressed in millimetres.

3.10 image distance, a': Distance between the image plane and the second principal point, expressed in millimetres.

3.11 object plane: Plane parallel to the reference plane containing an object point.

3.12 image plane: Plane parallel to the reference plane containing an image point.

3.13 ideal image height, h'_0: Image height without distortion, given by the geometry of perspective projection, expressed in millimetres.

3.14 ideal image field angle, ω'_0: Image field angle without distortion, given by the geometry of perspective projection, expressed in radians or degrees.

3.15 angular magnification, Γ: Limiting value of the equation

$$\Gamma = \lim_{\omega_p \to 0} \frac{\tan \omega'_p}{\tan \omega_p}$$

3.16 lateral magnification, m: Limiting value of the equation

$$m = \lim_{h \to 0} \frac{h'}{h}$$

4 Classes of application

4.1 Infinite object distance, finite image distance

The reference quantity is the image distance a', obtained as the limiting value of the equation

$$a' = \lim_{\omega_p \to 0} \frac{h'}{\tan \omega_p}$$

The absolute distortion is

$$V_a = h' - a' \tan \omega_p$$

and the relative distortion is

$$V_r = 100 \frac{h' - a' \tan \omega_p}{a' \tan \omega_p}$$

For telecentric imaging, the image distance a' is replaced by the distance of the telecentric stop from the first principal point.

If the image side focus lies in the image plane, then a' is the equivalent focal length. For photogrammetric lenses, the calibrated focal length is used instead of a' in the calculation of the absolute distortion V_a. The calibrated focal length is an adjusted value chosen to distribute the distortion within the image field in a specified manner.

4.2 Infinite object distance, infinite image distance

The reference quantity is the angular magnification Γ.

The relative distortion V_r is given by

$$V_r = 100 \frac{\dfrac{\tan \omega'_p}{\tan \omega_p} - \Gamma}{\Gamma}$$

4.3 Finite object distance, finite image distance

The reference quantity is the lateral magnification m.

The absolute distortion is

$$V_a = h' - hm$$

and the relative distortion is

$$V_r = 100 \frac{\dfrac{h'}{h} - m}{m}$$

4.4 Finite object distance, infinite image distance

The reference quantity is the object distance a, obtained as the limiting value of the equation

$$a = \lim_{h \to 0} \frac{h}{\tan \omega'_p}$$

The relative distortion is

$$V_r = 100 \frac{a \tan \omega'_p - h}{h}$$

For telecentric imaging, the distance of the telecentric stop from the second principal point replaces a.

5 Test methods

5.1 General

In order to determine the distortion, conjugate value pairs of object- and image-side coordinates must be measured. For the object side, the values concerned are the object pupil field angle ω_p or the object height h, and for the image side the image pupil field angle ω'_p or the image height h'. The terms object-side and image-side must be understood with reference to practical application.

When making measurements, the direction of radiation should be from the object side to the image side. If the opposite direction is to be applied for the convenience of measurement, it should be ensured that the aberrations of the optical system to be tested do not influence the accuracy of the measurement.

Illuminated reticles, an array of illuminated slits with known separations or a single illuminated slit whose displacement is measurable, serve the purpose of representing object positions of finite distance or, in the case of opposite direction of radiation, image positions of finite distance. Collimators are employed to represent objects at infinite distance whereas telescope lenses are used to render images at infinite distance measurable (or vice versa for the opposite direction of radiation).

For the measurement of the object or image pupil field angles, the collimator, or telescope, and the optical system to be tested (with its image or object plane) are displaced relative to each other in a way that the angles can be measured. The axis of rotation should pass through the middle of the entrance or exit pupil of the system to be tested in order to cover the full aperture of this system also in the case of larger field angles.

For the measurement of finite image or object heights, detection devices whose displacement is measurable or scales placed in the measuring plane are employed.

The distortion is calculated from the measured values in accordance with the formulae given in clause 4.

NOTE 1 In the case of opposite direction of radiation, care should be taken not to confuse image- and object-side quantities, as otherwise the distortion would be reversed in sign.

5.2 Apparatus

5.2.1 General requirements

The measurement setups shall be so designed that the reference plane of the optical system to be tested and the object or image plane can be aligned parallel to each other. In the case of infinite object or image distance, for the field angle $\omega_p = 0$ or $\omega'_p = 0$, the reference plane of the system to be tested shall be adjustable perpendicular to the direction of radiation. It is appropriate to use an autocollimator for the alignment instead of the collimator or telescope.

The instruments used for measuring the object and image pupil field angles and object and image height shall have accuracies such that the influence on the calculated distortion values is 5 to 10 times lower than the tolerance. For optical systems with very low permissible distortion, it may be not possible to achieve these instrument accuracies. In this case, the actual accuracy should be specified in the test report.

The general stability and precision of the measurement setup, in particular of the swivel bearings, shall be included in the error assessment.

The spectral characteristic of the measurement setup shall be adapted to the intended application of the optical system to be tested.

The coherency characteristics of the object illumination shall match those actually used for the optical system to be tested.

The mounting of appropriate diaphragms shall guarantee the limitations of the rays which correspond to the practical application of the optical system to be tested. Special attention is necessary in the case of magnifiers and eyepieces.

The illuminating optics shall be mounted in such a way that the principal rays correspond to practical applications.

If necessary, the illumination aperture shall be adapted to the intended application of the optical system to be tested.

Auxiliary optics used shall be sufficiently well corrected that they do not affect the measured values. Their pupils shall be large enough that the pupils of the optical system to be tested are not vignetted.

It shall be ensured that, during the measurement, the image plane corresponds as closely as possible to that of practical application. The application of given focusing criteria may be necessary for this purpose.

If high demands are made on the accuracy of measurement the application of criteria specified for the establishment of the image position may be necessary.

5.2.2 Infinite object distance, finite image distance

The measurement setup shall allow the measurement of conjugate value pairs of the object pupil field angle ω_p and the image height h'.

5.2.2.1 Camera setup

The object is represented by a mark in the focal plane of a collimator, preferably by an incoherently illuminated narrow slit. A device whose displacement can be measured is mounted in the image plane of the optical system to be tested in order to detect the image. It shall be possible to rotate the collimator and the optical system to be tested, with the detection device mounted in its image plane, relative to each other in such a way that the angle of rotation can be measured. It is of no importance which part is rotated and which part is stationary. The axis of rotation is perpendicular to the plane formed by the image height axis and the optical axis of the collimator and passes approximately through the middle of the entrance pupil of the optical system to be tested.

For measuring the object pupil field angle, a rotating stage with an angular scale or a theodolite may be employed. Rotation may be replaced by an array of several collimators arranged at different angles.

Prior to starting the measurement, the displacement direction of the image detector shall be aligned parallel to the reference plane of the optical system under test.

The collimator shall be aligned perpendicular to the reference plane. In this way, an object point is realized at the object pupil field angle $\omega_p = 0$; its image then indicates the coordinate origin in the image plane for the test procedure.

As an autocollimating telescope is necessary for this basic adjustment, it is expedient to design the collimator for use as an autocollimator. The detection device may be a microscope provided with a reticle mark arranged in the intermediate image plane, or a narrow slit with a photoelectric detector mounted behind it. In less critical cases, it may be sufficient to set up a plate provided with a graduated scale [see figure 1 a)].

It is also possible to mount a photographic test plate in the image plane and measure the image heights on the developed photoplate [see figure 1 b)]. In order to guarantee the necessary dimensional stability, glass plates should preferably be used. Image position displacements due to chemical influences during development shall be kept small and within the limits of the intended accuracy.

When the radiation is directed from the image to the object space, an illuminated reticle with appropriate marks, or an array of illuminated slits or a single illuminated slit whose displacement is measurable, is mounted in the image plane of the optical system to be tested instead of the detection device. Auxiliary optics with a detection device in the focal plane, such as a telescope with reticle mark in the intermediate image plane, replace the collimator. In order to facilitate the basic adjustment, it is expedient to employ an autocollimating telescope (see figure 2).

5.2.2.2 Nodal slide lens bench method

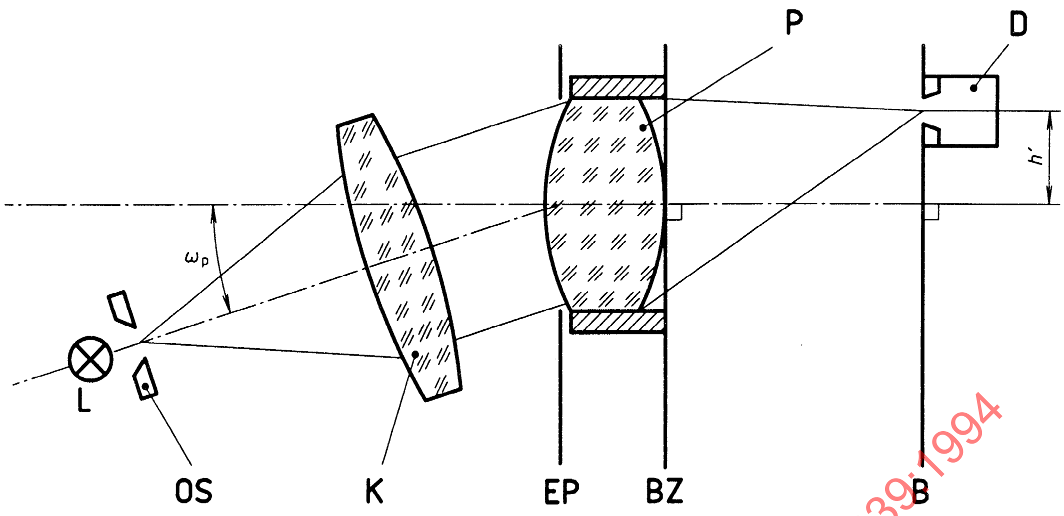
The object is represented by a mark in the focal plane of a collimator, preferably by an incoherently illuminated narrow slit. The aperture of the collimator shall be large enough to fill the aperture of the system to be tested at all field angles. The optical system to be tested is mounted on a rotating stage so that it can be rotated about a vertical axis. For $\omega_p = 0$, the reference plane of the system to be tested is perpendicular to the collimator. It shall be possible to shift the optical system to be tested in such a way that the axis of rotation passes through the second principal point of that system. In the image plane, a device is mounted for detecting the image, for example a microscope with reticle mark in the intermediate image plane (see figure 3).

The axis of rotation is adjusted to pass through the rear principal point, so that during rotation of the optical system through small angles, the image is stationary.

In order to measure the distortion, the optical system to be tested is displaced through the angle ω_p. The detection device must be refocused by the value

$$\Delta a' = \frac{a'(1 - \cos \omega_p)}{\cos \omega_p}$$

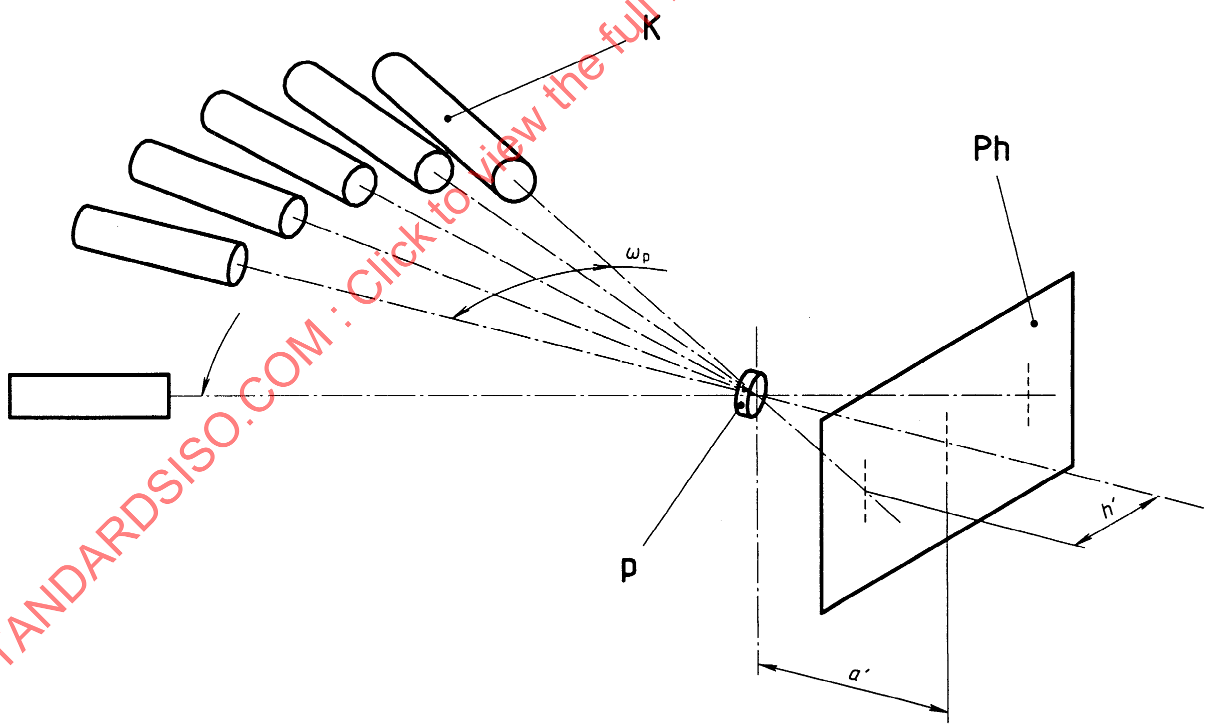
(away from the optical system to be tested).



Key

L	Illumination system	BZ	Reference plane of optical system to be tested
OS	Object slit	P	Optical system to be tested
ω_p	Object pupil field angle	B	Image plane
K	Collimator	D	Detection device
EP	Entrance pupil	h'	Image height

a) Camera setup

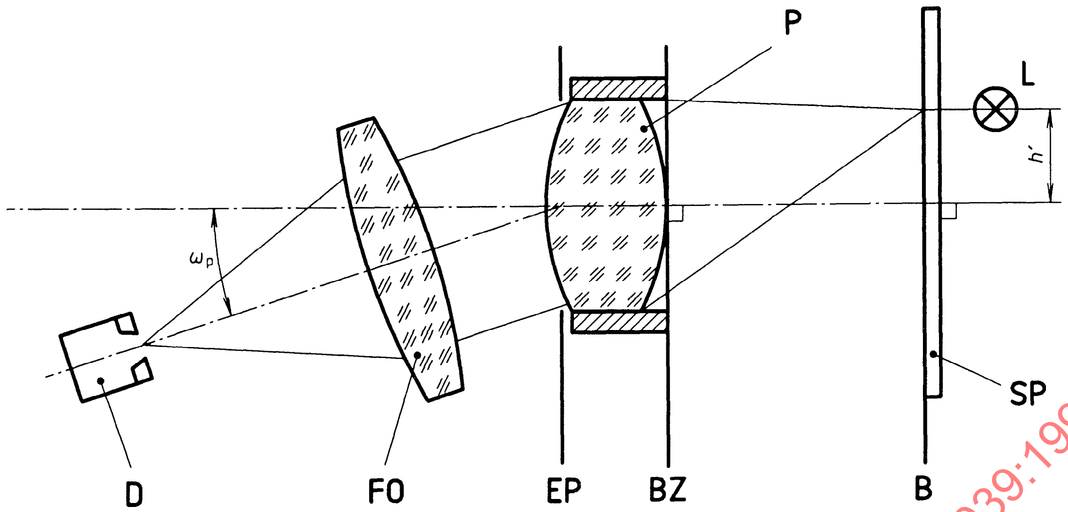


Key

K	Collimators	ω_p	Object pupil field angle
P	Optical system to be tested	Ph	Photo plate
a'	Distance of photo plate from the second principal point	h'	Image height

b) Example of a photographic test setup

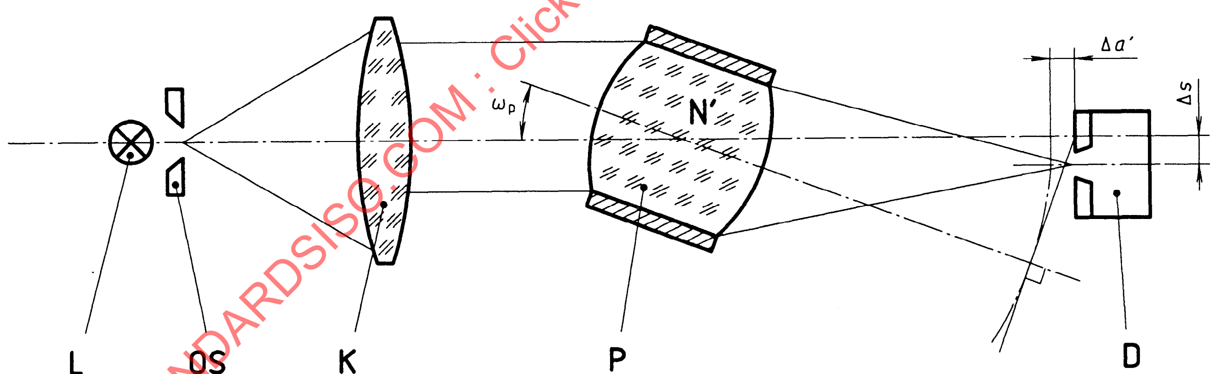
Figure 1 — Setup



Key

D	Detection device	BZ	Reference plane of optical system to be tested
ω_p	Object pupil field angle	P	Optical system to be tested
FO	Telescope lens	B	Image plane
EP	Entrance pupil	SP	Reticle
L	Illumination system	h'	Image height

Figure 2 — Camera setup with opposite radiation direction



Key

L	Illumination system	N'	Second principal point of optical system to be tested
OS	Object slit	D	Detection device
K	Collimator	Δs	Shift distance
ω_p	Object pupil field angle	$\Delta a'$	Refocusing distance
P	Optical system to be tested		

Figure 3 — Setup for nodal slide lens bench method

In order to make refocusing possible, a' must be known with sufficient accuracy. Refocusing is unnecessary when the measuring equipment includes a flat field bar. The detection device is shifted perpendicular to the collimator axis as far as the image position. From the shifted distance, Δs, we obtain the value V_a from

$$V_a = \frac{\Delta s}{\cos \omega_p}$$

The correct sign must be observed.

5.2.3 Infinite object distance, infinite image distance

The measurement setup shall allow the measurement of conjugate value pairs of the object pupil field angle ω_p and the image pupil field angle ω'_p.

The measurement setup is similar to the one described in 5.2.2.1. The moveable detection device in the image plane is replaced by a telescope lens which can be rotated about the exit pupil of the optical system to be tested and a detection device placed in its focal plane. For this purpose, a telescope with a reticle mark in the intermediate image plane may be employed or a telescope lens in whose focal plane there is a narrow slit with a photoelectric detector mounted behind it.

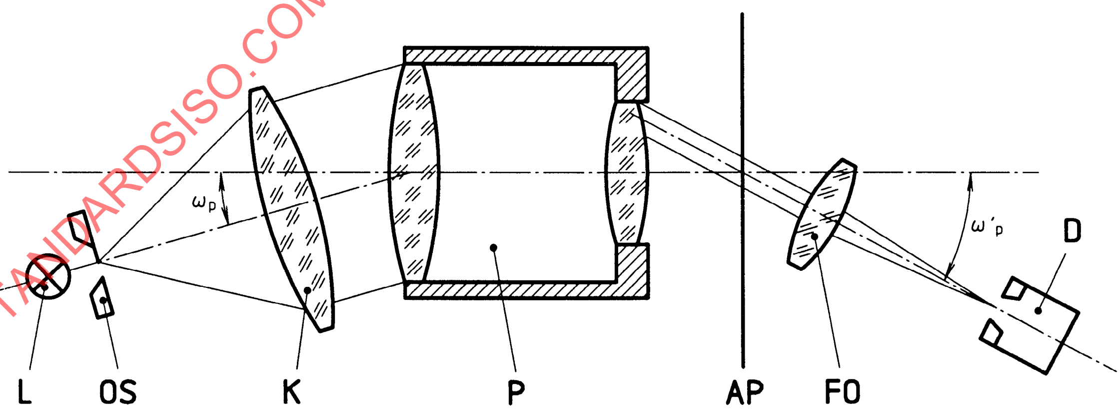
For the basic adjustment, first the measuring equipment shall be aligned to $\omega_p = 0$ and $\omega'_p = 0$ without the optical system under test being mounted. Then the optical system under test shall be placed into the measuring equipment and be adjusted in such a way that the image of the object mark appears at the same position in the detection device.

The axis of rotation between the optical system under test and the telescope lens shall be aligned parallel to the axis of rotation between the collimator and the optical system under test. It is of no importance which part of the measurement setup is stationary and which parts are displaced. Both displacement angles shall be measurable (see figure 4).

5.2.4 Finite object distance, finite image distance

The measurement setup shall allow the measurement of conjugate value pairs of the object height h and the image height h'.

When the radiation is directed from the object to the image space an illuminated reticle with fixed marks or an array of illuminated slits or a single illuminated slit whose displacement in the object plane can be measured, is mounted in the object plane. The optical system under test is stationary. A device for detecting the image, as described in 5.2.2 is arranged in the image plane of the optical system to be tested. The object plane, the reference plane of the optical system under test and the image plane shall be aligned parallel to each other (see figure 5).



Key

L	Illumination system	FO	Telescope lens
OS	Object slit	D	Detection device
K	Collimator lens	ω_p	Object pupil field angle
P	Optical system under test	ω'_p	Image pupil field angle
AP	Exit pupil		

Figure 4 — Setup for measuring the distortion of an afocal system

It is also possible to mount a photographic test plate in the image plane and measure the image heights on the developed plate, with the same precautions as described in 5.2.2.1.

When the radiation is directed from the image to the object space, the measuring marks are arranged in the image plane whereas the detection device is mounted in the object plane. The image geometry will indicate which direction of radiation is preferable in each individual case.

Both directions of radiation allow the projected image to be measured on a screen mounted in the respective plane. When the object distance required is too large to be realized in the test laboratory, the method described in 5.2.2 shall be followed in principle. The collimator or the telescope used shall be focused to the object distance required. It may be necessary to refocus the collimator or the telescope with increasing field angle. Care must be taken to ensure that no axial displacement occurs during refocusing. For this reason, it is advisable to use an alignment telescope.

5.2.5 Finite object distance, infinite image distance

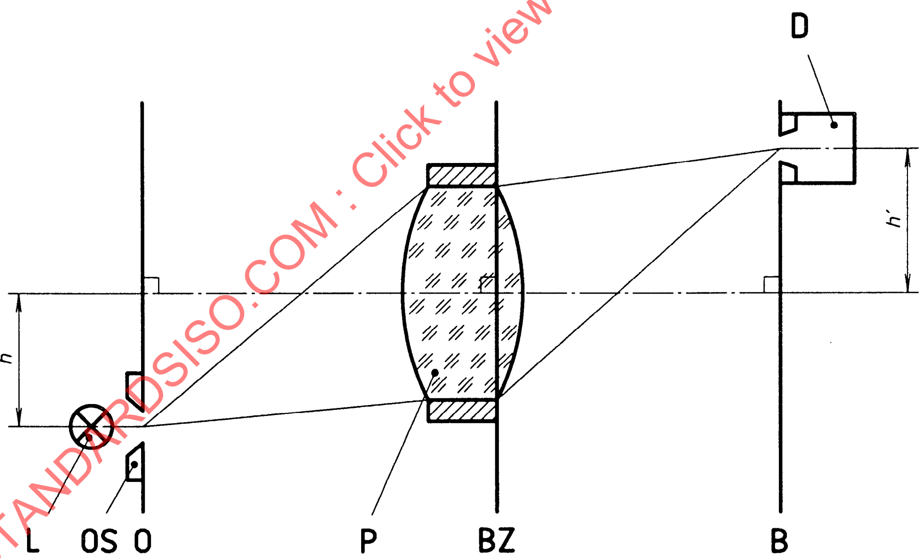
The measurement setup shall allow the measurement of conjugate value pairs of the object height h and the image pupil field angle ω'_p.

The measurement task is a reversal of the case in 5.2.2. The measurement setup is the same as the one described in 5.2.2, with opposite direction of radiation.

6 Procedure

6.1 Azimuth of the optical system to be tested

The measurement shall be carried out in two directions, mutually perpendicular, in the image plane. When the image format is square and the position of the optical system to be tested is known, the measurement shall be carried out in the actual diagonals of the format. The azimuth position of the measurement shall be indicated on the optical system to be tested.



Key

L	Illumination system	BZ	Reference plane of optical system to be tested
O	Object plane	B	Image plane
OS	Object slit	D	Detection device
P	Optical system to be tested	h	Object height
		h'	Image height

Figure 5 — Setup for measuring the distortion at finite object and image distance

6.2 Coordinate origin

For infinite object distance, the optical axis of the collimator employed during the measurement shall be aligned perpendicular to the reference plane. In the case of finite image distance, the point image generated by the collimator and the optical system to be tested indicates the coordinate origin of the image plane. In the case of infinite image distance, the zero-direction of the image pupil field angle is obtained in this way.

For finite object distance and finite image distance, the coordinate origin for infinite object distance shall be determined first as described above; this point shall then be transferred into the image plane for finite object distance by displacement perpendicular to the reference plane. If this is possible, as, for instance, in the case where the optical system to be tested cannot be employed for infinite object distance, the measurement is carried out for a coordinate origin whose position has only been estimated.

For finite object distance and infinite image distance the procedure described in the previous paragraph shall be followed, the coordinate origin being replaced by the zero-direction of the image field angle.

6.3 Selection of image heights

It is recommended that the measurement be carried out for the image heights or field angles given in table 1.

Table 1

Image heights	Field angles
$\pm h'_{max}$	$\pm \omega'_{p,max}$
$\pm 0{,}85\ h'_{max}$	$\pm 0{,}85\ \omega'_{p,max}$
$\pm 0{,}7\ h'_{max}$	$\pm 0{,}7\ \omega'_{p,max}$
$\pm 0{,}5\ h'_{max}$	$\pm 0{,}5\ \omega'_{p,max}$
$\pm 0{,}3\ h'_{max}$	$\pm 0{,}3\ \omega'_{p,max}$
$\pm 0{,}2\ h'_{max}$	$\pm 0{,}2\ \omega'_{p,max}$
$\pm 0{,}1\ h'_{max}$	$\pm 0{,}1\ \omega'_{p,max}$
NOTE — The above set of image heights or field angles should be used in intercomparisons. However, a different set of image heights may be selected for special applications.	

7 Evaluation

7.1 Calculation of the reference quantities a, a', m or Γ

The reference quantity is determined from the measured values for

$0{,}1\ h'_{max}$, $0{,}2\ h'_{max}$ and $0{,}3\ h'_{max}$ or

$0{,}1\ \omega'_{p,max}$, $0{,}2\ \omega'_{p,max}$ and $0{,}3\ \omega'_{p,max}$

in accordance with the respective formulae in clauses 3 and 4.

7.2 Calculation of the distortion

The distortion shall be calculated from the measured value pairs of the object and image-side coordinates in accordance with the formulae in clauses 3 and 4.

In general, the distortion values are not equal for diametrically opposite measuring points. The reasons for this may be:

a) lack of rotational symmetry of the optical system to be tested;

b) imperfection in the measurement setup; and

c) inaccurate choice of the coordinate origin.

Reason c) applies, in particular, in the case of a finite object and image distance.

The asymmetry of the distortion values shall then be minimized by shifting to a new zero point.

8 Presentation of the results

The distortion values thus determined shall be indicated with reference to the improved zero point. The deviation of this improved zero point from the zero point used during the measurement shall be indicated.

The measurement results for the two chosen azimuths shall be presented as a function of the image height or the field angle.

According to the application, a presentation of the values averaged over the four halves of the diagonals may be sufficient.

The presentation may be in the form of curves or tables.

9 Test report

For the presentation of the measured results, where applicable and necessary, the following variable parameters of the measurement or calculation, fixed parameters of the boundary conditions and specifications of the optical system to be tested shall be indicated in the following order:

a) laboratory or company carrying out the test;

b) statement whether the values are measured or calculated;

c) designation of the optical system tested (e.g. name, manufacturer, serial number);

d) additional elements;

e) test procedure;

f) environmental test conditions;

g) azimuth of the optical system tested, ψ;

h) wavelength or spectral evaluation function reference quantity, λ;

i) aperture, NA;

j) f-number;

k) lateral magnification, m;

l) angular magnification, Γ;

m) alignment chosen or focusing criterion.

Annex A
(informative)

Example for a method of shifting the zero point

A.1 Description of the method

For infinite object distance and finite image distance, a zero point can be determined for optimum symmetry by a shift of

$$d = \frac{1}{n} \sum_{i=1}^{n} \frac{V_{i1} - V_{i2}}{2 \tan^2 \omega_i}$$

where

d is the shifting value;

V_{i1} and V_{i2} are the opposite absolute distortion values measured for the field angle ω_i;

n is the number of the field angles measured.

Subsequently, the distortion values shall be converted to the new zero point for optimum symmetry.

For finite object and image distance, a zero point can be determined for improved symmetry by a shift of

$$d = \frac{1}{n} \sum_{i=1}^{n} \frac{h_{i1} - h_{i2}}{2}$$

If this method does not yield sufficient symmetry, within the limits of the measurement accuracy desired, a new series of measurements shall be carried out with the improved zero point. It shall be estimated whether the determined shift, d, causes a discrepancy of the distortion values beyond the measurement accuracy aimed at. Should this be the case, a new measurement shall be carried out.

A.2 Derivation of the formula used in the case of infinite object distance and finite image distance

In order to recalculate the measured distortion values for optimum symmetry, a shift, d_i, is calculated for each pair of image points whose positions are symmetrical about the origin. A similar derivation is valid in the case where the object pupil field angle is used as an independant variable instead of the image height. See figure A.1.

The distortion values for image points P_{i1} and P_{i2} with reference to 0 are

$$V_{i1} = h'_i - a' \tan \omega_{i1} \text{ and}$$

$$V_{i2} = h'_i - a' \tan \omega_{i2} \text{ respectively.}$$

With respect to S_i these distortions are as follows:

$$V_{i1,S} = (h'_i + d_i) - a' \tan (\omega_{i1} + \delta_i)$$

$$V_{i2,S} = (h'_i - d_i) - a' \tan (\omega_{i2} - \delta_i)$$

$$\tan \delta_i = \frac{d_i}{a'}$$

Neglecting $\tan^2 \delta_i$ and using the approximation

$$\frac{1}{1 \pm x} \approx 1 \pm x$$

this leads to

$$V_{i1,S} \approx V_{i1} - d_i \tan^2 \omega_{i1}$$

$$V_{i2,S} \approx V_{i2} + d_i \tan^2 \omega_{i2}$$

and with $\omega_{i1} \approx \omega_{i2} \approx \omega_i$, this yields

$$d_i \approx \frac{V_{i1} - V_{i2}}{2 \tan^2 \omega_i} \approx \frac{\Delta V_i}{2 \tan^2 \omega_i}$$

Since the distortion values of all measured image points on one diagonal have to be related to a common reference point the arithmetic mean of all shifts d_i has to be calculated in order to get a mean shift d and a new origin for optimum symmetry S. Thus

$$d = \frac{1}{n} \sum_{i=1}^{n} \frac{V_{i1} - V_{i2}}{2 \tan^2 \omega_i}$$

where n is the number of measured image points on one half diagonal. Recalculations of V_{i1} and V_{i2} are done by inserting d instead of d_i into the equations for $V_{i1,S}$ and $V_{i2,S}$ given above.

EXAMPLE

Calculation of a'

	$i = 1$	$i = 2$	$i = 3$
h'_i	2	4	6
measured ω_i	2,253°	4,499°	6,730°
tan ω_i	0,039 34	0,078 68	0,118 00
$a'_i = \dfrac{h'_i}{\tan \omega_i}$	50,833	50,838	50,850

$$a' = \frac{a'_3 h'^2_1 h'^2_2 \left(h'^2_2 - h'^2_1\right) - a'_1 h'^2_3 h'^2_2 \left(h'^2_2 - h'^2_3\right) + a'_2 h'^2_1 h'^2_3 \left(h'^2_1 - h'^2_3\right)}{\left(h'^2_1 - h'^2_3\right)\left(h'^2_2 - h'^2_1\right)\left(h'^2_2 - h'^2_3\right)}$$

$$a' = 50{,}832$$

Calculation of V_i

	Left side				Right side			
h'_i	21	18	15	11	11	15	18	21
measured ω_i	21,995°	19,274°	16,357°	12,191°	12,204°	16,394°	19,319°	22,085°
tan ω_i	0,403 92	0,349 68	0,293 50	0,216 05	0,216 28	0,294 20	0,350 57	0,405 75
V_i	0,468	0,225	0,081	0,018	0,006	0,045	0,180	0,375

Recalculation for optimum symmetry

V_i	0,093	0,045	0,036	0,012	$d = 0{,}202$			
$\tan^2 \omega_i$	0,163	0,122	0,086	0,046 6				
$V_{i,\mathrm{S}}$	0,435	0,200	0,064	0,008 6	0,015	0,062	0,204	0,408