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INTERNATIONAL
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SURFACE VEHICLE RECOMMENDED PRACTICE

Submitted for recognition as an American National Standard

SAE J1750

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DESCRIBING AND EVALUATING THE TRUCK DRIVER'S VIEWING ENVIRONMENT

1. ***Scope and Purpose***—This SAE Recommended Practice establishes two uniform methods for describing and evaluating the truck driver's viewing environment, the Polar Plot and the Horizontal Planar Projection. The Polar Plot presents the entire available field of view in an angular format, onto which items of interest may be plotted, whereas the Horizontal Planar Projection presents the field of view at a given elevation chosen for evaluation.

These methods are based on the Three Dimensional Reference System described in SAE J182a. This document relates to the driver's exterior visibility environment and was developed for the heavy truck industry (Class B vehicles, class 6, 7, 8 vehicles) although the projection principles presented in this document can be applied to any class of motor vehicles.

This document is intended to complement SAE J1050a and provides a visual format that can describe the driver's entire viewing environment. This environment can then be analyzed to determine what the driver is capable of seeing. It should be noted that one of the most important factors affecting the driver's field of view and the ability to make valid vehicle/design comparisons is the location of the driver's eyepoint. SAE J941 defines the Eyellipse which forms the basis for eyepoints chosen as the origin for Polar Plots and Horizontal Planar Projections. Both the Horizontal Planar Projection and Polar Plot create monocular evaluations. Projections/plots of multiple eyepoints must be overlaid to create binocular or ambinocular evaluations.

Analytical methods for creating Polar Plots and Horizontal Planar Projections for direct and indirect vision (planar and spherical convex mirrors) are presented. Note that it is possible to create plots and projections for other mirror surfaces and vision devices if the equations for determining reflection points are provided. It is also possible to create Polar Plots and Horizontal Planar Projections using experimental and graphical methods. These are beyond the present scope of this document.

2. ***References***

2.1 **Applicable Publications**—The following publications form a part of the specification to the extent specified herein. Unless otherwise indicated the latest revision of SAE publications shall apply.

2.1.1 SAE PUBLICATIONS—Available from SAE, 400 Commonwealth Drive, Warrendale, PA 15096-0001.

SAE J182a—Motor Vehicle Fiducial Marks
SAE J264—Vision Glossary
SAE J941—Motor Vehicle Drivers' Eye Locations
SAE J1050a—Describing and Measuring the Driver's Field of View
SAE J1100—Motor Vehicle Dimensions
SAE J1516—Accommodation Tool Reference Point

3. Definitions

3.1 Driver's Viewing Environment—The environment surrounding the driver as described relative to the driver and his/her vision abilities.

3.2 Polar Plot—A two-dimensional plot which represents the angle of sight lines tangent to items of interest within the driver's viewing environment relative to the horizontal and vertical plane passing through the origin of the sight line.

3.3 Horizontal Planar Projection—A two-dimensional plot which represents the intersection of sight lines with a ground plane or a specified plane parallel to the ground plane. The sight lines are tangent to items of interest within the driver's viewing environment. The Horizontal Planar Projection maps only the limits of items of interest (in contrast to the Polar Plot). The effect is as if a lamp at the driver's eyes casts a shadow of a window, mirror or other item onto the plane.

3.4 Eyellipse—The contraction of the words "Eye" and "Ellipse" and is so named because of the elliptical shape of the driver's eye range. "Eyellipse" is the statistical model defined in SAE J941.

3.5 Monocular Field of View—The field of view that can be seen by one eye. (Reference SAE J1050a)

3.6 Binocular Field of View—The total field of view that can be seen by both eyes simultaneously. (Reference SAE J1050a)

3.7 Ambinocular Field of View—The total field of view that can be seen by both eyes separately. This includes the binocular field as well as the monocular field visible to the right eye but not the left eye and vice versa. (Reference SAE J1050a)

3.8 Sight Line—A line representing the driver's line of sight from an eye point or a V point to a target point or at a given angle.

3.9 Vision Reference Points—Points from which sight planes/lines may be constructed. (V point)

3.10 Vision Opening Line—What the driver actually perceives as the D.L.O. (day light opening). It is the intersection of a surface of interest and a sight line from a vision reference point, tangent to the first body component obstructing these sight lines (i.e., window moldings, seals, ceramic paint, concealed wipers, front end surface, etc.)

3.11 Polar Plot/Horizontal Planar Projection Origin Point—Selected by the plot originator. It can be the eyellipse centroid, left or right eyellipse centroid, or a vision reference point.

3.12 Item of Interest—An item which the plot originator chooses to define as part of the driver's viewing environment. It can be vehicle vision opening lines, other vehicles, field of view targets, lane markers, etc. Note that items of interest may be unique to a specific eye point. For example, an item of interest such as the top edge of the dash may disappear as the eye point moves upward. The new item of interest might be the hood.

3.13 Variable/Coordinate System Definitions—The three-dimensional vehicle coordinate system is that established in J182. The positive X axis is to the rear, the positive Y axis to the driver's right and the positive Z axis is up. A second coordinate system is used in the Polar Plot and Horizontal Planar Projection calculations. In this system the axis directions remain the same, but the origin is moved to the eye point. Table 1 is presented to assist in the definition of the variables in both systems:

TABLE 1—COORDINATE SYSTEM NOTATION

	Vehicle Coordinate System	Eye Coordinate System	Conversion
Point on Item of Interest	Tx, Ty, Tz	tx, ty, tz	tx = Tx - Ex ty = Ty - Ey tz = Tz - Ez
Point on Mirror	Mx, My, Mz	mx, my, mz	mx = Mx - Ex my = My - Ey mz = Mz - Ez
Intercept Point on Horizontal Plane	Nx, Ny	nx, ny	nx = Nx - Ex ny = Ny - Ey

3.14 Class B Vehicles—Those vehicles having an H point height (H30) between 405 and 520 mm and steering wheel diameters (W9) between 450 and 560 mm. This class of vehicles includes heavy trucks and some buses and multipurpose vehicles. (Reference SAE J1516)

3.15 Accommodation Heel Reference Point—Refer to SAE J1516

3.16 Accommodation Tool Reference Point—A point on the Accommodation Tool Reference Line at height H30 (as specified by the vehicle manufacturer). (Reference SAE J1516)

3.17 Accommodation Tool Reference Line—Refer to SAE J1516.

3.18 Seat Back Angle (L40)—Refer to SAE J1100.

***4. Polar Plot Procedure*—**Polar plots are found by computing the angles from the eye point to points within the field (items of interest).

4.1 Generation of Polar Angles for the Direct Viewing Environment

4.1.1 DEFINE THE ORIGIN—The polar plot origin (Ex, Ey, Ez) in the vehicle coordinate system may be the Eyellipse centroid, left or right Eyellipse centroid or any vision origin point defined by the user.

4.1.2 DEFINE EACH POINT TO BE GRAPHICALLY REPRESENTED—Define points on each item of interest using the eye coordinate system (tx, ty, tz). (See 3.13)

$$\begin{aligned} tx &= Tx - Ex \\ ty &= Ty - Ey \\ tz &= Tz - Ez \end{aligned} \quad \text{(Eq. 1)}$$

4.1.3 COMPUTE THE AZIMUTH AND ELEVATION ANGLES—Convert the point (–tx, ty, tz) to polar coordinates θ (azimuth) and ϕ (elevation) using the following equations in Figure 1:

NOTE—In this conversion, the negative of the X coordinate is used to provide a presentation with +θ to the right and –θ to the left.

	ty > 0	ty < 0
tx < 0	$\theta = -\arctan\left(\frac{ty}{tx}\right)$	
tx = 0	$\theta = 90$	$\theta = -90$
tx > 0	$\theta = 180 - \arctan\left(\frac{ty}{tx}\right)$	$\theta = -180 - \arctan\left(\frac{ty}{tx}\right)$

FIGURE 1—EQUATIONS

$$\phi = \arctan\frac{tz}{\sqrt{tx^2 + ty^2}} \quad \text{(Eq. 2)}$$

4.2 Generation of Polar Angles for the Indirect Viewing Environment - Planar Mirrors—The periphery of the mirror is plotted as points in the direct field using 4.1. Other target points viewed in the mirror are plotted using methods in this section.

4.2.1 DEFINE THE ORIGIN—The polar plot origin (Ex, Ey, Ez) may be the Eyellipse centroid, left or right Eyellipse centroid or any vision origin point defined by the user.

4.2.2 DETERMINE THE MIRROR LOCATION AND ANGLE—The mirror location and angle are defined by finding the equation of the surface. Select three points on the mirror surface not in a single line and preferably apart. Using these points, define the equation of the mirror plane in the vehicle coordinate system in the form:

$$A*Mx + B*My + C*Mz + D = 0 \quad \text{(Eq. 3)}$$

One method of finding the coefficients in given in Appendix A.

4.2.3 DEFINE EACH POINT TO BE GRAPHICALLY REPRESENTED—Define a point on the item of interest in the vehicle coordinate system as Tx, Ty, Tz.

4.2.4 FIND THE REFLECTION POINT OF TX, TY, TZ—Use the following equations with the coefficients from 4.2.2 to solve for the reflection point (Rx, Ry, Rz) for the item of interest (Tx, Ty, Tz).

$$\begin{aligned} Rx &= Tx + 2*A*k \\ Ry &= Ty + 2*B*k \\ Rz &= Tz + 2*C*k \end{aligned} \quad \text{(Eq. 4)}$$

where:

$$k = \frac{-D - A*Tx - B*Ty - C*Tz}{A^2 + B^2 + C^2}$$

Convert the point Rx, Ry, Rz to the eye coordinate system rx, ry, rz as follows:

$$rx = Rx - Ex \quad ry = Ry - Ey \quad rz = Rz - Ez \qquad \text{(Eq. 5)}$$

4.2.5 COMPUTE THE ANGLES—Convert the point (–rx, ry, rz) to polar coordinates θ (azimuth) and Φ (elevation) using the table and equation in 4.1.3 substituting –rx, ry, rz for –tx, ty, tz.

4.3 Generation of Polar Angles for the Indirect Viewing Environment—Spherical Convex Mirrors—The periphery of the mirror is plotted as points in the direct field using 4.1. Other target points viewed in the mirror are plotted using methods in this section.

4.3.1 DEFINE THE ORIGIN—The polar plot origin (Ex, Ey, Ez) may be the Eyellipse centroid, left or right Eyellipse centroid, or any vision origin point defined by the user.

4.3.2 DETERMINE THE MIRROR LOCATION AND ANGLE—The mirror location and angle are defined by finding the equation of the mirror surface in the vehicle coordinate system. Select four points on the mirror surface not in a single line and preferably apart. Using these points, define the equation of the mirror plane in the form:

$$A*Mx + B*My + C*Mz + D + F*(Mx^2 + My^2 + Mz^2) = 0 \qquad \text{(Eq. 6)}$$

One method of finding the coefficients in given in Appendix B.

4.3.3 DEFINE EACH POINT TO BE GRAPHICALLY REPRESENTED—Define a point on an item of interest in the vehicle coordinate system as Tx, Ty, Tz.

4.3.4 FIND THE REFLECTION POINT OF TX, TY, TZ—The point on the mirror at which the target point is seen (reflection point) is most easily found iteratively. Select a mirror point, test if the eye will see the target at that point, and if not, select another. A number of methods may be used to do this. One is presented in Appendix C.

Convert the point Rx, Ry, Rz to the eye coordinate system as rx, ry, rz. (See 4.2.4)

4.3.5 COMPUTE THE ANGLES—Proceed as in 4.2.5.

5. Horizontal Planar Projection Procedure—The Horizontal Planar Projection is meant to map the limits of windows or other items onto a horizontal plane. The effect is as if a lamp at the driver's eyes cast a shadow of the item onto the plane.

5.1 Generation of Intercepts for the Direct Viewing Environment

5.1.1 DEFINE THE EYE POINT—Define the eye point in the vehicle coordinate system as (Ex, Ey, Ez). This may be the Eyellipse centroid, left or right Eyellipse centroid or any vision origin point defined by the user.

5.1.2 DEFINE THE LOCATION OF THE HORIZONTAL PLANE AT GZ—Define the location of the horizontal plane to which the items of interest will be projected by its Z coordinate, Gz, in vehicle coordinates. This plane can be the ground or any elevation of interest.

5.1.3 DEFINE THE POINT TO BE PROJECTED—Define the point to be projected in the vehicle coordinate system as Tx, Ty, Tz. For a point to be projected it must lie between the eye point and the plane.

$$Gz < Tz < Ez \text{ or } Gz > Tz > Ez \qquad \text{(Eq. 7)}$$

5.1.4 CALCULATE THE INTERCEPT POINT—Calculate the Ratio R.

$$R = \frac{Gz - Ez}{Tz - Ez} \qquad \text{(Eq. 8)}$$

If R is less than or equal to zero, then no intercept exists. If R is greater than zero then calculate the intercept points Nx and Ny. These points are in the vehicle coordinate system.

$$\begin{array}{ll} \text{IF } R \le 0 & \text{NO INTERCEPTS} \\ \text{IF } R > 0 & Nx = Ex + R*(Tx - Ex) \\ & Ny = Ey + R*(Ty - Ey) \end{array} \qquad \text{(Eq. 9)}$$

If the eye point is to form the origin of the Horizontal Planar Projection, convert the points Nx, Ny to the eye coordinate system as nx, ny as follows:

$$\begin{array}{l} nx = Nx - Ex \\ ny = Ny - Ey \end{array} \qquad \text{(Eq. 10)}$$

5.1.5 CONVERT TO DESIRED COORDINATE SYSTEM—If desired, the intercept points can be converted to another coordinate system with origin at Ox, Oy, Oz. This origin may be at the Eyellipse centroid, a vision point or any other suitable location.

$$\begin{array}{l} nx = Nx - Ox \\ ny = Ny - Oy \\ nz = Nz - Oz \end{array} \qquad \text{(Eq. 11)}$$

5.2 Generation of Intercepts for the Indirect Viewing Environment

5.2.1 DEFINE THE ORIGIN—The horizontal planar projection origin (Ex, Ey, Ez) may be the Eyellipse centroid, left or right Eyellipse centroid or any vision origin point defined by the user.

5.2.2 DEFINE THE LOCATION OF THE HORIZONTAL PLANE AT GZ—Define the location of the horizontal plane to which the items of interest will be projected by its Z coordinate, Gz, in vehicle coordinates. This plane can be ground or any elevation of interest.

5.2.3 DETERMINE THE MIRROR LOCATION AND ANGLE—The mirror location and angle are defined by finding the equation of the surface.

5.2.3.1 *Planar Mirror*—Select three points on the mirror surface not in a single line and preferably apart. Using these points, define the equation of the mirror plane in the vehicle coordinate system in the form:

$$A*Mx + B*My + C*Mz + D = 0 \qquad \text{(Eq. 12)}$$

One method of finding the coefficients is given in Appendix A.

5.2.3.2 *Spherical Convex Mirror*—Select four points on the mirror surface not in a single line and preferably apart. Using these points, define the equation of the mirror plane in the vehicle coordinate system in the form:

$$A*Mx + B*My + C*Mz + D + F*(Mx^2 + My^2 + Mz^2) = 0 \qquad \text{(Eq. 13)}$$

One method of finding the coefficients in given in Appendix B.

5.2.4 DEFINE THE POINT TO BE PROJECTED—Define the point to be projected as Tx, Ty, Tz in vehicle coordinates.

5.2.5 CALCULATE THE INTERCEPT POINT—Using the coefficients of the equation of the mirror surface, calculate Qx, Qy and Qz. Note that for a planar mirror, F = 0.

$$Qx = Ex - 2*(2*F*Mx + A)*K$$
$$Qy = Ey - 2*(2*F*My + B)*K$$
$$Qz = Ez - 2*(2*F*Mz + C)*K \quad \text{(Eq. 14)}$$

$$K = \frac{(2*F*Mx + A)(Ex - Mx) + (2*F*My + B)(Ey - My) + (2*F*Mz + C)(Ez - Mz)}{A^2 + B^2 + C^2 - 4*F*D} \quad \text{(Eq. 15)}$$

5.2.5.1 Calculate the Ratio R

$$R = \frac{Gz - Mz}{Mz - Qz} \quad \text{(Eq. 16)}$$

If R is less than or equal to zero, then no intercept exists. If R is greater than zero, calculate the Intercept points Nx and Ny. These points are in the vehicle coordinate system.

$$\text{IF } R \le 0 \quad \text{NO INTERCEPTS} \quad \text{(Eq. 17)}$$
$$\text{IF } R > 0 \quad Nx = Mx + R*(Mx - Qx)$$
$$Ny = My + R*(My - Qy)$$

If the eye point is to form the origin of the Horizontal Planar Projection, convert the points Nx, Ny to the eye coordinate system as nx, ny as follows:

$$nx = Nx - Ex \quad \text{(Eq. 18)}$$
$$ny = Ny - Ey$$

5.2.6 CONVERT TO DESIRED COORDINATE SYSTEM—If desired, the intercept points can be converted to another coordinate system with origin at Ox, Oy, Oz. This origin may be at the Eyellipse centroid, a vision point or any other suitable location.

$$nx = Nx - Ox \quad \text{(Eq. 19)}$$
$$ny = Ny - Oy$$
$$nz = Nz - Oz$$

6. Plot Format

6.1 Key Input Parameters—All Polar Plots and Horizontal Planar Projections are highly dependent upon a few key vehicle and driver parameters which have the potential to greatly vary the characteristics of the plots and their interpretation. Care must be taken to define these parameters in order to ensure plot repeatability. These Key Input Parameters are as follows:

6.1.1 ACCOMMODATION HEEL REFERENCE POINT HEIGHT FROM GROUND—Due to the wide variety of chassis configurations available even from a given manufacturer, the Accommodation Heel Reference Point height from ground must be specified. Care must be taken in evaluating competitors and vehicles within one's own product line to avoid misleading results. Vehicles should be specified as equivalently as possible.

6.1.2 CHASSIS ATTITUDE TO GROUND—Similar to the Accommodation Heel Reference Point height the chassis attitude to ground must be specified and declared as horizontal about the front axle, unloaded, fully loaded, etc.

6.1.3 ACCOMMODATION TOOL REFERENCE POINT—The location of the Accommodation Tool Reference Point must be specified for each plot to avoid confusion as to how the eyellipse was located.

6.1.4 SEAT BACK ANGLE (L40)—The Seat Back Angle used must be specified for each plot to avoid confusion as to how the eyellipse was located.

6.1.5 ORIGIN POINT—The plot origin point (eye point) must be labeled so it can be examined relative to the SAE eyellipse position.

6.1.6 HORIZONTAL INTERCEPT PLANE—The height of the plane chosen for evaluation in a Horizontal Planar Projection relative to ground must be specified for each plot.

6.1.7 MIRROR SIZE, POSITION AND ATTITUDE—The size, position and attitude of the plane and spherical convex mirrors used in the plots must be specified. The radius of curvature for each spherical convex mirror must also be indicated. In an indirect polar plot the direct plot of the mirror periphery should be shown to indicate what can/cannot be seen by the driver in the mirror. The plane mirror attitude should be such that it picks up the edge of the trailer since Federal Motor Vehicle Safety Standard #111 specifies that the mirror must provide a clear view down the side of the trailer. The spherical convex mirror attitude should represent how the mirror is used in the field.

6.2 Plot Format and Example - Polar Plot

6.2.1 PLOT AXES—Plot the azimuth angle (θ) on the horizontal axis with $+\theta$ to the right and the elevation angle (ϕ) on the vertical axis with $+\phi$ up. (The chosen eyepoint forms the origin of the plot.)

6.2.2 PLOT LABELS—The particular vehicle shown must be indicated. The Key Input Parameters in 6.1 must be completely defined in an attached Data Sheet for the plot. The Data Sheet must be included/published with the plot itself in order for the Polar Plot to meet this document. Labeling the chosen items of interest is also recommended.

6.2.3 EXAMPLE PLOT—See Figures 2 and 3 for an example of a Polar Plot and associated Data Sheet that complies with this Recommended Practice.

6.3 Plot Format and Example - Horizontal Planar Projection

6.3.1 PLOT AXES—Plot the calculated intercept points along X, Y axes (orientation per SAE J182) in the eye, vehicle, or other chosen coordinate system.

6.3.2 PLOT LABELS—The particular vehicle shown must be indicated. The Key Input Parameters in 6.1 must be completely defined in an attached Data Sheet for the plot. The Data Sheet must be included/published with the plot itself in order for the Horizontal Planar Projection to meet this Recommended Practice. Labeling the chosen items of interest is also recommended.

6.3.3 ADDITIONAL INFORMATION—Additional information, such as an outline of the tested vehicle, an outline of another vehicle, lane markers or the like may be included on the plot. If included, they should be clearly labeled.

6.3.4 DIFFERENTIATING AREAS—Areas on the plot should be clearly differentiated as to those areas visible through window or mirrors and those that are not visible. For example, those areas visible may be white and those not visible may be cross-hatched.

6.3.5 EXAMPLE PLOT—See Figures 4 and 5 for an example of a Horizontal Planar Projection and associated Data Sheet that complies with this document.

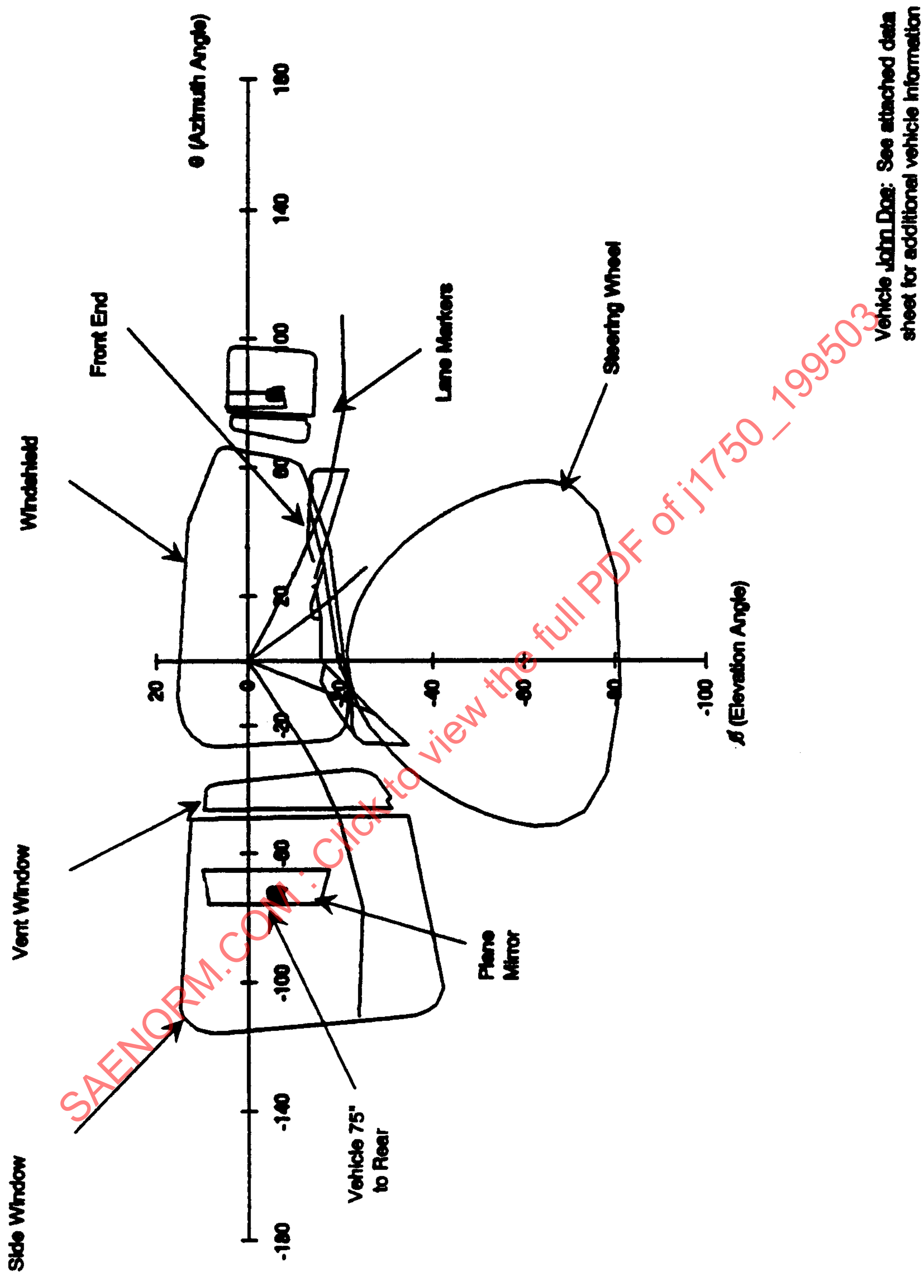

FIGURE 2—POLAR PLOT EXAMPLE WHICH MEETS SAE J1750

1. ACCOMMODATION HEEL REFERENCE POINT HEIGHT FROM GROUND ____________

2. CHASSIS ATTITUDE TO GROUND ____________
(Angle to Ground Plane)

Chassis Load Assumption: ____________

3. ACCOMMODATION TOOL REFERENCE POINT (X, Y, Z) ____________

4. SEAT BACK ANGLE (L40) ____________

5. POLAR PLOT ORIGIN POINT (X, Y, Z) ____________

6. MIRROR REFLECTIVE SURFACE SIZE

Planar Mirror (Height/Width) ____________

Spherical Convex Mirror (Diameter/Radius of Curvature) ____________

7. MIRROR LOCATION (VEHICLE COORDINATES)

Planar Mirror (Geometric Center) ____________

Spherical Convex Mirror (Geometric Center through plane of mirror periphery) ____________

8. MIRROR ATTITUDE ____________
(Longitudinal Plan View Angle)

* Vehicle Dimensional Coordinate System per SAE J182a.

FIGURE 3—DATA INPUT SHEET-POLAR PLOT*

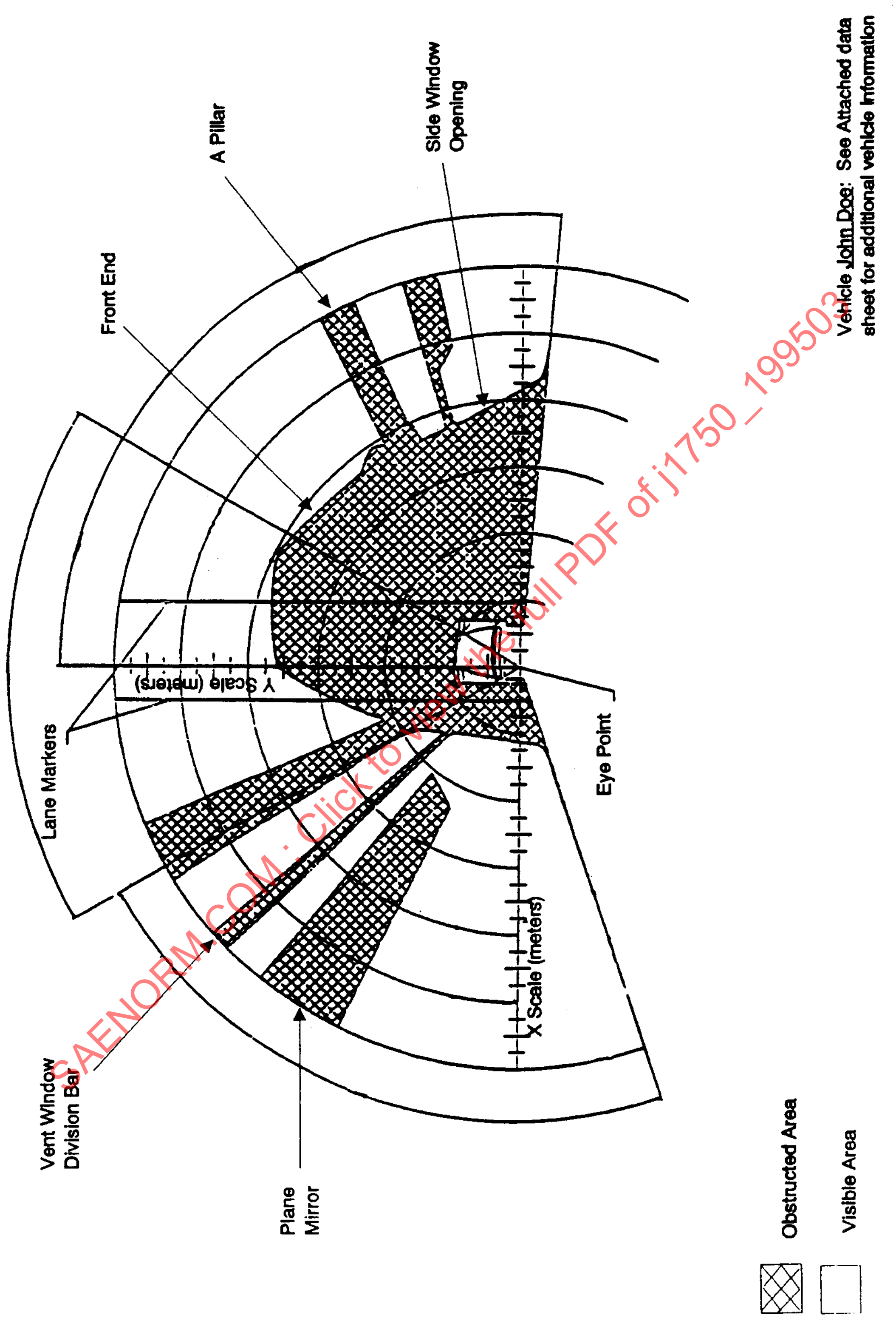

FIGURE 4—HORIZONTAL PLANAR PROJECTION EXAMPLE WHICH MEETS SAE J1750

1. ACCOMMODATION HEEL REFERENCE POINT HEIGHT FROM GROUND ____________

2. CHASSIS ATTITUDE TO GROUND ____________
   (Angle to Ground Plane)

   Chassis Load Assumption: ____________

3. ACCOMMODATION TOOL REFERENCE POINT (X, Y, Z) ____________

4. SEAT BACK ANGLE (L40) ____________

5. POLAR PLOT ORIGIN POINT (X, Y, Z) ____________

6. MIRROR REFLECTIVE SURFACE SIZE

   Planar Mirror (Height/Width) ____________

   Spherical Convex Mirror (Diameter/Radius of Curvature) ____________

7. MIRROR LOCATION (VEHICLE COORDINATES)

   Planar Mirror (Geometric Center) ____________

   Spherical Convex Mirror (Geometric Center through plane of mirror periphery) ____________

8. MIRROR ATTITUDE ____________
   (Longitudinal Plan View Angle)

* Vehicle Dimensional Coordinate System per SAE J182a.

FIGURE 5—DATA INPUT SHEET—HORIZONTAL PLANAR PROJECTION*

PREPARED BY THE SAE TRUCK AND BUS VISIBILITY SUBCOMMITTEE OF THE
SAE TRUCK AND BUS CAB AND OCCUPANT ENVIRONMENT COMMITTEE

APPENDIX A

METHOD FOR DETERMINING COEFFICIENTS OF EQUATION OF PLANAR MIRROR SURFACE

A.1 Select three points on the mirror surface not in a single line and preferably apart and define them as:

$$\begin{matrix} Mx_1, My_1, Mz_1 \\ Mx_2, My_2, Mz_2 \\ Mx_3, My_3, Mz_3 \end{matrix} \qquad \text{(Eq. A1)}$$

NOTE—Points are defined in vehicle coordinates.

A.2 The equation of the mirror plane per 4.2.2 is:

$$A*Mx + B*My + C*Mz + D = 0 \qquad \text{(Eq. A2)}$$

and can be expressed in matrix form as:

$$\begin{vmatrix} A & B & C & D \\ Mx_1 & My_1 & Mz_1 & 1 \\ Mx_2 & My_2 & Mz_2 & 1 \\ Mx_3 & My_3 & Mz_3 & 1 \end{vmatrix} = 0 \qquad \text{(Eq. A3)}$$

A.3 Therefore, the following matrices solve for the coefficients A, B, C, D:

$$A = \begin{vmatrix} My_1 & Mz_1 & 1 \\ My_2 & Mz_2 & 1 \\ My_2 & Mz_3 & 1 \end{vmatrix} \qquad B = \begin{vmatrix} My_1 & Mz_1 & 1 \\ My_2 & Mz_2 & 1 \\ My_2 & Mz_3 & 1 \end{vmatrix} \qquad \text{(Eq. A4)}$$

$$C = \begin{vmatrix} My_1 & Mz_1 & 1 \\ My_2 & Mz_2 & 1 \\ My_2 & Mz_3 & 1 \end{vmatrix} \qquad D = \begin{vmatrix} My_1 & Mz_1 & Mz_1 \\ My_2 & Mz_2 & Mz_2 \\ My_2 & Mz_3 & Mz_3 \end{vmatrix}$$

APPENDIX B

METHOD FOR DETERMINING COEFFICIENTS OF EQUATION OF SPHERICAL CONVEX MIRROR SURFACE

B.1 Select four points on the mirror surface not in a single line and preferably apart and define them as:

$$\begin{matrix} Mx_1, My_1, Mz_1 \\ Mx_2, My_2, Mz_2 \\ Mx_3, My_3, Mz_3 \\ Mx_4, My_4, Mz_4 \end{matrix} \qquad \text{(Eq. B1)}$$

NOTE—Points are defined in vehicle coordinates.

B.2 The equation of the mirror surface per 4.3.2 is:

$$A*Mx + B*My + C*Mz + D + F*(Mx^2 + My^2 + Mz^2) = 0 \qquad \text{(Eq. B2)}$$

$$(\text{where } F \neq 0)$$

and can be expressed in matrix form as:

$$\begin{vmatrix} A & B & C & D & F \\ Mx_1 & My_1 & Mz_1 & 1 & (Mx_1^2 + My_1^2 + Mz_1^2) \\ Mx_2 & My_2 & Mz_2 & 1 & (Mx_2^2 + My_2^2 + Mz_2^2) \\ Mx_3 & My_3 & Mz_3 & 1 & (Mx_3^2 + My_3^2 + Mz_3^2) \\ Mx_4 & My_4 & Mz_4 & 1 & (Mx_4^2 + My_4^2 + Mz_4^2) \end{vmatrix} \qquad \text{(Eq. B3)}$$